

RADIATION SHIELDING

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Volume 2

RADIATION SHIELDING

*The International Series of Monographs
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RADIATION SHIELDING

by

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ATOMIC ENERGY RESEARCH ESTABLISHMENT
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CONTENTS

	PAGE
Authors' Preface	ix
1. THE BIOLOGICAL HAZARDS OF NUCLEAR RADIATIONS	
1.1. Introduction	1
1.2. Ionization as a measure of biological damage	3
1.3. Relative biological efficiency	5
1.4. Mechanism of damage by neutrons	6
1.5. Recommended maximum permissible levels for external whole-body irradiation	7
1.6. Recommended maximum permissible levels of radioactive contamination of drinking water and air	12
Appendix: Definitions of obsolete units	14
References	15
2. ATTENUATION OF GAMMA RAYS AND HIGH ENERGY ELECTRONS	
2.1. Introduction	17
2.2. Cross-section and related quantities	19
INTERACTIONS OF GAMMA RAYS WITH MATTER (ELEMENTARY PROCESSES)	
2.3. The photoelectric effect	21
2.4. Pair production	22
2.5. Scattering of gamma radiation	24
INTERACTIONS OF GAMMA RAYS WITH MATTER IN BULK	
2.6. The attenuation of narrow beams of gamma rays	32
2.7. The attenuation of broad beams of radiation; build-up factors	40
2.8. Back-scattering and air scattering of gamma radiation	56
SOURCES OF GAMMA RADIATION	
2.9. Gamma radiation produced in nuclear reactors	61
2.10. Gamma rays from fission products	62
2.11. Shielding of electrons of energies less than a few million electron-volts	71
2.12. Gamma radiation (bremsstrahlung) produced during the stopping of high-energy electrons	81
2.13. Measurements of high-energy gamma ray intensity	90
References	95
3. SHIELDING AGAINST NEUTRONS: NEUTRON PHYSICS	
3.1. General principles	99
3.2. The neutron	103
3.3. The nucleus	107

	PAGE
TYPES OF REACTION BETWEEN NEUTRONS AND NUCLEI	
3.4. Scattering	116
3.5. Radiative capture	130
3.6. Reactions involving the emission of charged particles	137
3.7. The $(n, 2n)$ reaction	141
3.8. Artificial radioactivity resulting from neutron reactions	141
SOURCES OF NEUTRONS	
3.9. Fission neutrons	146
3.10. (α, n) sources	149
3.11. (γ, n) sources	150
References	156
4. NEUTRON ATTENUATION IN THICK SHIELDS	
4.1. Introduction	158
4.2. Diffusion theory	159
4.3. The solution of the diffusion equation in plane geometry	162
4.4. The solution of the diffusion equation in spherical geometry	166
4.5. The physical meaning of diffusion length	168
4.6. The slowing-down of neutrons	169
4.7. Age theory	171
4.8. The attenuation of the fast component	175
4.9. Build-up of neutrons of lower energies	186
4.10. The moment method	189
MISCELLANEOUS TOPICS	
4.11. The escape of neutrons from a surface	192
4.12. Effect of ducts and voids in shields	197
References	210
5. USEFUL MATHEMATICAL FORMULAE	
5.1. Exponential integrals and secant integral functions	212
5.2. Solutions for standard cases involving plane and slab sources	218
5.3. Line sources and cases which can be considered as superpositions of line sources	225
5.4. Cylindrical sources	227
5.5. Spherical sources	228
References	229
6. THE SHIELDING OF REACTORS AND RADIOACTIVE MATERIALS	
6.1. Introduction	230
HEAT GENERATION IN SHIELDS	
6.2. Heating of the shield by gamma rays from core and reflector	235
6.3. Heating by neutrons	245
6.4. The determination of the temperature distribution in the shield	248
6.5. Thermal shields	251

BULK SHIELDS

6.6. Concrete shields	258
6.7. Minimum cost bulk shields	278
6.8. Metal-hydrogen bulk shields	284

MISCELLANEOUS TOPICS

6.9. Activation induced by irradiation in a neutron flux	296
6.10. Activity induced in cooling circuits	309
6.11. Beam traps	314
6.12. Shielding windows	318
6.13. Shielding required for radioactive materials during transport by normal public services	324
References	328

APPENDICES

I. Data useful in shielding calculations	331
II. Thermal neutron scattering and absorption cross-sections	333
III. Thermal neutron activation cross-sections	336
INDEX	337

AUTHORS' PREFACE

FOR nearly ten years after the construction of the first reactor, shielding remained the Cinderella of the nuclear technologies. This was partly because the processes involved were imperfectly understood; partly because the attention being paid to the reactor core left little effort that could be applied elsewhere; but most of all because there were simple empirical methods of designing a shield which would at least work and be safe, even though it might be unnecessarily heavy and expensive. However, in the last few years two new factors have emerged. At sea the first mobile reactors are already operating, while on land nuclear power has reached the stage of commercial exploitation. How much a shield weighs, and what it costs, have become matters of some importance. These new developments have provided the incentive that was previously lacking, so that today shielding has begun to pass out of the purely empirical phase, and the basis for a theoretical understanding has been created.

Because of the still incomplete state of the subject, and also because of our natural inclination as physicists, we have chosen to write a textbook, rather than to compile a ready-reckoner. Our aim has been to give the reader the "feel" of the subject. With this in mind we have laid great stress on physical principles, and on how these principles influence the engineering design of the shield; we have emphasized the importance of minimizing costs; we have introduced the reader to some of the "fringe topics" which he is bound to meet, sooner or later, in the course of his work; and we have tried to choose our references so that he will be guided through the confusing jungle of published reports. To keep the book to a reasonable length the discussion has been confined to reactors, betatrons, synchrotrons, and linear accelerators, and the interesting and somewhat different problems encountered in the shielding of very high-energy accelerating machines have had to be omitted.

This book could not have been written without the generous assistance of many of our colleagues in the United Kingdom Atomic Energy Authority. We particularly wish to thank H. J. Dunster, W. S. Eastwood, R. H. McK. Hyder, J. S. Story, and J. B. Sykes for their careful and critical reading of the manuscript; and all those in America and Great Britain who so kindly sent us results in advance of publication.

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CHAPTER ONE

THE BIOLOGICAL HAZARDS OF NUCLEAR RADIATIONS

1.1. INTRODUCTION

THE history of biological damage by ionizing radiations is almost as old as the history of X-rays. The first case was reported in July, 1896, within a few months of their discovery by RÖNTGEN, and two years before radium had been separated in a pure state by the Curies. Attempts were soon made to set up safety rules for the newly-discovered radiations, though there were serious difficulties in drawing conclusions from the known cases of damage, owing to the uncontrolled conditions of much of the early work. Probably the first of these attempts was by ROLLINS,⁽¹⁾ who suggested that "if a photographic plate is not fogged in seven minutes, the radiation is not of harmful intensity." This proposal (which incidentally hints at the difficulties of measuring the intensity of X-radiation at that time) is now thought to be insufficiently stringent by a factor of about one hundred. Cases of radiation damage continued to occur, and, by the first decade of this century, the accumulated evidence began to show that the damage was not confined to the skin, but affected deep-seated organs such as the bone-marrow; while the frequent occurrence among radiation workers of malignant growths showed that the radiation was a powerful carcinogenic agent.

The greatly increased hazard to the population arising from the expansion of the radium dial-painting industry during the 1914-1918 war led RUSS⁽²⁾ to propose an organized system of protection. Unfortunately, very little was done to put his suggestions into practice, and as a result a large number of cases of aplastic anaemia were reported in the years immediately following the armistice. In the United States the number of cases of radium poisoning among dial painters became so large that public opinion was profoundly disturbed. Most of the cases arose from the practice of pointing the brushes with the lips—a method which resulted in the ingestion and retention of radium by the body, which was thereafter subjected to a continuous internal bombardment by nuclear radiation.

The British X-ray and Radium Protection Committee was formed in 1921, with the purpose of establishing adequate standards of protection for all persons exposed to radiation hazards; it published its first recommendations in July of the same year. An American committee formed by the American Röntgen Ray Society in 1920 published a somewhat similar set of recommendations in 1922. From then on the apathy towards the problems of protection disappeared.^(3g) The First International Radiological Congress was held in London in 1925, and set up two commissions to report on units and protection. As a result of their recommendations, the second congress (Stockholm, 1928) adopted the first internationally recognized unit of dosage—the röntgen. In

the remaining eleven years before the outbreak of the Second World War a mass of evidence regarding permissible exposures was accumulated;⁽³⁻⁸⁾ subsequent experience of X- and gamma radiations has so far led only to minor modifications of the 1939 standards.

The development of chain-reacting piles in 1942, and the rapid growth of atomic energy industries in many countries, have increased enormously the number of persons exposed to nuclear radiations. Just as remarkable is the increase in the quantity of radioactive material available. In 1939 about 1000 grams of separated radium represented the total resources of the entire world. This quantity is completely dwarfed by the output of radioactive materials from a power-producing nuclear reactor: a reactor produces about one gram of radioactive fission products per megawatt-day of operation; moreover, because of their relatively short half-lives, these fission products are immensely more radioactive, weight for weight, than radium. If a country the size of Great Britain relied solely on nuclear power for its electricity production (which totalled 6×10^{10} kilowatt hours in 1953), about 10 *tons* of fission products would have to be disposed of without hazard to the population every year.

Fortunately, the magnitude of the task has itself produced the climate of opinion necessary for the efficient operation of protective measures. Apart from a few isolated accidents, caused by disregard of normal safety rules, the entire development of atomic energy has been carried through without injury to the staff. This excellent record of safety is due in no small measure to the emergence of a new profession, that of "health physicist" (a term which appears to have originated in Los Alamos in 1942). According to PARKER: "Health physics is a borderline subject surrounded by industrial medicine, radiobiology, industrial safety, public health, physics, chemistry, and engineering... In general it is concerned with the physics and biophysics involved in the interaction of radiation with the human body, with special emphasis on the protection of radiation workers against the potential hazards of their occupation."

The dangers from which the worker with radioactive materials has to be protected are of two kinds: external irradiation, and the more insidious danger arising from eating contaminated food, or breathing contaminated air. The first (which is the main concern of this book) is the preoccupation of the physicist and design engineer, who are jointly responsible for seeing that an adequate shield is constructed round any reactor or accelerating machine. However, even when the efficiency of the shield has been demonstrated in practice, one has not necessarily finished with the problem: holes through shields have removable plugs, and if they are taken out intense beams of radiation can escape; some shields are built so that they can be partly dismantled; and sometimes one is forced to carry out maintenance work on highly active equipment. Nevertheless, on the whole it is not difficult to guard against overexposure to external radiation, especially if there is some obvious alteration in the shielding which puts one on one's guard.

There is not necessarily any warning of an ingestion or breathing hazard. This is a continuing danger, demanding good initial design of chemical plants, reactor fuel elements, isotope containers, and "hot" laboratories; careful and correct manipulation of apparatus, and scrupulous avoidance of smoking,

eating, or drinking while in an active laboratory; regular monitoring of equipment, and in some cases medical surveillance of the individual.⁽⁹⁾ A description of the techniques necessary to overcome this second type of hazard lies outside the scope of this book; but since it is essential that everyone who works with radiation should be aware of the hazards of eating, drinking, or breathing radioactive material, a table of permissible levels of contamination is given in the last section of this chapter.

The calculation of the thickness of the shield necessary to give adequate protection against an external source of radiation requires a knowledge of three factors: the types and intensities of radiation emitted by the source, the rate of attenuation of the radiations by the shielding material (including the effect of geometry or the production of additional secondary radiation during the attenuation process), and the intensity that is acceptable at the outer surface of the shield. The first two of these three factors are matters of physics which will be dealt with later in the book. The third is intimately connected with the "permissible weekly dose of radiation," which will now be discussed.

1.2. IONIZATION AS A MEASURE OF BIOLOGICAL DAMAGE

When nuclear radiations (such as gamma rays, fast electrons and beta particles, protons, alpha particles, and neutrons) pass through matter, they produce ionization. This may happen either directly, as in the stopping of beta particles, or indirectly through the formation of recoil electrons and heavy ions. Some of the processes involved are described in detail in Section 1.4 and Chapters 2 and 3; for the moment, however, we need only observe that recoiling electrons or heavy ions are able to cause chemical changes in organic materials, which, if the irradiation is sufficiently intense and prolonged, can completely modify the properties of the material. For instance, polymethyl methacrylate ("perspex" or "plexiglass") placed in a pile neutron flux of 10^{12} neutrons $\text{cm}^{-2} \text{sec}^{-1}$, takes on a pale yellow colour after one day's irradiation; if the irradiation is continued, bubbles of gas begin to form in the material; and finally, after about a year, the material disintegrates into a powder. Living tissue is affected by quantities of radiation far smaller than those needed to produce detectable changes in inanimate matter: the same flux of neutrons would cause fatal injuries to a small mammal in no more than a few hundredths of a second.

The Unit of X- and Gamma Radiation—the Röntgen

As might be expected, it is found that the extent of radiation-induced biological damage depends on the energy liberated in the tissue in the form of ionization. Some types of damage increase more or less linearly with the quantity of ionization, suggesting that the process is of a "target" or "hit or miss" type, and that the damage, once caused, is permanent; for other effects the body is provided with a repair mechanism, which, to a considerable extent, can prevent the appearance of any symptoms until a certain threshold dose-rate is reached. The damaging effect on the organism as a whole increases roughly linearly with increasing ionization. For this reason, the amount of X- or gamma radiation is usually measured in terms of a unit quantity which will liberate a

standard amount of ionization in a standard substance, air. Since equal masses of air and tissue absorb X- and gamma rays with about equal effectiveness (Fig. 2.6.6), a given amount of radiation, measured in this way, corresponds roughly to a particular amount of damage, irrespective of the quantum energy of the radiation. Thus the one unit can be given a dual usage, and can be used both as a measure of dosage and of quantity of radiation.

The unit—the röntgen—is defined as the “quantity of X- or gamma radiation such that the associated corpuscular emission per 0.001293 gram of air produces, in air, ions carrying one electrostatic unit of quantity of electricity of either sign” (International Recommendations on Radiological Units, 1953⁽¹⁰⁾). The mass referred to is that of one cubic centimetre of standard air. The phrase “associated corpuscular emission” includes the fast photo- and Compton electrons produced by the interaction of the incident radiation with the measuring substance (air) and, in addition, all the secondary electrons (or delta rays) liberated in the slowing down of these fast particles by the air. Since the energy lost by a fast electron in liberating one ion pair in air is about 32.5 eV,^{(11, 12)*} and since 1 e.s.u. of charge is equivalent to 2.083×10^9 singly charged ions, it follows that one röntgen corresponds to the liberation of 5.24×10^{13} eV, or 83.8 ergs per gram of air. The energy necessary to liberate one ion pair is almost independent of the energy of the ionizing photo- or Compton electron, and consequently this numerical relationship holds over a wide range of X and gamma quantum energies.

Shielding calculations usually yield results expressed in photons per cm² per second, rather than in the form of ergs per gram of air per second; it is therefore convenient to know the flux† of photons corresponding to a dose of one röntgen per unit time.

Let ϕ be the photon flux (photons cm⁻² sec⁻¹), E MeV the quantum energy of the radiation, ΔE the energy liberated per second in one gram of air, and χ_a the mass energy absorption coefficient (in cm² per gram of air) given in Table 2.5.3. Then $\Delta E = \phi E \chi_a$ MeV g⁻¹ sec⁻¹. Since 1 MeV is equal to 1.602×10^{-6} ergs, the dose rate D is given by the equation

$$D = \phi E \chi_a \left[\frac{1.6 \times 10^{-6}}{83.8} \right] \text{ röntgens sec}^{-1}. \quad (1.2.1)$$

Fig. 1.5.1, which is derived from this equation using WHITE's data (Table 2.5.3), shows the flux of photons giving the normal maximum permissible weekly dose, for occupational exposure, of 0.3 röntgen in a period of 40 working hours, as a function of the energy of the radiation.^(14, 15)

For reasons discussed in Section 2.13, a number of difficulties of a practical nature arise when one attempts to use the röntgen to measure quantities of gamma radiation of quantum energy greater than about 3 MeV. For higher energies it is preferable to quote the actual fluxes in photons cm⁻² sec⁻¹ or to express gamma ray intensities in units such as watts cm⁻². At the same time it

* Although this figure is usually quoted, and is conventionally used in dosimetry calculations, a more recent determination⁽¹³⁾ gives a value of 35.0 ± 0.6 eV.

† Flux: See definition, page 20.

is necessary to introduce a new unit of biological dose; such a unit would in any case be necessary, since the röntgen, which was originally defined before the discovery of the neutron, is specifically limited to X- and gamma radiation.

The Unit of Absorbed Dose: the Rad

The international unit of absorbed dose is the rad, equal to 100 ergs per gram; this unit was first adopted by the Seventh International Congress of Radiology, Copenhagen, 1953.⁽¹⁰⁾ The *absorbed dose* of any ionizing radiation is defined as the amount of energy imparted to matter by ionizing particles per unit mass of irradiated material at the place of interest. The *integral absorbed dose*, the integrated value of the energy absorbed throughout a given region of interest, is expressed in gram rad (1 gram rad = 100 ergs). Note that the röntgen was specifically retained by the 1953 Congress as the unit for X- and gamma radiation, and that in tissue the absorbed dose per röntgen is very nearly equal to one rad (actually 0.93 rad).

Prior to the adoption of the rad a number of other units were in general use. Since they are liable to cause confusion when some of the original papers are being studied, their definitions are given in the appendix to this chapter.

1.3. RELATIVE BIOLOGICAL EFFICIENCY

Besides depending on the quantity of ionization, the amount of radiation damage is also found to be affected by the density of ionization. Particles such as slow protons and naturally occurring alpha particles, which have a high specific ionization (measured in ion pairs per unit length of track), are more effective than the more lightly ionizing electrons and gamma rays in promoting changes which depend on the liberation of many ions in a small volume simultaneously. For instance, cataract of the eye is much more readily induced by the dense column of ionization left by a proton than by an equal quantity of the more diffuse ionization resulting from beta or gamma irradiation. The more heavily ionizing radiations are said to have a higher *relative biological efficiency* (r.b.e.) for this kind of damage. The damaging effect on the entire organism is also found to increase with increasing ion density.

The relative biological efficiency of a given type of radiation in causing a given effect is defined as the following ratio:

$$\text{r.b.e.} = \frac{\text{Absorbed dose (ergs per gram) required to produce a given effect under gamma ray irradiation}}{\text{Absorbed dose (ergs per gram) required to give the same effect when the irradiation is by the particle being studied}}$$

The most reliable information on protection is based on the occupational exposure of radiologists to ordinary X-rays of up to 250 keV energy, and it is therefore customary to base the scale of relative biological efficiencies on the effect of radiations of this type which, in water, liberate ~100 ion pairs per micron thickness. However, since it is found in practice that the biological efficiency of a dose delivered to tissue by X- or gamma rays does not depend to any

marked extent on quantum energy, the r.b.e. of all energies can be taken as equal to unity. It follows from the mechanism by which energy is transferred to the absorbing medium that the same value must also apply to positrons and electrons of practically all energies. The much higher mass of other ionizing particles causes their trajectory to be more or less straight; unlike electrons, which are easily scattered, they produce a compact column of ionization, with a mean linear ion density which is proportional to the square of the charge, and which decreases rapidly with increasing velocity v (roughly as $1/v^2$), until $v \approx 0.96$ times the velocity of light, when a roughly constant value, equal for all particles of equal charge, is attained. Table 1.3.1. shows how the r.b.e. of heavy particles depends on their specific ionization.

Table 1.3.1. Effect of Specific Ionization on r.b.e. of Heavy Ionizing Particles⁽¹⁸⁾

Average specific ionization (ion pairs per micron of water)	Average linear energy transfer to water (keV per micron)	r.b.e.
< 100	< 3.5	1
100-200	3.5-7.0	1-2
200-650	7.0-23	2-5
650-1500	23-53	5-10
1500-6000	53-175	10-20

For practical purposes, an r.b.e. of 10 is applicable to protons up to 10 MeV and (for reasons explained in the following section) to fast neutrons of up to 10 MeV; the same value also applies to internal irradiation by naturally occurring alpha particles. A value of 20 is assumed for irradiation by heavy nuclei recoiling after being struck by a fast neutron.

In order to take the r.b.e. into account, a unit of dosage known as the rem (röntgen-equivalent-man) is in common use. As currently employed, it means the product of the dose in rads and the r.b.e. Since it is extremely difficult to measure the relative biological efficiency accurately, this method of expressing the dose is at best only semi-quantitative, and has not been officially recognized by the International Commission on Radiological Units. It has advantages, however, when adding doses caused by the different components of mixed radiations.

1.4. MECHANISM OF DAMAGE BY NEUTRONS

The permissible fluxes of neutrons, quoted in the following section, are calculated values, inferred from the manner in which neutrons interact with the nuclei which make up human tissue. Being themselves uncharged, neutrons cannot cause ionization directly, but must first liberate a charged particle in a

nuclear reaction. The ionization produced when *thermal* neutrons (q.v.) fall on tissue arises principally from the two reactions $H(n, \gamma)D$ and $N^{14}(n, p)C^{14}$. In the reaction with hydrogen a 2.2 MeV gamma ray is emitted, and this will in turn give rise to Compton recoil electrons; while the N^{14} isotope gives a proton of energy 0.6 MeV, with a range in tissue of about 0.01 millimetre. The relative contributions of these two processes can be calculated from the known cross-sections of these nuclei, and from the percentage composition of wet tissue, which has been given⁽⁷⁾ as: Oxygen 73% by weight, carbon 12%, hydrogen 10%, nitrogen 4%, remaining 1% mainly sodium, magnesium, phosphorus, sulphur, chlorine, potassium, calcium. In calculating the permissible flux it is also necessary to know the density distribution of thermal neutrons in the body; this is assumed to be similar to that found in experiments with paraffin wax and water.⁽¹⁷⁾ The permissible exposure level can then be calculated from the energy absorbed in tissue, due allowance being made for the biological efficiency factors (10 and 20 respectively) of the nitrogen protons and the recoiling daughter nuclei. Such calculations have been made by MITCHELL⁽¹⁸⁾ and SNYDER.⁽¹⁹⁾

When *fast* neutrons strike hydrogen nuclei they lose on the average about two-thirds of their energy in a single collision; the energy acquired by the hydrogen is easily sufficient to free it from any chemical bonds, and it recoils as a heavily ionizing proton. In being slowed down in tissue from 1 MeV to thermal energies ($\sim 1/40$ eV) the neutron suffers on the average about 20 such collisions, and travels a distance of the order of 5 cm from its starting-point. It also makes a number of collisions with the other constituents of tissue, but in doing so loses relatively little energy: for neutron energies between 0.5 and 5 MeV $\sim 90\%$ of the energy given up by the neutron is lost in collisions with hydrogen nuclei. Once the neutron has been slowed down to thermal energies it diffuses for a few centimetres before being captured by a hydrogen or nitrogen nucleus. Calculations of the permissible fluxes of fast neutrons have been made by TAIT,⁽²⁰⁾ SNYDER,⁽²¹⁾ MITCHELL,⁽²²⁾ and BIRAM.⁽²³⁾

Since it is relatively difficult to design portable equipment suitable for monitoring fast neutron fluxes, they are potentially a more serious hazard to the radiation worker than gamma radiation. It should be particularly borne in mind that fast neutrons are effective promoters of cataract of the eye. A number of young American cyclotron workers, who had received doses of up to 200 rads, were unfortunately injured before this danger was sufficiently appreciated (see the report of the U.S. Radiation Cataract Survey, 1949).^(24, 25)

1.5. RECOMMENDED MAXIMUM PERMISSIBLE LEVELS FOR EXTERNAL WHOLE-BODY IRRADIATION*

A reliable estimate of the quantity of externally administered radiation which is certainly acceptable can be made from consideration of the combined effects of cosmic rays and naturally occurring radioactivity (Table 1.5.1.). At the other end of the scale, the quantity of radiation which, administered in a short time, would be lethal to 50% of the population, is also known with sufficient

* Permissible levels of contamination of drinking water and air are given in Section 1.6.

Table 1.5.1. *Radiation Doses to Which the Population is Liable to be Subjected*

External exposure to natural radiations and cosmic rays ⁽²⁶⁾	0.0002 rem/day
Internal exposure to naturally occurring radium and K^{40} ⁽²⁶⁾	0.0004 to 0.0014 rem/day
Internal exposure to K^{40} alone ⁽²⁷⁾	0.0001 rem/day
Home television set—recommended maximum value for dose at surface of cathode-ray tube ⁽¹⁶⁾	0.002 r/hr
Chest X-ray (best) ⁽²⁸⁾	0.006 r
(average) ⁽²⁹⁾	0.2 r
(fluoroscopic examination) ⁽²⁸⁾	~10 r
Shoe-fitting fluoroscope	3–8 r/min
Local dose during treatment of tumours ⁽²⁶⁾	3000 to 7000 r
Human skin erythema (hard gamma rays) ⁽³⁰⁾	~1000 r
Internationally agreed maximum level for normal occupational exposure	0.3 rad/week
Median lethal dose for man (whole-body irradiation) ⁽³⁰⁾	~500 r

accuracy—it is about 500 r. The question of where between these two extremes the upper limit of safety lies is not easily answered, since (except for very large doses) the effects of over-irradiation may have extremely long incubation periods; this naturally makes it difficult to correlate cause and effect with any precision. The present internationally accepted recommendations are based on the accumulated experience with X- and gamma radiations over the last 50 years, together with some supporting evidence from biological experiments. Although they are subject to periodical review by the International Commission on Radiological Protection, no reason has been found for making any important change in the maximum permissible level that was adopted in 1939.

The maximum permissible weekly doses for occupational exposure, listed below, are taken from the recommendations of the I.C.R.P., which considers that they are such as to involve a risk which is small compared with the other hazards of life. However, in addition to these limitations on the rate at which radiation may be received, it is also necessary to set further limits on the integrated dose received by a worker over a period of many years. One limit is set by the effect of radiation in causing deleterious changes in the genetic structure of an individual. It is extremely difficult to make a satisfactory quantitative estimate of this limit, but the report⁽³³⁾ of a committee set up by the British Government in 1956 recommends that no individual should accumulate a dose of more than 50 r before the age of thirty (and adds that this figure may need to be revised if more than about 2 per cent of the population are so exposed). A second limit is that an integrated “whole-body” dose of 200 r should not be exceeded during the lifetime of any worker, since it is known that beyond this point the normally very small incidence of the fatal disease, leukaemia, is significantly increased. These two limits control the long term average exposure to radiation, while the levels given below control the maximum rates of irradiation over much shorter periods. In addition to laying down these levels for individual exposure, the British committee also strongly recommended that the dose due to man-made radiation, averaged over a whole population, should not be allowed to exceed twice the “background” level

from natural radioactivity. This statement clearly implies that all unnecessary exposure to radiation must be avoided. In this connection it should be remembered that ionizing radiations are encountered to an ever-increasing extent in everyday life, and that the doses received from such devices as shoe-fitting fluoroscopes are not entirely negligible, as Table 1.5.1 shows.

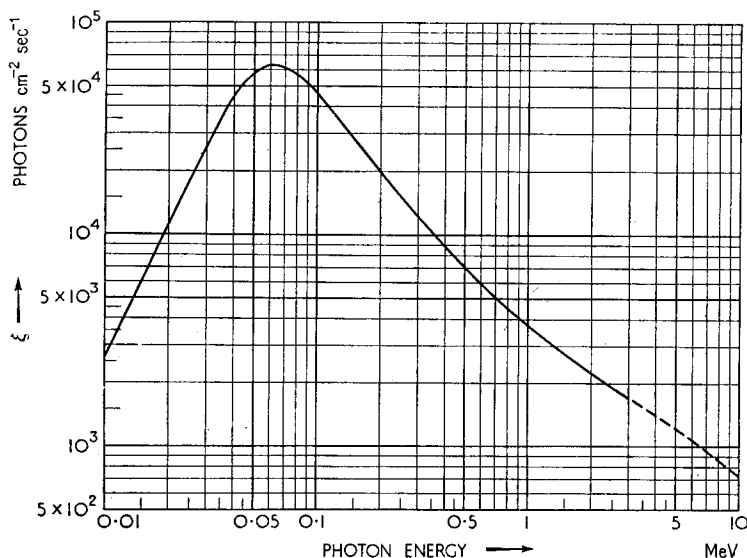


Fig. 1.5.1. Flux of photons ξ giving the permissible weekly occupational dose (0.3 r) in 40 hours. The broken portion of the curve is a reminder that the use of the röntgen is restricted to energies below 3 MeV.

Permissible Weekly Doses (Occupational Exposure)

(A) *Basic permissible weekly doses for the critical organs. (Whole-body exposure to ionizing radiations of any type or types):*

- (i) Whole-body dose: 0.3 rem per week in blood-forming organs, the gonads, and the eyes;
- (ii) Surface dose: 0.6 rem per week in the basal layer of the epidermis (which is situated at a depth of about 7 mg cm^{-2} below the passive horny layer of the skin).

These figures are to be regarded as the maximum permissible weekly doses for occupational exposure, except where modifying factors apply. Doses due to different types of radiation are additive. By whole-body exposure is meant exposure of the trunk, and not necessarily the exposure of the extremities.

Note (a) that for the penetrating radiations encountered in nuclear engineering condition (i) is limiting; (b) that for gamma radiation of quantum energy greater than a few MeV the maximum dose occurs at an appreciable distance below the surface of the body (see § 2.13, Fig. 2.13.4).

(B) For X-rays of energy less than 3 MeV, for which the röntgen is specifically retained as the unit of dose, the difference between rems and röntgens is small enough to be neglected for these purposes ($1 \text{ r} = 93 \text{ erg/gram of tissue}$; while for X- and gamma rays, of r.b.e. = 1, one rem = 100 erg/gram). The flux of photons giving one permissible level of radiation is shown in Fig. 1.5.1.

(C) Neutrons. One weekly dose is given by exposure for 40 hours to the following neutron fluxes:

Table 1.5.2. Permissible Neutron Fluxes for Exposure of 40 hours per week

Neutron energy	Neutron flux ($\text{n cm}^{-2} \text{ sec}^{-1}$), measured in air
0.025 eV (Thermal neutrons)	2000
10 eV	2000
10 keV	1000
0.1 MeV	200
0.5 MeV	80
1 MeV	60
2 MeV	40
3–10 MeV	30

(D) Beta and alpha particles. The number of incident β particles per cm^2 per sec giving a dose to the surface of the body of 0.6 rem per 40 hours is shown in Fig. 1.5.2. For energetic beta-particles, the permissible flux is not a rapidly

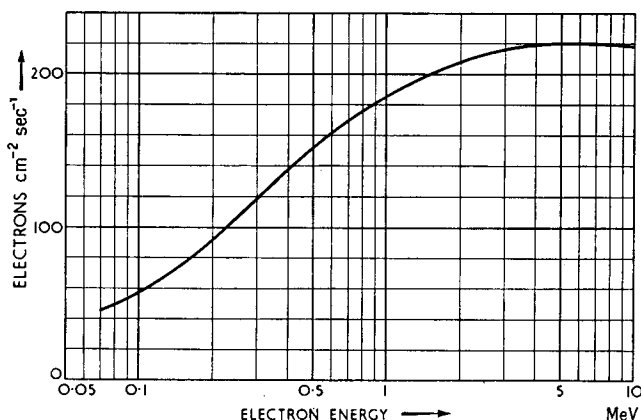


Fig. 1.5.2. Collimated flux of monoenergetic electrons, giving a dose of 0.6 rem per 40 hours. Below 0.07 MeV beta particles are a hazard only when emitted internally. (After MORGAN⁽²⁶⁾ and STEPHENSON⁽³¹⁾.)

changing function of the beta ray energy, and consequently it does not greatly matter whether one takes the mean energy or the end-point energy when dealing with the continuous energy spectrum emitted by a radioactive substance.

For the same reason the energy degradation occurring in a thick source also has a relatively small effect on the biological dose at the surface of the body per incident electron. The tissue which is critical as far as external irradiation by beta particles is concerned is the basal layer of the epidermis. This critical layer is not reached by soft beta rays, of energy less than about 0.07 MeV, or by any naturally

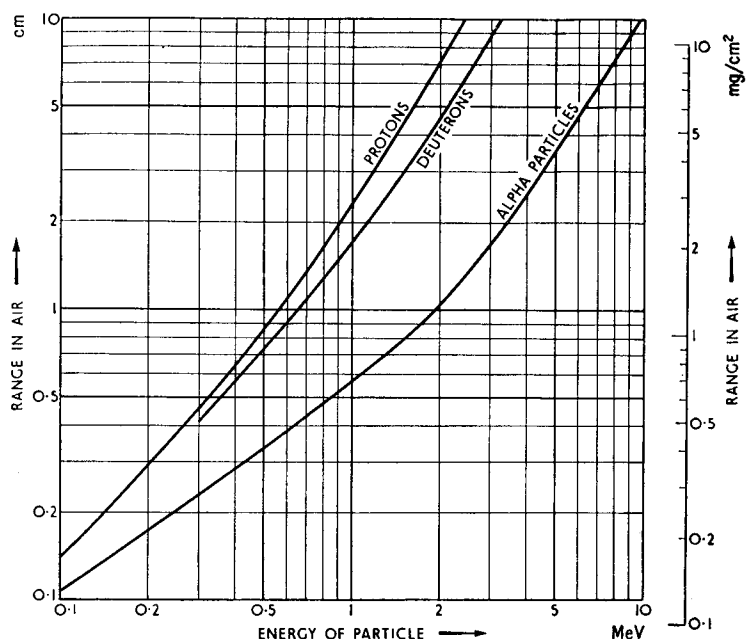


Fig. 1.5.3. Range of protons, deuterons and alpha particles in air at 15°C and 760 mm pressure. The range in mg/cm^2 of tissue is roughly equal to the range in mg/cm^2 of air (LIVINGSTON and BETHE⁽³²⁾.) For electrons see Fig. 2.11.1.

occurring alpha particles (Fig. 1.5.3.). Consequently, these radiations will be without effect on the body (apart from the lens of the eye), except when they are sufficiently intense to cause burning; or when emitted internally, as a result of eating, drinking or breathing radioactive contamination; or when, as sometimes happens, beta and alpha emitters are absorbed into the skin.

The ease with which beta particles can be shielded tends to give the impression that they are intrinsically less dangerous than gamma radiation or neutrons. As a result strong open beta sources are often handled in the fingers. It is worth bearing in mind, therefore, that a 1 millicurie source gives 3000 rad per hour at a distance of 3 mm. Gloves afford some protection, though they cannot conveniently be made thick enough to stop high-energy beta particles. Standard heavy PVC gloves have a thickness of 125 mg cm^{-2} , equivalent to the maximum range of beta particles of 0.4 MeV; clothing such as trousers averages 50 mg cm^{-2} .

(E) *Factors modifying the basic permissible weekly doses.* Practical considerations make it desirable to use maximum permissible doses which differ

appreciably from the basic permissible doses, where this may be done without appreciably increasing the risk. The I.C.R.P. have considered some special cases, for which they recommend the following values.

- (i) *Radiation of very low penetrating power* (Half-value layer less than 1 mm of water)
 - Whole body exposure: 1.5 rem/week
 - Lens of eye: 0.3 rem/week
- (ii) *Irradiation of a limited portion of the body.*
 - Hands and forearms: 1.5 rem/week in the skin, less in the deeper tissues.
 - Feet and ankles: 1.5 rem/week in the skin, less in the deeper tissues.
 - Head and neck: 1.5 rem/week in the skin, less in the deeper tissues, and
0.3 rem/week in the lens of the eye.

(F) *Permissible dose rate.* It is believed that the radiation damage suffered by the human system does not depend at all critically on the dose-rate, although the evidence is still incomplete. However, because of the difficulty of keeping adequate control of exposure times, exposure to dose rates above 10 rads per hour should be deliberately incurred only in exceptional circumstances.

(G) *Emergency exposure.* In a properly controlled laboratory or industrial installation there is no difficulty in arranging that these permitted levels of radiation are not often exceeded.^(3c) Nevertheless, emergencies do occasionally arise in which exposure to considerably greater amounts of radiation is unavoidable; the best-known instance is probably the rebuilding during 1953-4 of the damaged Canadian NRX reactor. In such cases an individual is usually allowed to continue working as long as his total exposure does not exceed 3 r in any 13-week period. Any person who accidentally receives a bigger dose must subsequently be given work involving considerably less exposure to radiation, for a period necessary to bring his average exposure down to the permitted level. One should not regard an averaging procedure of this kind as a justification for treating occasional large doses of radiation as unimportant.

1.6. RECOMMENDED MAXIMUM PERMISSIBLE LEVELS OF RADIOACTIVE CONTAMINATION OF DRINKING WATER AND AIR

Radioactive material can enter the body by ingestion, inhalation, or by absorption through the skin. Ingestion may occur through drinking contaminated water, by eating contaminated food, or by smoking in an active laboratory without having checked that the hands are free from radioactivity. Inhalation of particulate material, a danger particularly associated with the manipulation of fine powders, can be minimized by providing adequate ventilation, and by careful design of "dry-boxes" and other equipment. Absorption through the skin can easily occur through cuts on the fingers; it is desirable to suspend work with materials for which the permissible body-burden is very small, such as plutonium, when the skin is in any way damaged. Absorption may also occur

from splashing with contaminated lipoid solvents (i.e. fat-dissolving liquids). These dangers can be very largely avoided by skilled operation, by proper supervision and ruthless stamping out of slovenliness, by wearing protective clothing for the more hazardous operations, and by the co-operation of an efficient health physics monitoring team, provided with accurate and quick-reading instruments.

Since it is extremely difficult to detect the accumulation of active particles in the lung or the deposition of radio-elements in the organs of the body before irreparable damage has been done, the danger from the ingestion of active material must be considered as the most serious of all the hazards associated with radioactivity. If, in spite of all precautionary measures, ingestion is known to have occurred, medical advice should be sought as a matter of urgency: immediate attention can in some cases help to ensure that the bulk of the radioactive material is excreted.

The metabolic history of ingested material depends very markedly on its chemical nature. Some elements, such as sodium, are rapidly excreted; others are selectively absorbed in particular organs (e.g. iodine in the thyroid), or retained for considerable periods in the liver; while radium, strontium, and plutonium can be deposited and permanently fixed in the bone-structure. The hazard associated with the radioactive contamination of the body by a particular substance depends on several factors: (a) its *biological half-life*, or the average time which elapses before 50% of the material is eliminated from the body; (b) its *radioactive half-life*, since this determines the total number of disintegrations that will take place within the lifetime of the individual; (c) *the specific ionization of the radiations emitted*, since this determines the relative biological efficiency, just as for external radiation; (d) *the energy liberated per disintegration*; (e) the way in which it is distributed in the body.

These factors have been taken into account by the International Subcommittee on Permissible Doses for Internal Radiation in their calculations of the maximum permissible concentrations of various isotopes in air and drinking water. A short abstract from the very complete tables prepared by the subcommittee is given in Table 1.6.1. It is believed that an individual may be exposed to the permitted levels of contamination for long periods, for 24 hours per day, without suffering any injury. It is necessary, however, to make a specific exception in the case of some beta emitters in chemical forms insoluble in lung fluids; in such cases the maximum permissible concentrations in air are lower than the figures quoted in the table. The handling of materials in this category should be specially supervised by a competent health physics authority.

The figures quoted were calculated on the assumption of a daily intake of 2.5 litres of water, and 20 cubic metres of air. No particular harm is to be expected if the recommended values are exceeded for a short period of time. For instance, exposure of individuals for a few days to water and air concentrations ten times those listed would not be harmful, provided the average concentration over a year did not exceed the recommended values. When exposure is limited to the working week of 40 hours, the figures quoted may be increased by a factor of three (*not* $168/40 = 4.2$, because a relatively greater intake occurs during working hours). To be consistent with the limitation of lifetime dose

Table 1.6.1. Maximum permissible concentrations, for occupational exposure, of water and air contamination

Nuclide	Continuous m.p.c. ($\mu\text{c ml}^{-1}$)*		40 h week m.p.c. ($\text{dis min}^{-1} \text{m}^{-3}$)
	Water	Air	Air
H ³ (as H ₂ O)	0.2	10 ⁻⁵	7 × 10 ⁷
C ¹⁴ (as CO ₂)	3 × 10 ⁻³	10 ⁻⁵	7 × 10 ⁷
Na ²⁴	8 × 10 ⁻³	10 ⁻⁶	7 × 10 ⁸
P ³²	2 × 10 ⁻⁴	10 ⁻⁷	7 × 10 ⁵
A ⁴¹	5 × 10 ⁻⁴	5 × 10 ⁻⁷	3 × 10 ⁶
Ca ⁴⁵	10 ⁻⁴	8 × 10 ⁻⁹	5 × 10 ⁴
Co ⁶⁰	4 × 10 ⁻⁴	8 × 10 ⁻⁸	5 × 10 ⁵
Cu ⁶⁴	5 × 10 ⁻³	9 × 10 ⁻⁷	6 × 10 ⁶
Sr ⁸⁹	7 × 10 ⁻⁵	2 × 10 ⁻⁸	10 ⁵
Sr ⁹⁰ (+ Y ⁹⁰)	8 × 10 ⁻⁷	2 × 10 ⁻¹⁰	10 ³
Zr ⁹⁵ (+ Nb ⁹⁵)	6 × 10 ⁻⁴	8 × 10 ⁻⁸	5 × 10 ⁵
Ru ¹⁰⁶ (+ Rh ¹⁰⁶)	10 ⁻⁴	2 × 10 ⁻⁸	10 ⁵
I ¹³¹	6 × 10 ⁻⁵	6 × 10 ⁻⁹	4 × 10 ⁴
Cs ¹³⁷ (+ Ba ¹³⁷)	2 × 10 ⁻³	2 × 10 ⁻⁷	1 × 10 ⁶
Ce ¹⁴⁴ (+ Pr ¹⁴⁴)	10 ⁻⁴	2 × 10 ⁻⁹	10 ⁴
Au ¹⁹⁸	6 × 10 ⁻⁴	10 ⁻⁷	7 × 10 ⁵
Po ²¹⁰	3 × 10 ⁻⁶	10 ⁻¹⁰	700
Rn ²²² + daughter	—	10 ⁻⁷	7 × 10 ⁵
Ra ²²⁶ + daughter	4 × 10 ⁻⁸	(total μc) 8 × 10 ⁻¹²	(total disintegrations) 50
Th natural	5 × 10 ⁻⁷	3 × 10 ⁻¹¹	400
U natural	2 × 10 ⁻⁶	3 × 10 ⁻¹¹	(total disintegrations) 400
U enriched	4 × 10 ⁻⁶	6 × 10 ⁻¹¹	(total disintegrations) 400
U ²³³	(total μc) 3 × 10 ⁻⁶	(total μc) 3 × 10 ⁻¹¹	(total disintegrations) 200
Pu ²³⁹	3 × 10 ⁻⁶	2 × 10 ⁻¹²	10
Any fission product mixture	10 ⁻⁷	10 ⁻⁹	7 × 10 ³
Any mixture of α emitters	10 ⁻⁷	5 × 10 ⁻¹²	30

Note: Where disintegrating nuclei also have radioactive daughter products (e.g. Ra) the activities quoted are those of the parent, except where otherwise stated.

* The curie and microcurie (μc) are units of radioactive source strength, defined in Section 6.9.

to 200 rem the long term average exposures to contaminated air should be kept a factor of 3 below the quoted levels. The figures quoted for unidentified mixtures of fission products and of alpha emitters should not be used for periods of more than a few months unless it is certain that the concentrations of radium 226, plutonium 239, and strontium 90 are all below the permitted levels.

APPENDIX

DEFINITIONS OF OBSOLETE UNITS

The *röntgen-equivalent-physical* (abbreviated "rep"). The energy absorbed per unit mass of tissue exposed to a dose of one rep is taken (in the more recent

literature) to be equal to the energy absorbed per gram of tissue exposed to one röntgen, i.e. 93.1 ergs. The unit known as the *energy unit* is essentially similar. Prior to the redefinition of the rep by the Chalk River Conference (September 1949), the somewhat different value of 84 ergs per gram per rep was in general use.

The *gram-röntgen* is the energy absorbed per gram of air per röntgen, i.e. 84 ergs.

The "n unit" has been fairly widely used for fast neutron dosimetry in the United States. It is based on the commercial availability of the 100 r Victoreen condenser ionization chamber, which has a hydrogenous (bakelite) wall. Since the effect of fast neutrons on tissue is principally due to the heavily ionizing recoil protons formed when hydrogen atoms are struck, a dose measurement (in arbitrary units) can be made by exposing this type of chamber to a fast neutron beam. The dose of fast neutrons which produces in the 100 r Victoreen chamber the same effect as 1 röntgen of gamma rays is called an "n unit." Although this unit has the great advantage of providing a simple method of monitoring fast neutron dosage, it is by no means easy to interpret in terms of the more fundamental unit, the rad. AEBERSOLD and ANSLOW⁽³⁰⁾ found that $1 \text{ n} \approx 1.9 \text{ rad}$.

The "N unit" is defined similarly, except that in this case the chamber to be used is the 25 r Victoreen chamber. The assumption is usually made that $1 \text{ N} = 1 \text{ n}$.

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CHAPTER TWO

ATTENUATION OF GAMMA RAYS AND HIGH-ENERGY ELECTRONS

2.1. INTRODUCTION

PENETRATING electromagnetic radiations are emitted in many different types of nuclear reaction: for example, in fission, in the capture of neutrons, in the annihilation of a positron with an electron, or by the decay of many radioactive materials. They are also produced by the stopping of high-energy electrons, a process which has been in common use for over fifty years as a source of X-rays. The properties of these radiations, including their power of penetrating dense media, are very dependent on the wavelength. In the optical region electromagnetic radiation possesses properties that are predominantly wavelike. This remains the case as the wavelength decreases through the ultra-violet and soft X-ray regions; but on approaching the range of wavelengths emitted in nuclear reactions the radiation behaves more and more as though it were composed of particles rather than waves. These pseudo-particles, which are described as *photons*, or *quanta*, are each associated with an amount of energy $h\nu$, where ν is the frequency of the radiation, and h is Planck's constant, 6.624×10^{-27} erg-sec. They share with material particles the property of possessing momentum, and behave in many respects as independent entities. The duality of the properties of the radiation is perhaps most marked in the hard X-ray region, where the wavelength is of the order of 0.1 to 0.5 Å. Individually the photons behave as particles, but the statistical behaviour of a large assembly of photons is described by their wave properties. Thus a beam of X-rays can be diffracted by a crystal, and can show the diffraction intensity patterns typical of waves; but a suitable detector is nevertheless able to detect the arrival of each individual photon.

The ranges of quantum energies which are of interest in various fields of work are shown diagrammatically in Fig. 2.1.1. The term *gamma ray* is commonly applied to all radiation of energy greater than about 0.1 MeV,* and particularly to radiations originating in nuclear disintegrations, while *X-ray* usually refers to radiation produced by a machine of not more than about 500 kilovolts peak voltage; but there is no really hard and fast rule that determines which term is to be used. It will be seen from the figure that the gamma ray energies of importance in the field of nuclear engineering and in the design of accelerators are in the range where the particle-like properties predominate, and where the process of attenuation by a shield is therefore most easily pictured in terms of the life-histories of the individual photons.

* Mega electron-volt, the most convenient unit for quantum energies in this field, equal to 1.602×10^{-6} ergs.

Before a gamma ray shield can be designed three things must be known: the intensity and quantum distribution of the source to be shielded, the rate of attenuation of these photons by the material chosen for the shield, and the level to which it is desired to reduce the radiation. The energy distribution and intensity of the gamma radiation emitted by an X-ray machine or other electron accelerator can be calculated with an accuracy which is usually sufficient for shielding purposes. There is no method of calculating what radiations are emitted by artificially or naturally radioactive materials, but here a massive body of

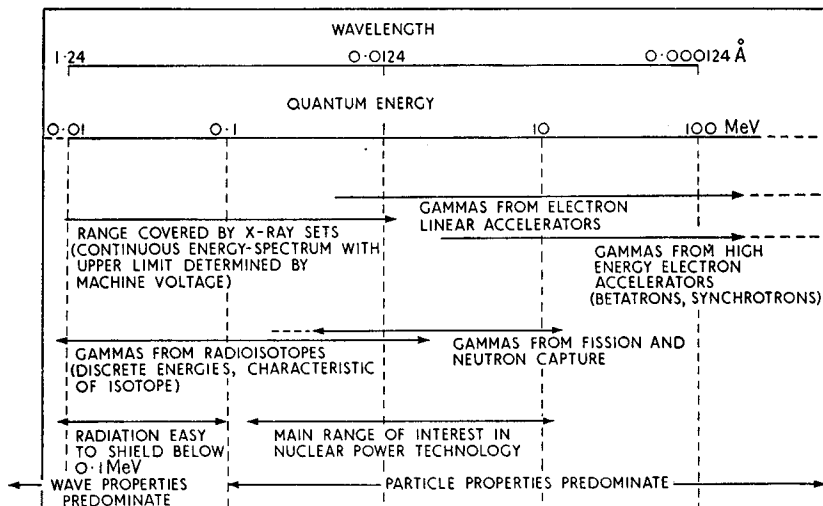


Fig. 2.1.1. Ranges of quantum energies of interest in various fields of work.

experimental information is available. The gamma rays emitted in the course of many nuclear reactions are also known in some detail; and when the information is not available it is sometimes possible to consider the energy-balance of the reaction, and so to set an upper limit to the energy emitted in the form of penetrating radiation.

The processes by which gamma rays are attenuated in a shield are well understood, and, provided complicated geometries are avoided, the amount of shielding required to reduce the intensity by a given factor can be calculated to a satisfactory degree of accuracy. Three main processes are involved: pair production, the photoelectric effect, and Compton scattering. The first two depend critically on the atomic number of the absorber, and are particularly important for the heavy elements. The third process involves interactions with orbital electrons; since equal masses of all materials (except hydrogen) contain very roughly the same number of electrons, the contribution made by Compton scattering depends very little on atomic number, and is mainly determined by the mass per unit area of the shield. The energy regions in which the three processes predominate are roughly as follows:

	<i>Photo-electric effect</i>	<i>Compton scattering</i>	<i>Pair production</i>
Light elements (aluminium)	Below 50 keV	50 keV–15 MeV	Above 15 MeV
Heavy elements (lead)	Below 0.5 MeV	0.5 MeV–5 MeV	Above 5 MeV

The process of attenuation proceeds roughly exponentially with distance. The ten-folding-length—the distance in which the intensity of the radiation is reduced by a factor of ten—is a function of gamma ray energy which, for the heavy elements used in reactor shields, has a maximum value at energies of about 3 MeV. This maximum corresponds to an energy region where the most important process is Compton scattering, and therefore has roughly the same value of about 60 g cm^{-2} for all materials (Note that in this subject it is often convenient to express shield thicknesses in g cm^{-2}). Since a considerable proportion of the gamma radiation emitted by a nuclear reactor is in the region of this maximum, and since the intensity is such that attenuations of 10^8 or 10^9 are commonly required, it follows that shields for nuclear reactors are inevitably massive affairs, with weights of the order of 0.5 kg cm^{-2} (or $\sim 0.5 \text{ ton ft.}^{-2}$) of shield area. The exponential law of attenuation makes it pointless, on occasion, to introduce too much refinement in a shielding calculation. For instance, an attenuation of 10^6 may be required for the shield of an electron accelerator, but if the calculation were in error by a factor of two, this would be compensated by a change in shield thickness of only 5%.

The emphasis in this chapter is on the physical principles of gamma ray attenuation, and no effort has been made to deal with problems of mechanical design, since these are more easily considered along with the other practical matters dealt with in Chapter 6. The three main elementary attenuation processes are described in §§ 2.3–2.5, which are followed by a discussion of the attenuation by thick shields (§ 2.6, § 2.7). Some data on backscattering of gamma radiation are presented in § 2.8. An account is given in §§ 2.9, 2.10 and 2.12 of the physics of some practically important types of gamma ray source, but discussion of the gamma radiation accompanying neutron capture and inelastic scattering, or from neutron-induced radioactivity, is deferred until the following chapter. For convenience the isolated topic of beta ray shielding is dealt with in § 2.11 as an introduction to the discussion of bremsstrahlung (§ 2.12). The last section (2.13) explains the necessity (already referred to in Chapter 1) of abandoning the röntgen as a unit of intensity for high-energy gamma radiation.

2.2. CROSS-SECTION AND RELATED QUANTITIES

The probability that a photon, in traversing a slab of material, will be affected by any process such as pair production or Compton scattering is most simply described in terms of a *cross-section* for that process.

Assume that ϕ_0 photons $\text{cm}^{-2} \text{ sec}^{-1}$ are normally incident on a slab of material

in which there are N atoms per cm^3 . The quantity ϕ_0 is usually referred to as the incident *flux*; it would be more correct to describe it as the *flux density*, but the abbreviated term is the one in common use. Note that the term flux is reserved for the quantity of radiation crossing unit area placed normal to the direction of the photons. For an uncollimated beam the flux is equal to the number of photons threading a sphere of unit cross-sectional area per second.

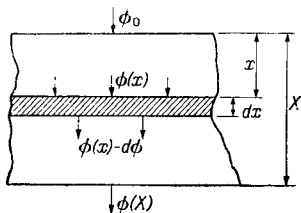


Fig. 2.2.1.

Let $\phi(x)$ be the flux at depth x of photons which have not suffered any collision or interaction in their passage through the slab; $\phi(x)$ is usually called the *uncollided flux*. Let the probability that a photon, in traversing a thickness dx , suffers a collision of the type i , be dp_i ; and assume, for the moment, that no other type of

collision is possible. Then σ_i , the cross-section per atom for the process, is defined by the equation:

$$dp_i = N\sigma_i dx = -\frac{d\phi}{\phi}$$

On integrating, the uncollided flux $\phi(X)$ emerging through a slab of thickness X is found to be

$$\phi(X) = \phi_0 e^{-N\sigma_i X} \quad (2.2.1)$$

If more than one process is involved, σ_i in equation (2.2.1) must be replaced by the sum of the component cross-sections, σ_{tot} . The total cross-section depends markedly on both the atomic number of the absorber and on the photon energy. Atomic cross-sections are expressed in units of 10^{-24} cm^2 , or “barns.”

The product $N\sigma_{\text{tot}}$ is frequently described as the *macroscopic total cross-section of the material per cm^3* , to distinguish it from the *microscopic* cross-section σ_{tot} . The inverse of the macroscopic total cross-section, $(N\sigma_{\text{tot}})^{-1}$, which has the dimensions of length, is known as the *mean free path* for the photons in the material. In describing the attenuating properties of a shield, reference is often made to the *absorption coefficient* of the material. This sometimes means $N\sigma_{\text{tot}}$, and sometimes $N\sigma_{\text{tot}}/\rho$, where ρ is the density of the absorber; in the latter case it is conveniently measured in cm^2 per gram. It is usually obvious from the context which quantity is meant. In this book the term *linear absorption coefficient* will be used in the sense of $N\sigma_{\text{tot}}$, and will be denoted by μ , while $N\sigma_{\text{tot}}/\rho$ will be described as the *mass absorption coefficient*, χ . Different subscripts will be used to denote the component absorption coefficients and cross-sections: e.g. μ_{PE} , σ_C , χ_{PP} .

The term *relaxation length* is often encountered in papers dealing with the shielding of both gamma rays and neutrons. It is used to describe the parameter p characterizing the fall-off of any quantity q (e.g. flux, energy density, biological dose) which decreases exponentially with distance x according to the law $q = q_0 e^{-x/p}$. In the particular case of gamma radiation the relaxation length in

a thick shield happens to be not very different from the mean free path defined above, and as a result the two terms are sometimes confused with each other.

INTERACTIONS OF GAMMA RAYS WITH MATTER (ELEMENTARY PROCESSES)

2.3. THE PHOTOELECTRIC EFFECT

The ejection from an atom of an orbital electron by a photon is important historically because of the rôle it played in suggesting the quantum nature of electromagnetic radiation. In the photoelectric interaction the whole of the quantum energy $h\nu$ is transferred to an atomic electron, which emerges from the atom with an energy $E = h\nu - I$, where I is its ionization energy. The photoelectric cross-section per atom (σ_{PE}) varies roughly as the fourth power of the atomic number, and decreases rapidly with increasing ν —roughly as $h\nu^{-3}$ —for large values of the photon energy. At lower photon energies, however, the dependence on $h\nu$ is not monotonic: it exhibits sharp discontinuities (the $K, L \dots$ absorption edges) corresponding to the threshold energies at which a photon can just eject an electron from the $K, L \dots$ electron shells of the atom. These discontinuities are evident in the curves for uranium, lead and tungsten in Fig. 2.6.4. As the photon energy is raised beyond each absorption edge the cross-section drops again smoothly. In the region between the discontinuities the dependence of σ_{PE} on atomic number and $h\nu$ (illustrated in Fig. 2.5.3) may be given a plausible explanation based on quite simple classical-physics arguments. In order for the electron to resonate under the influence of the high-frequency electromagnetic field of the incident photon it must be tightly bound; that is, a large electrostatic force holding the electron to the nucleus is required, and so Z , the atomic number, must be large. In addition, the frequency of the photon must not be too high. The same consideration suggests that the most tightly bound electrons would be those predominantly concerned in the photoelectric process. One may also put the argument somewhat differently: an electron can absorb electromagnetic radiation only when it is bound. One would therefore expect that the tighter the binding the more the electrons can contribute to the absorption of energy. These arguments are, of course, in no sense explanations, but they provide an easy way of remembering the main features of the process. The expectation that the most important contribution would be from the inner electron shells is confirmed experimentally: at photon energies considerably above the K absorption edge (i.e. where ionization of all the electronic shells is energetically possible) the inner (K) shell is ionized in 80% of all ionizing events.

The calculation of the cross-section of the K shell per atom (${}_K\sigma_{PE}$) has been carried out by HEITLER,⁽¹⁾ using a non-relativistic description of the interaction, and other simplifying assumptions. He obtains

$${}_K\sigma_{PE} = \frac{8\pi r_0^2}{3} \cdot \frac{Z^5}{137^4} \cdot 4\sqrt{2} \left(\frac{mc^2}{h\nu} \right)^{7/2} \quad (2.3.1)$$

where r_0 is the classical radius of the electron, 2.82×10^{-13} cm, and mc^2 is its

rest energy (0.51 MeV). If the photon energy is denoted by E_0 MeV, and $_{K}\sigma_{PE}$ is in barns, this expression simplifies to

$$_{K}\sigma_{PE} = 1.0 \times 10^{-9} Z^5 E_0^{-7/2} \quad (2.3.1a)$$

Since K shell ionization occurs in 80% of all ionizing events, a rough value for the atomic cross-section may be obtained by multiplying $_{K}\sigma_{PE}$ by 5/4. More elaborate calculations have been carried out by HALL,⁽²⁾ HULME *et al.*,^(3,4) SAUTER,⁽⁵⁾ and STOBBE⁽⁶⁾; they lead to expressions which are much more complicated than formula (2.3.1), but confirm the general dependence on Z and E_0 which this formula predicts.

It is found experimentally⁽⁷⁻⁹⁾ that in the region above the K absorption edge the cross-section is approximately proportional to Z^n/E_0^3 , where values of the exponent n suggested by different observers vary from 3.94 to 4.4. However, all such simple formulae are satisfactory only over small ranges of Z and E_0 , and break down as the K absorption edge is approached. Semi-empirical formulae derived by VICTOREEN⁽⁹⁾ summarize the data⁽⁷⁾ much more satisfactorily. Detailed discussions of the available theoretical and experimental work are given in WHITE's review of gamma-ray cross-sections,⁽¹⁰⁾ and by HEITLER.⁽¹⁾

Fluorescent radiation. When an electron has been ejected by the photoelectric process, usually from the K shell, the vacancy may be filled in one of two ways. An electron from the L or M shells may make a transition to the K shell, with the emission of a secondary photon (in the first case the "fluorescent" radiation is described as K_α radiation, and in the second case as K_β radiation). Alternatively, an electron (Auger electron) may be ejected directly from one of the outer shells without the emission of a photon. (This may be pictured as the filling of the K shell by an electron, with the emission of a photon, which is immediately re-absorbed in an outer shell, causing the liberation of a photoelectron; however, this description cannot be considered as more than a convenient mnemonic, since it fails to predict the correct probability of the process.) The ratio of the number of atoms which emit fluorescent radiation to the number of quanta absorbed photoelectrically is described as the *fluorescent yield* of the material. It is a function of the atomic number Z of the medium, and increases as Z increases. To a good approximation the K shell fluorescent yield is given⁽¹¹⁾ by the equation

$$\Phi_K = [1 + 1.12 \times 10^6 Z^{-4}]^{-1}. \quad (2.3.2)$$

The fluorescent radiation is always less energetic than the photon which was originally absorbed, and therefore has a smaller penetrating power. As a result, its contribution to the total dose at the surface of a gamma ray shield is never more than one or two per cent,⁽¹²⁾ which is small enough to be disregarded. Thus for all practical purposes the photoelectric effect represents a pure absorption of the gamma ray beam.

2.4. PAIR PRODUCTION

This is the phenomenological description given to the process in which a photon of energy greater than twice the rest mass of an electron (i.e. > 1.02 MeV) can be converted into a pair of positive and negative electrons. The process cannot occur except in the presence of matter, and is more probable the higher

is the atomic number of the absorber, and the more energetic is the gamma ray. The equations for the conservation of energy and momentum in the process can be satisfied simultaneously only if another particle takes part. Although this particle is not itself affected—it acts as a kind of catalyst—this restriction means that the process cannot occur in free space. The theory given by HEITLER and SAUTER,⁽¹³⁾ and BETHE and HEITLER,⁽¹⁴⁾ shows that the cross-section for pair production in the presence of a nucleus of charge Z should vary as Z^2 , and should increase rapidly with E_0 , the energy of the photon, until a constant value is reached at high energies (see Fig. 2.5.3). Although these calculations are approximate they are found⁽¹⁰⁾ to give an adequate description of the experimental results (within about 10%) for all elements heavier than aluminium that have been studied, and for gamma ray energies up to 350 MeV.

It was pointed out by PERRIN⁽¹⁵⁾ that pair production should not be confined to interactions with nuclei, but should also take place in the neighbourhood of the Z orbital electrons. However, the latter effect is relatively unimportant, as it has a threshold energy (2.04 MeV) higher than that for nuclear pair production, and a cross-section smaller in the ratio $1/Z$.

Just as in the case of the photoelectric process, pair production is followed by the emission of relatively soft gamma radiation, though for a different reason. The free positrons produced in the process are unstable in the presence of negative electrons, with which they rapidly recombine. The disappearance of the positron in this annihilation process is accompanied by the emission of an equivalent amount of gamma ray energy: the annihilation process is in fact the reverse of the pair creation process.

The most probable type of annihilation process is a collision between a positron and a free negative electron. Energy and momentum can in this case be conserved only if two quanta are emitted; and as this type of annihilation is more probable for slowly moving than for fast positrons the usual result is the emission of two quanta of energy 0.51 MeV. This has been satisfactorily confirmed by the very precise experiments of DUMOND *et al.*⁽¹⁶⁾

The rarer type of annihilation, with the emission of a single quantum, takes place only in collision with a bound electron. Even in the heaviest elements the one-quantum process is a factor of five less probable than the two-quanta process, and for light elements it is still less important. The single process differs from the double process in being most probable for fairly high positron energies, of about 0.5 MeV, and being small for slow positrons.⁽¹⁾ The length of time during which positrons can diffuse in a solid medium without suffering annihilation is of the order of 10^{-10} sec in lead.

The emission of these relatively soft secondary quanta, following the absorption of the primary gamma ray, is usually of little account from the point of view of shielding, and is frequently ignored. A quantitative estimate of how much they contribute to the total dose existing at the surface of a shield has been made by GOLDSTEIN,⁽¹²⁾ using the moment method of calculation (Section 2.7). Annihilation radiation is found to be most important for high-energy gamma ray sources in the range 6 to 10 MeV and for medium to heavy elements ($Z = 25$ to 85). In the worst case considered it accounted for 7% of the total dose, which is immaterial for most purposes. It has a very marked effect, however, on the

spectrum of the scattered radiation in the low-energy region, below 500 keV, and must therefore be considered when the softer radiations are of interest.

2.5. SCATTERING OF GAMMA RADIATION

In addition to the two purely absorptive processes just mentioned, there are also a number of ways in which gamma rays can be scattered. The most important of these is *Compton scattering*, another effect which is understandable

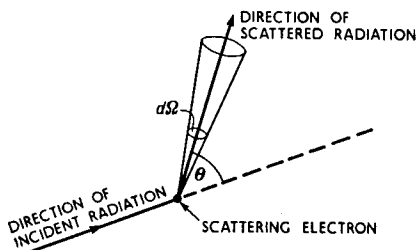


Fig. 2.5.1.

only with the aid of the quantum hypothesis: if a gamma ray photon, which according to this theory has energy $h\nu$ and momentum $h\nu/c$ (where c is the velocity of light, 3×10^{10} cm/sec), collides with an electron—which for the moment is assumed to be free to recoil—then some of the momentum will be transferred from the photon to the electron. Since the usual momentum and energy conservation laws apply to the collision, the photon must also have

lost energy. In other words, $(h\nu)_{\text{final}}$ is less than $(h\nu)_{\text{initial}}$: the frequency of the radiation has been lowered by the collision. In this respect the process differs from classical scattering of light, which takes place without appreciable change of frequency. Compton scattering provides a method of lowering the energy of photons by successive collisions, until they have been sufficiently degraded to be absorbed photoelectrically.

The equation describing the change of photon energy during a Compton collision is most simply written in terms of the wavelength $\lambda = c/\nu$. If the photon is deflected through an angle θ (Fig. 2.5.1), then it is easily shown from considerations of energy and momentum that

$$\Delta\lambda = \lambda_{\text{final}} - \lambda_{\text{initial}} = \lambda_0(1 - \cos \theta) \quad (2.5.1a)$$

where $\lambda_0 = h/mc = 0.0242 \text{ \AA}$ is a constant, known as the Compton wavelength, and m is the mass of the electron. Expressed in terms of photon energy, this equation becomes

$$\gamma_1 = \gamma_0[1 + \gamma_0(1 - \cos \theta)]^{-1} \quad (2.5.1b)$$

where $\gamma = h\nu/mc^2 = (\text{energy of photon in MeV})/0.51$, and the suffixes 0 and 1 refer to the incident and scattered photons respectively. Glancing collisions ($\theta \approx \text{zero}$) therefore transfer very little energy, just as in billiard-ball mechanics: a photon must be deflected through a large angle for its energy to be appreciably degraded in a single collision. Large-angle scattering of high-energy photons ($\gamma \gg 1$) results in a scattered photon of energy $\approx 0.51/(1 - \cos \theta)$ MeV. Thus high-energy photons scattered through a right angle will have energies of about 0.5 MeV, and those back-scattered through 180° will have energies of about 0.25 MeV. Values of the scattered energy are given in Table 2.5.1 for various incident energies and angles of deflection.

Table 2.5.1. *Energy of Photons Deflected Through an Angle θ by Compton Scattering*
All energies in MeV

Angle of scattering θ (degrees)	Source energy			
	0.5	1.25	3.0	6.0
	Energy of scattered radiation			
0	0.500	1.250	3.000	6.000
5	0.498	1.238	2.933	5.738
10	0.4925	1.204	2.749	5.075
15	0.4835	1.152	2.490	4.258
20	0.4715	1.086	2.203	3.481
25	0.4572	1.013	1.920	2.824
30	0.4408	0.936	1.663	2.300
35	0.4234	0.861	1.439	1.893
40	0.4051	0.789	1.249	1.576
45	0.3866	0.722	1.087	1.329
50	0.3683	0.660	0.955	1.134
55	0.3505	0.605	0.843	0.981
60	0.3332	0.556	0.749	0.857
65	0.317	0.511	0.672	0.757
70	0.3016	0.472	0.606	0.674
75	0.2871	0.438	0.551	0.606
80	0.2736	0.408	0.504	0.549
85	0.2615	0.381	0.463	0.501
90	0.2501	0.357	0.428	0.461
100	0.2298	0.318	0.374	0.397
110	0.2134	0.287	0.332	0.350
120	0.1998	0.263	0.300	0.316
130	0.1893	0.245	0.276	0.289
140	0.1806	0.231	0.258	0.270
150	0.1743	0.220	0.246	0.256
160	0.1699	0.214	0.238	0.248
170	0.1674	0.210	0.233	0.240
180	0.1665	0.208	0.211	0.239

(Computation by BILLINGS⁽¹⁷⁾)

The Compton differential cross-section per electron $C(\theta) \cdot d\Omega$ —the probability that an unpolarized photon of initial energy γ_0 should be scattered through an angle θ into a solid angle $d\Omega$ —is given by the Klein-Nishina formula

$$C(\theta) d\Omega = r_0^2 d\Omega \frac{1 + \cos^2\theta}{2} \frac{1}{[1 + \gamma_0(1 - \cos \theta)]^2} \times \left\{ 1 + \frac{\gamma_0^2 (1 - \cos \theta)^2}{(1 + \cos^2\theta)[1 + \gamma_0(1 - \cos \theta)]} \right\} \quad (2.5.2)$$

The differential cross-section per electron predicted by this formula is shown in Fig. 2.5.2 as a function of θ . For high-energy photons the scattering is predominantly forward, but as the low-energy X-ray region is approached, the

Klein-Nishina formula goes over smoothly into the $(1 + \cos^2 \theta)$ dependence predicted by the classical Thomson formula. The latter expression is derived by considering (i) the oscillation of a free electron under the oscillating accelerating force of the incident electromagnetic wave, and (ii) the energy which such an oscillating electric current will necessarily "reradiate," according to the classical laws of electro-dynamics.

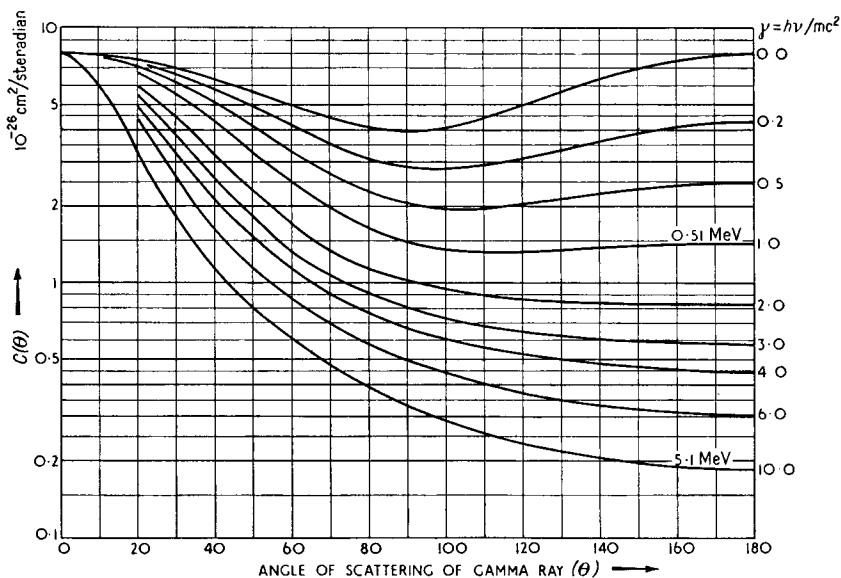


Fig. 2.5.2. Compton differential scattering cross-section per electron as a function of energy and angle of scattering.

The total Compton scattering cross-section per electron for unpolarized radiation, ζ_C , (obtained by integrating equation (2.5.2) over all angles) is equal to

$$\zeta_C = 2\pi r_0^2 \left\{ \frac{1 + \gamma_0}{\gamma_0^3} \left[\frac{2\gamma_0(1 + \gamma_0)}{1 + 2\gamma_0} - \ln(1 + 2\gamma_0) \right] + \frac{1}{2\gamma_0} \ln(1 + 2\gamma_0) - \frac{1 + 3\gamma_0}{(1 + 2\gamma_0)^2} \right\}. \quad (2.5.3)$$

For high-energy gamma rays (i.e. $\gamma_0 \gg 1$) this reduces to

$$\zeta_C \approx \frac{\pi r_0^2}{\gamma_0} (\ln 2\gamma_0 + \frac{1}{2}) \quad (2.5.3a)$$

Fig. 2.5.3 shows that over a considerable range of gamma ray energies, which includes much of the radiation emitted by a nuclear reactor, the Compton cross-section predominates over the photoelectric and pair production cross-sections, for all except the very heaviest elements. The Compton process is an electronic process, and since equal masses of all materials except hydrogen

contain roughly the same number of electrons ($\approx 3 \times 10^{23}$) per gram, it follows that in this energy region equal masses of all materials will be about equally effective as gamma ray attenuators.

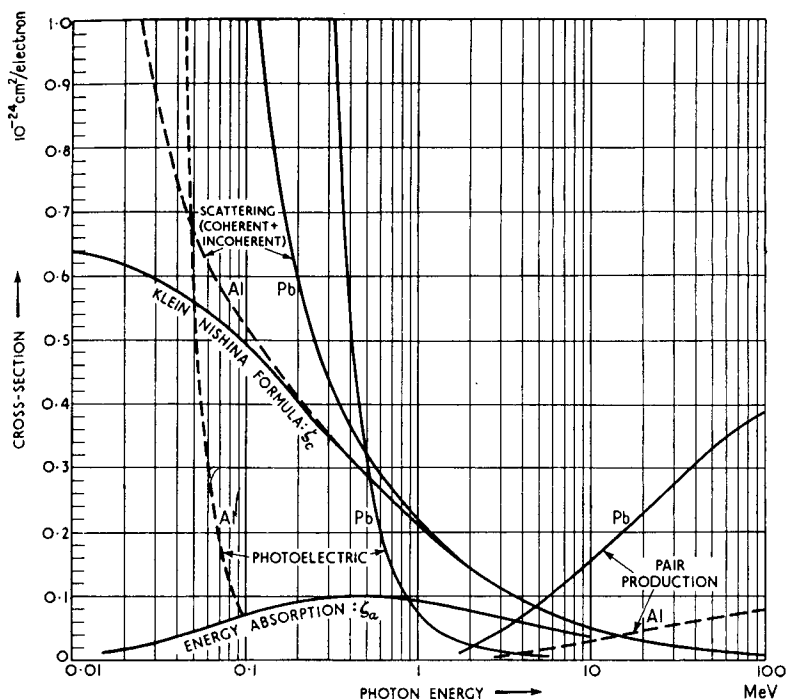


Fig. 2.5.3. Scattering and absorption cross-sections per electron for light and heavy elements as a function of photon energy. (From WHITE⁽¹⁰⁾.) Coherent and incoherent scattering are discussed on page 30.

In dosimetry calculations use is frequently made of the Compton energy absorption cross-section per electron ζ_a , which expresses the probability of transfer of energy from a photon to an electron by the Compton process. It is equal to $\zeta_c \cdot f$, where f is the fraction of the gamma ray energy which is converted to kinetic energy of the recoil electrons in a single collision, averaged over all directions of electron recoil. Since the range of the recoil electrons is small (except in gases or for high-energy gamma radiation), ζ_a is a measure of the total energy communicated locally to the absorbing medium by the Compton process. It may be shown by integration of the Klein-Nishina equation that

$$\zeta_a = \pi r_0^2 [(\gamma_0^{-1} - 2\gamma_0^{-2} - 3\gamma_0^{-3}) \ln(1 + 2\gamma_0) - 8\gamma_0^2/3(1 + 2\gamma_0)^3 + (6 + 22\gamma_0 + 18\gamma^2 - 2\gamma_0^3)/(\gamma_0^2(1 + 2\gamma_0)^2)] \quad (2.5.4)$$

Free-electron values of ζ_c , ζ_a and f are given in Table 2.5.2.

By making a similar correction to the photoelectric and pair-production

Table 2.5.2. Scattering Cross-section and Energy Absorption Cross-section for Free Electrons, as given by the Klein-Nishina Formula

Photon energy (MeV)	Total scattering cross-section ζ_{σ} (barns per electron)	Energy absorption cross-section ζ_a (barns per electron)	$f = \zeta_a/\zeta_{\sigma}$
0.010	0.637	0.0077	0.012
0.015	0.627	0.0138	0.022
0.020	0.616	0.0196	0.032
0.030	0.596	0.0295	0.049
0.040	0.578	0.0380	0.066
0.050	0.561	0.0451	0.080
0.060	0.546	0.0509	0.093
0.080	0.517	0.0610	0.118
0.100	0.4929	0.0685	0.139
0.150	0.4436	0.0812	0.183
0.200	0.4066	0.0886	0.218
0.300	0.3535	0.0985	0.271
0.400	0.3167	0.0982	0.310
0.500	0.2892	0.0986	0.341
0.600	0.2675	0.0984	0.368
0.800	0.2350	0.0959	0.408
1.0	0.2112	0.0929	0.440
1.5	0.1716	0.0849	0.495
2.0	0.1464	0.0777	0.531
3.0	0.1151	0.0664	0.577
4.0	0.0960	0.0582	0.606
5.0	0.0828	0.0519	0.627
6.0	0.0732	0.0471	0.643
8.0	0.0599	0.0399	0.666
10	0.0510	0.03487	0.684
15	0.03773	0.02679	0.710
20	0.03024	0.02201	0.728
30	0.02199	0.01643	0.747
40	0.01746	0.01327	0.760
50	0.01456	0.01121	0.770
60	0.01254	0.00973	0.776
80	0.00988	0.00776	0.785
100	0.00820	0.00651	0.794

(From WHITE⁽¹⁰⁾)

cross-sections, to allow for the energy re-emitted as fluorescent or annihilation radiation, we may derive an *energy absorption cross-section per atom* σ_a :

$$\sigma_a = Z \cdot \zeta_a + \sigma_{PP} \left(\frac{h\nu - \Phi I}{h\nu} \right) + \sigma_{PE} \left(\frac{h\nu - 2mc^2}{h\nu} \right) \quad (2.5.5a)$$

The quantity σ_a is sometimes called the true absorption cross-section.

The energy absorption cross-section as defined by equation (2.5.5a) is a measure of the energy communicated to the electrons of the medium only as long as the absorber has dimensions which are small compared with the mean free path of the annihilation and fluorescent radiations. The fluorescent radiation in particular has a very small penetrating power, and is re-absorbed close to the

Table 2.5.3. Mass Absorption Coefficient χ , Mass Energy Absorption Coefficient χ_a , and the Corresponding Linear Absorption Coefficients μ and μ_a for Air at 20°C

Composition of air assumed for calculation: 78% N₂, 21% O₂, 1% A.

Photon energy (MeV)	χ (cm ² /g)	χ_a (cm ² /g)	μ (cm ⁻¹)	μ_a (cm ⁻¹)
0.01	4.55	4.20	54.9×10^{-4}	50.7×10^{-4}
0.015	1.45	1.18	17.5	14.3
0.02	0.712	0.479	8.59	5.77
0.03	0.335	0.139	4.04	1.67
0.04	0.239	0.0616	2.88	0.742
0.05	0.203	0.0376	2.45	0.453
0.06	0.185	0.0287	2.23	0.346
0.08	0.166	0.0236	2.00	0.284
0.10	0.155	0.0232	1.87	0.279
0.15	0.136	0.0251	1.64	0.302
0.20	0.123	0.0269	1.48	0.324
0.30	0.107	0.0288	1.29	0.347
0.40	0.0953	0.0295	1.15	0.356
0.50	0.0869	0.0296	1.05	0.357
0.60	0.0804	0.0296	0.969	0.356
0.80	0.0706	0.0288	0.851	0.348
1.0	0.0635	0.0279	0.766	0.336
1.5	0.0516	0.0256	0.622	0.309
2.0	0.0443	0.0237	0.543	0.286
3.0	0.0357	0.0211	0.430	0.254
4.0	0.0307	0.0193	0.369	0.233
5.0	0.0275	0.0182	0.331	0.219
6.0	0.0252	0.0174	0.304	0.209
8.0	0.0222	0.0162	0.267	0.195
10	0.0204	0.0155	0.246	0.187
15	0.0180	0.0147	0.217	0.177
20	0.0169	0.0145	0.204	0.175
30	0.0163	0.0146	0.196	0.176
40	0.0160	0.0148	0.193	0.179
50	0.0161	0.0151	0.194	0.182
60	0.0162	0.0154	0.195	0.185
80	0.0166	0.0160	0.199	0.192
100	0.0169	0.0164	0.204	0.197

(From WHITE⁽¹⁰⁾)

point of origin, so that a better indication of the local energy deposition in thick absorbers would be given by the somewhat larger quantity

$$\sigma_a' = Z \cdot \zeta_a + \sigma_{PP} + \sigma_{PE}. \quad (2.5.5b)$$

Examples may be found in the literature of the use of both these expressions for dosimetry calculations. The difference between them is not large for materials such as air or tissue, which are composed mainly of light elements, since the fluorescent yield Φ is then extremely small. For heavy elements, however, for which the fluorescent yield approaches unity, the two quantities are quite

different for low-energy radiation, and care must be taken to select whichever is appropriate. The energy absorption coefficients listed in Table 2.5.3 were calculated using the expression (2.5.5b).

We have so far made the assumption that the electrons taking part in the Compton process are unbound. This is an oversimplification which breaks down^(1, 10, 18) at low energies, for then the motion imparted by the photon to the electron is not large compared with its initial motion within the atom. The scattered photon then no longer has a unique energy (for a given angle of scattering), but may have any energy lying within a distribution peaked at a value close to that for free-electron scattering. In addition, some part of the radiation will now be scattered by electrons which behave as though they were bound. The substantial loss of energy which is the most important characteristic of the Compton process is primarily due to the small mass of the electron; had this mass been greater a smaller electron recoil energy would have been permitted by the energy and momentum conservation laws, and the scattering would have been more nearly without change of photon wavelength. Since bound electrons effectively have the same mass as that of the whole atom, they are incapable of taking up more than a small fraction of the photon energy, and the scattering will be to all intents and purposes without change of wavelength. Moreover, because all the atomic electrons will behave similarly in this respect, the radiation scattered by the individual bound electrons of a given atom will be *coherent*, that is, capable of showing constructive or destructive interference. This behaviour contrasts strongly with the free-electron Compton scattering, where the independent behaviour of the electrons precludes any interference effects; free-electron scattering is therefore referred to as *incoherent*. The transition from free-electron scattering to scattering from bound electrons without appreciable energy-loss proceeds smoothly as the energy of the incident photon is reduced (the intensity of the incoherent portion of the radiation gradually decreasing, and the coherent radiation becoming correspondingly more pronounced). As the electrons are more tightly bound in a heavy atom than in a light one, this transition sets in at higher gamma ray energies the higher is the atomic number. For lead, about 10% of the scattering of 0.5 MeV photons is coherent;⁽¹⁰⁾ for iron the same fraction is not reached until the gamma ray energy is below 150 keV. The loss of stopping power due to the partial disappearance of incoherent scattering is of little practical importance in shielding, since it occurs only in a relatively low-energy region, where photoelectric absorption is beginning to predominate.

Other types of scattering have a practical importance which is insignificant compared with that of Compton scattering. There are three types of scattering in which the atom can recoil as a whole, and in which the scattering is therefore substantially without change of energy:⁽¹⁹⁾

- (a) scattering by bound electrons, without the ejection of any electron from the atom (Rayleigh scattering),
- (b) nuclear Thomson scattering (barely detectable),
- (c) nuclear resonance scattering (extremely small cross-section).

Rayleigh scattering is the name given to the coherent type of scattering mentioned in the previous paragraph, which is essentially forward scattering.

For small angles of scatter the differential Rayleigh cross-section per unit solid angle can be one or two orders of magnitude greater than the differential atomic Compton cross-section, the exact ratio between the two depending on the gamma ray energy and the atomic number of the scatterer (Fig. 2.5.4). However,

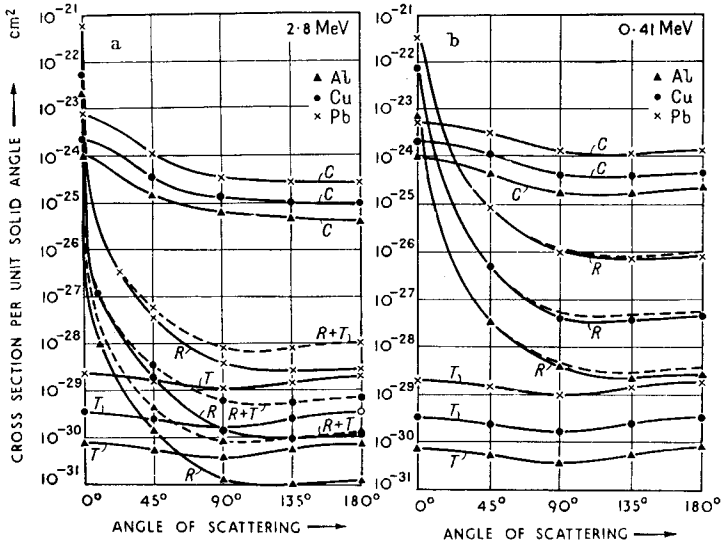


Fig. 2.5.4. Calculated cross-sections for scattering of 2.8 MeV and 0.41 MeV gamma-rays by aluminium, copper, and lead. C = Compton scattering. T = Thomson scattering. R = Rayleigh scattering. (After MOON⁽¹⁹⁾.)

since the Rayleigh cross-section is large only for a very small range of angles, the *total* Rayleigh cross-section σ_{Rayleigh} is very much smaller than the total Compton cross-section per atom. The ratio between the two is given by the approximate formula

$$\frac{\sigma_{\text{Rayleigh}}}{Z \cdot \zeta_C} \approx 1.1 \times 10^{-5} \frac{Z^{5/3}}{E_0} \quad (E_0 \text{ in MeV}) \quad (2.5.6)$$

As far as shielding calculations are concerned Rayleigh scattering is unimportant, since for all practical purposes the additional elastic scattering of photons through such small angles makes no difference to the numbers penetrating the shield.

Nuclear Thomson scattering. The cross-section for nuclear Thomson scattering may be obtained by inserting the nuclear mass M and charge Z instead of the electronic mass and charge in Thomson's classical formula. The total cross-section, $8\pi Z^4 e^4 / 3M^2 c^4 \approx 2(Z^4/A^2) \times 10^{-7}$ barns, is negligible compared with the Rayleigh and Compton cross-sections. This type of scattering is coherent with Rayleigh scattering; constructive interference between the two types of radiation would give rise to a small change in the effective cross-section by an amount indicated by the pecked lines in Fig. 2.5.4.

Nuclear resonance scattering, the nuclear analogue of the phenomenon of fluorescence, has a very low probability, and for all practical purposes can be ignored. Among the other processes which are too improbable to have any influence on shielding considerations are: (i) Delbruck scattering, the scattering of photons by the electric fields surrounding charged particles, (ii) The Double Compton process, in which the scattering of an incident photon by an electron is accompanied by the formation of a second photon, (iii) meson production by high-energy gamma radiation.

Before concluding the discussion of the basic gamma-ray absorption processes reference must be made to photoneutron production—the ejection of a neutron from a nucleus by a high-energy photon. The cross-section for the process is too small for it to make a significant contribution to the rate at which gamma rays are attenuated: even for heavy nuclei and radiation of the order of 20 MeV the cross-section is only about one barn. However, the process is of practical importance owing to its use as a neutron source in electron accelerator targets, and it will be in this connection that we shall refer to it again in Chapter 3. As far as the shielding of nuclear reactors is concerned it can usually be disregarded; for although it is true that it provides a source of penetrating neutrons in the material of the shield itself, the strength of this source is nearly always negligible compared with the neutrons leaking through the shield from the reactor core.

INTERACTIONS OF GAMMA RAYS WITH MATTER IN BULK

2.6. THE ATTENUATION OF NARROW BEAMS OF GAMMA RAYS

The rate at which gamma rays are attenuated is determined primarily by the atomic number and density of the shielding material, and to a lesser extent by the geometrical configuration. For reasons which will become clear as the discussion proceeds, the case of most practical importance—the illumination of a large area of an extended thick shield by an uncollimated beam of photons—is also the most difficult to treat theoretically. It is simpler to take as a starting-point the collimated, narrow-beam laboratory type of experiment, from which

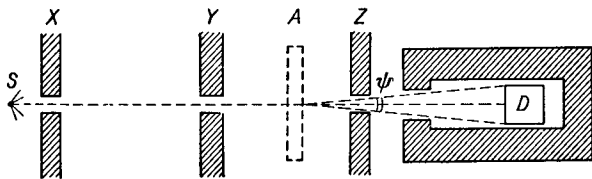


Fig. 2.6.1. Typical narrow-beam or "good-geometry" absorption measurement.

the mass absorption coefficients given in Fig. 2.6.4 are derived. The term "good geometry" is frequently used to describe such experiments, and "bad geometry" to denote broad beam conditions. Fig. 2.6.1 shows a typical arrangement for a good-geometry absorption coefficient measurement. Gamma rays from a source S are collimated into a narrow beam by the lead stops X , Y , and Z , and are recorded by a detector D , which is surrounded by shielding to

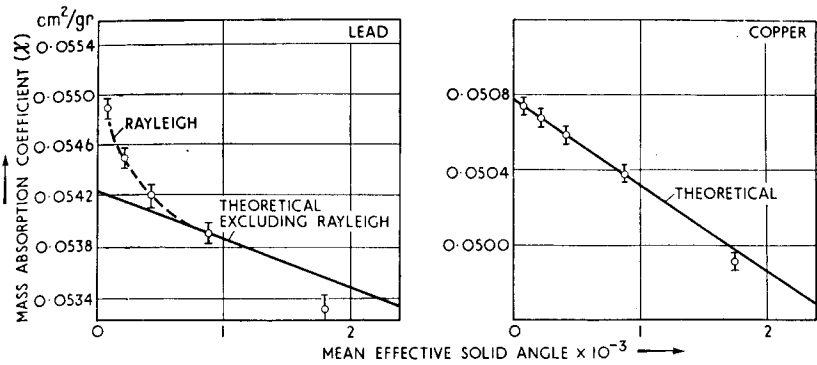


Fig. 2.6.2. Dependence of measured mass-absorption coefficient on the solid angle subtended by the detector, for Co^{60} (≈ 1.2 MeV) gamma rays. (After COLGATE⁽²⁰⁾.)

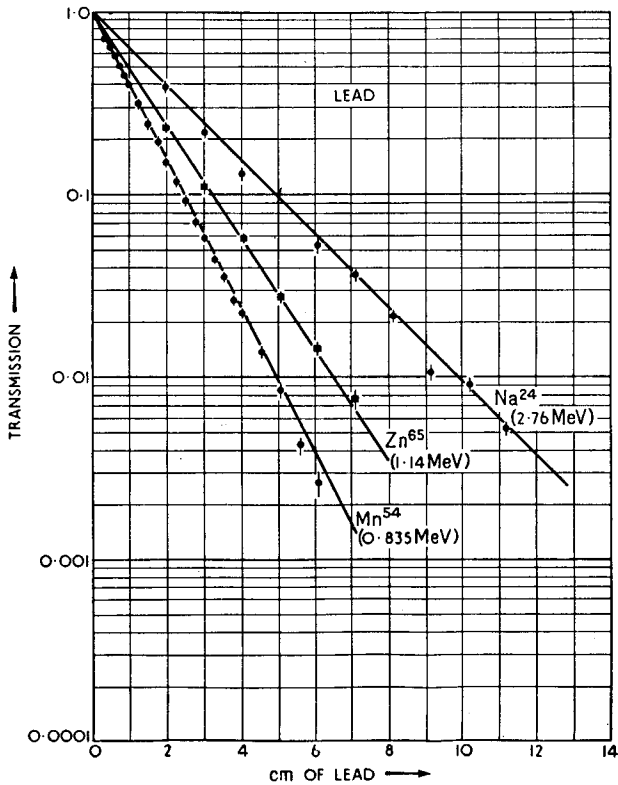


Fig. 2.6.3. Experimental attenuation in lead of gamma rays of various energies, for narrow-beam conditions. (After DAVISSON and EVANS⁽²¹⁾.)

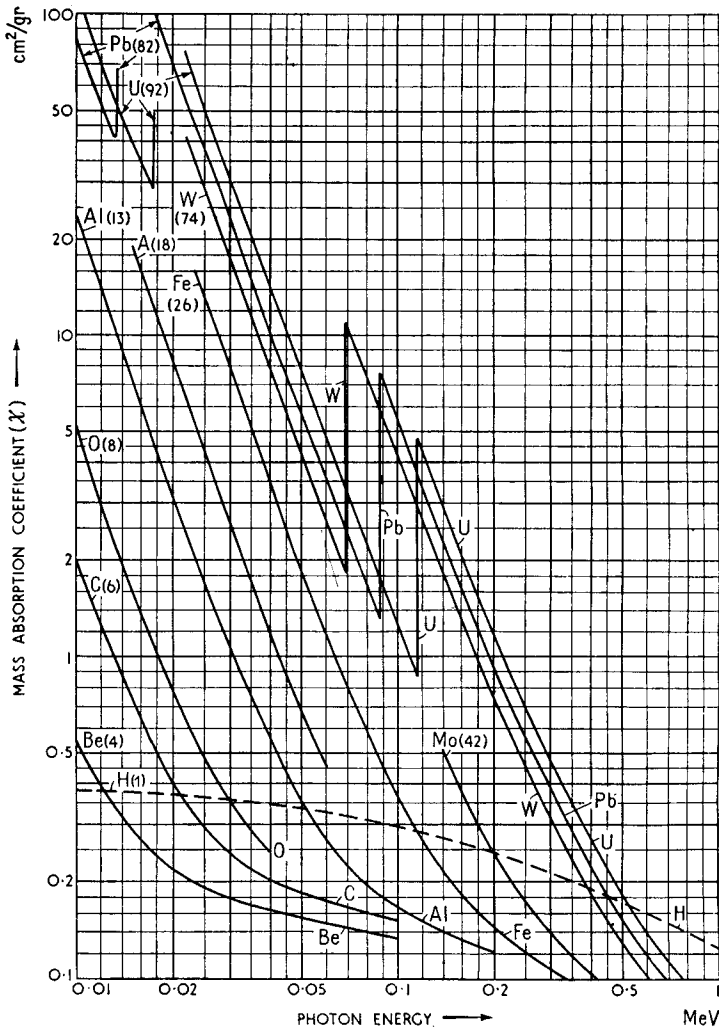


Fig. 2.6.4(a).

prevent it “seeing” any unwanted background radiation. The absorber being studied, A , is then interposed, and the corresponding reduction in intensity measured. Provided the angle ψ subtended at A by the detector is extremely small, one can assume that all photons that have been scattered by the absorber will miss the detector. The observed atomic total cross-section σ_{tot} is then the sum of the component atomic cross-sections:

$$\sigma_{\text{tot}} = \sigma_{PP} + \sigma_C + \sigma_{PE} + \sigma_{\text{Rayleigh}} \quad (2.6.1)$$

where

$$\sigma_C = Z \cdot \zeta_C$$

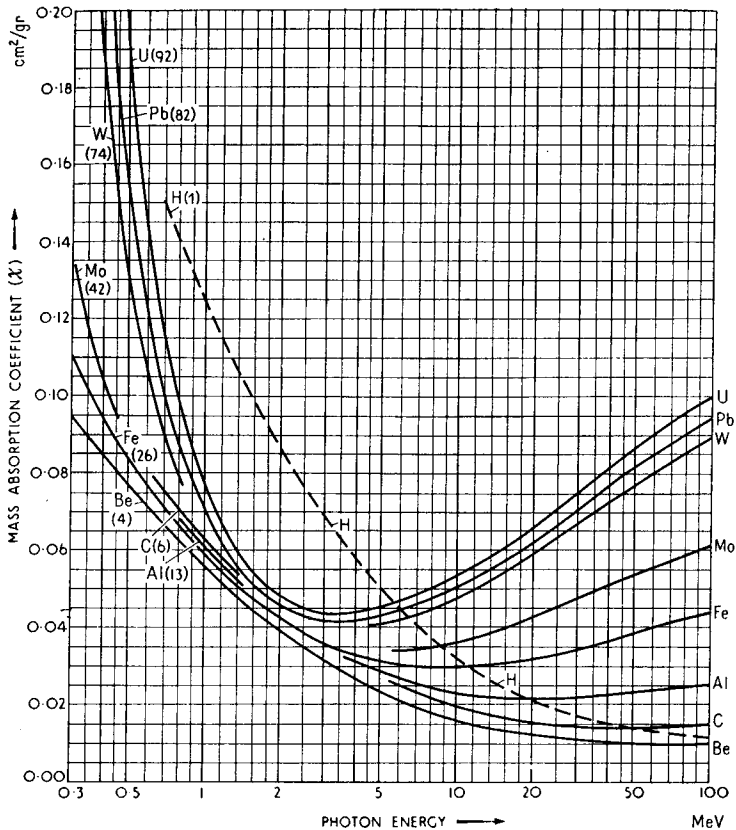


Fig. 2.6.4(b).

Fig. 2.6.4. Narrow-beam mass absorption coefficients as a function of atomic number and photon energy. Mass energy absorption coefficients are given in Fig. 2.6.6. (Drawn from tabulations by WHITE^{(10).})

In practice, the degree of collimation which can be used is limited by the necessity for an adequate intensity at the detector. As ψ increases from zero, and the experimental arrangement departs more and more from the ideal narrow-beam geometry, some of the radiation scattered through small angles by the absorber A will no longer miss the detector, but will be counted. The observed cross-section will then be lower than for the ideal narrow-beam case. This is illustrated in Fig. 2.6.2, which shows how the determination of the ideal good-geometry absorption coefficient depends on the extrapolation of the results to zero solid angle. However, in performing this extrapolation, one may justifiably ignore the rapid change of slope at very small values of ψ (which is due to the failure of Rayleigh-scattered photons to reach the detector), since, as already mentioned, for all practical purposes Rayleigh scattering has no effect

on the attenuation of gamma radiation. In any case it introduces only a small correction, which is never more than 6% even in lead. In what follows its existence will be ignored, except where otherwise stated.

For the case of perfect geometry the attenuation is found to be accurately exponential:

$$\phi(x) = \phi_0 e^{-\mu x} \quad (2.6.2)$$

where ϕ_0 is the incident intensity, $\phi(x)$ is the intensity observed through an absorber of thickness x , and $\mu \text{ cm}^{-1}$ is the linear absorption coefficient defined in Section 2.2. In the case of a mixture we must use the mean absorption coefficient, $\bar{\mu} = \sum_r (\mu_r F_r)$, where F_r is the fraction of the total volume occupied by the r th component of the mixture. The results of some typical measurements illustrating this exponential attenuation are given in Fig. 2.6.3.

Values of the mass absorption coefficient $\chi = \mu/\rho$, correct to about 3%, are plotted in Fig. 2.6.4. The curves show the general behaviour to be expected from the preceding discussion. At low energies the removal of photons from the beam is mainly due to the photoelectric effect, which becomes more important as the atomic number Z of the absorber is increased, and as the quantum energy goes down. At high energies, and for high Z , pair production predominates. In between there is a region in which the Compton scattering process is the most important, and in which equal masses of all materials are therefore about equally effective. This region, (0.5 to 10 MeV approximately), happens to be the one which is most important in reactor technology, for falling within it are the penetrating radiations from fission, capture, and inelastic scattering, as well as from the decay of many fission products. Over most of the energy range covered in Fig. 2.6.4 the variation of χ with atomic number is smooth, and values for elements not shown in the figures can be obtained by interpolation. However, this is not the case for high atomic number and low gamma ray energy, where interpolation is rendered very uncertain by the sharp discontinuities in the photoelectric part of the cross-section; if accurate values are required, reference should be made to the extensive tabulations of ALLEN,⁽⁷⁾ LATYSHEV,⁽²²⁾ and VICTOREEN,⁽⁹⁾ or to WHITE's review article.⁽¹⁰⁾ For heavy elements it will be noticed that there is a fairly sharp minimum in the mass absorption cross-section at about 3 MeV, while for light elements this minimum occurs only at very high energies. As we shall see in the next section the attenuation of broad beams of radiation depends somewhat on whether the incident gamma ray has an energy lying above or below this minimum. The apparently anomalous behaviour of hydrogen (shown as a dotted line in Fig. 2.6.4) is due to the fact that it contains about twice as many electrons, weight for weight, as any other material; its Compton scattering cross-section per unit mass is therefore also twice as great. It behaves as a nearly pure Compton scatterer, since its atomic number is so small that the contribution from the photoelectric effect at 0.01 MeV is only 1% of the total, while pair production amounts to no more than 6% at 10 MeV.

Fig 2.6.5 presents the information already given in Fig. 2.6.4 in a form more suited to rapid calculation.

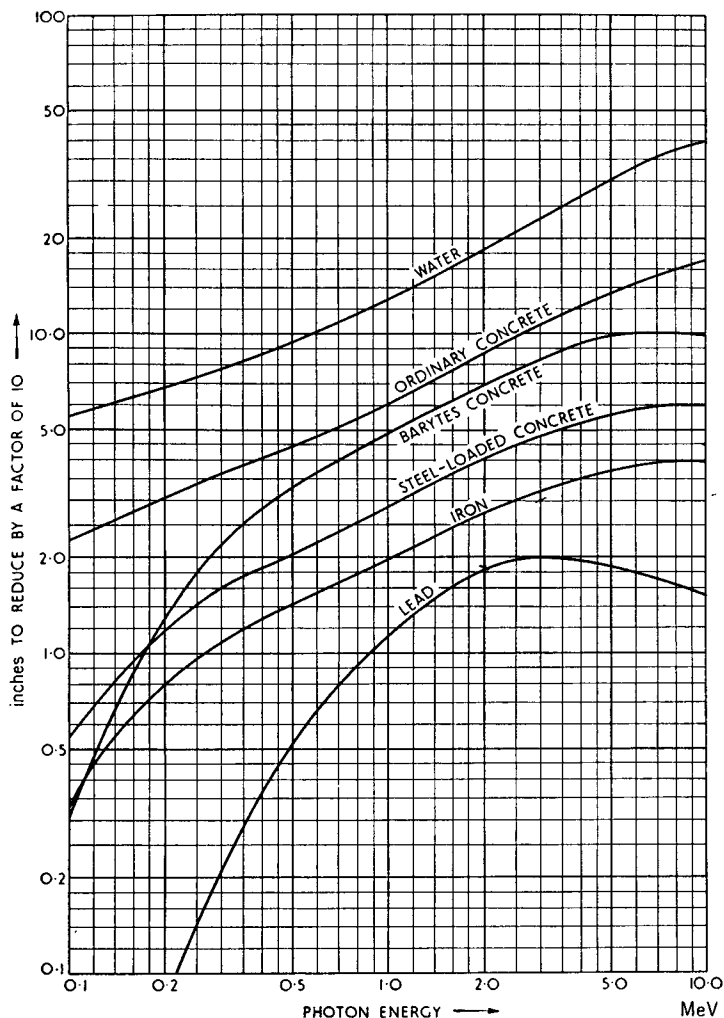


Fig. 2.6.5. Ten-folding lengths for common shielding materials; e -folding lengths may be obtained by dividing by $\ln_e 10 = 2.306$.

The curves are based on narrow-beam coefficients and do not include any allowance for multiple scattering. They were calculated assuming the following densities and compositions:

Shot concrete: Density 5.3 g cm^{-3} (79.5% Fe by weight)

Ordinary concrete: Density 2.35 g cm^{-3} . Composition: (wt. per cent)

Ca: 8.6, Si: 35.8, Fe: 1.2, Al: 2.0, Na: 0.33, H: 0.63, C: 0.4, O: 51.1

Barytes concrete: Density 3.1 g cm^{-3} . Composition: Ba: 35.8, Ca: 7.4, S: 9.0, Si: 8.9, Fe: 1.5, H: 0.44, C: 1.1, O: 35.4.

Lead: Density 11.4 g cm^{-3}

Iron: Density 7.8 g cm^{-3}

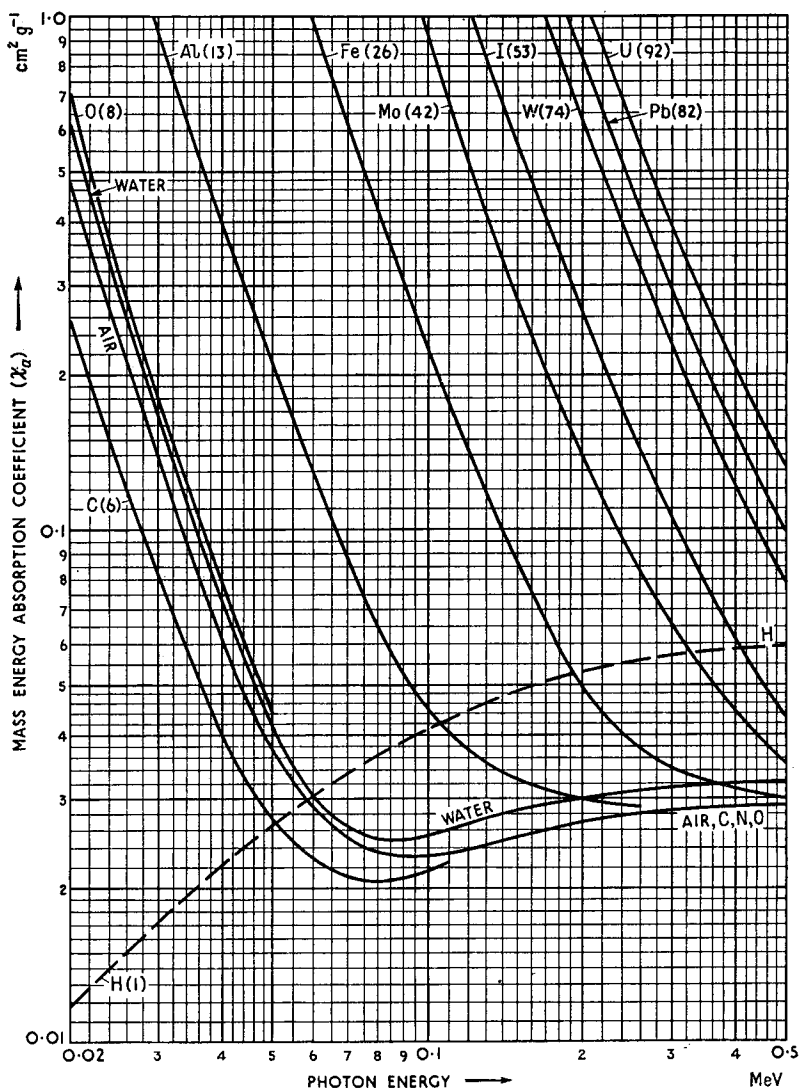


Fig. 2.6.6(a).

Fig. 2.6.6. Mass energy absorption coefficients χ_a as a function of photon energy

At high energies the coefficients have been calculated using equation (2.5.5b). At lower energies a correction for the coherent part of the total scattering cross-section has been included for $Z > 26$. For most purposes the value of χ_a for tissue may be assumed to be equal to that for water. (From tabulations by WHITE⁽¹⁰⁾ and SNYDER and POWELL⁽¹⁰⁴⁾.)

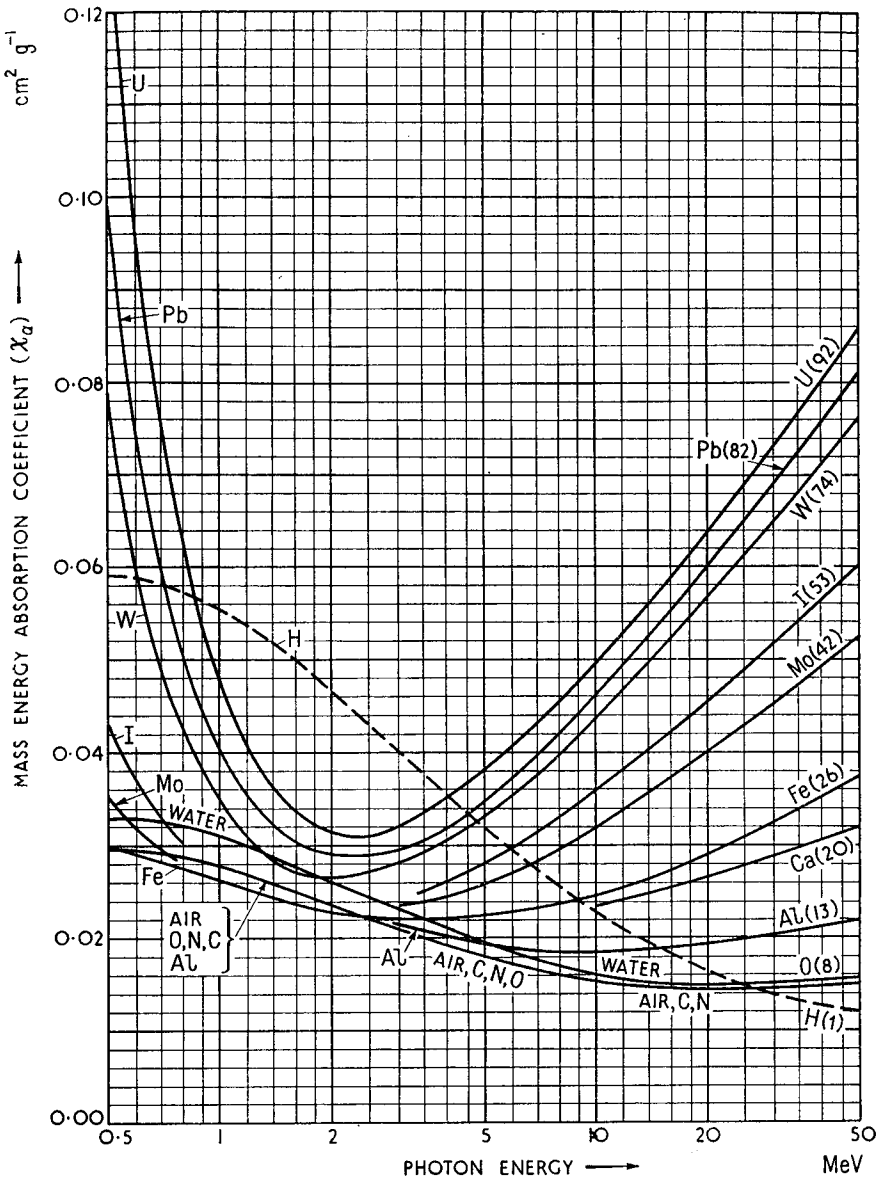


Fig. 2.6.6(b).

2.7. THE ATTENUATION OF BROAD BEAMS OF RADIATION; BUILD-UP FACTORS

Two of the three main processes discussed above—the photoelectric effect and pair production—effectively represent pure absorption of the gamma radiation, since the secondary radiation to which they later give rise is always much softer than the primary radiation. If these were the only processes, there would be no difficulty in applying the results of good-geometry measurements

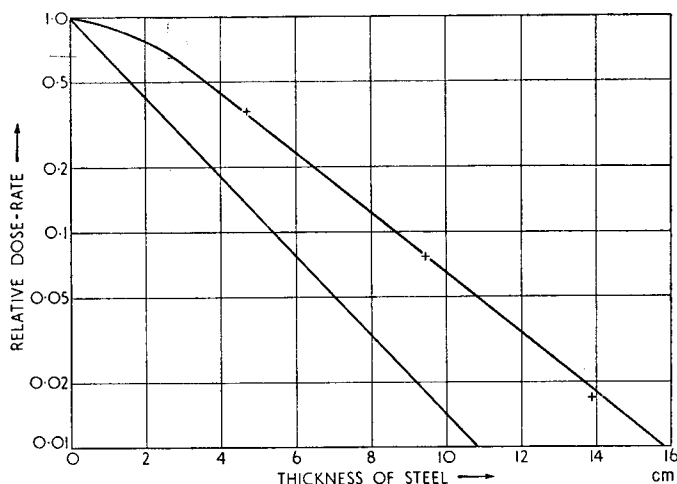


Fig. 2.7.1. Relative dose-rate from a plane isotropic source of Co^{60} gamma rays (≈ 1.2 MeV), when observed through a plane steel slab absorber under broad-beam conditions. The upper curve was observed by HALMSHAW and KNAPP.⁽⁴⁰⁾ The lower curve is calculated on the basis of narrow-beam absorption coefficients, without allowance for build-up, using the data of Fig. 2.6.4. The crosses were calculated by making an approximate allowance for build-up, based on the data for plane collimated sources given in Fig. 2.7.3(b).

to the case of broad beams of radiation. It is the existence of the third process, Compton scattering, that complicates the issue, for the scattered photons which are not counted in good-geometry experiments still have to be reckoned with in shielding problems. Moreover, Compton scattering of high-energy gamma rays is largely forward scattering (Fig. 2.5.2), so that the scattered photon may continue with its direction and penetrating power not greatly changed. A considerable number of scattering collisions may be needed before the energy of the photon is sufficiently reduced for it to be photoelectrically absorbed. If a broad beam of gamma rays is normally incident on a slab of absorber, the total amount of radiation transmitted will therefore be greater than that given by equation (2.6.2).

The results shown in Fig. 2.7.1 are typical of those obtained when the incident radiation is monoenergetic, and has an energy E_0 below the energy $E_{\min}$ at which the material is least absorbant (see Fig. 2.6.4). The curve relating the logarithm of the attenuation to the thickness is not a straight line, as it would be in the

narrow-beam case, but has a slope which becomes progressively steeper as the depth of penetration increases. The slope is everywhere less than would be expected on the basis of good-geometry attenuation coefficients, particularly in the first few mean free paths. As the distance into the shield increases, the curve becomes almost a straight line with a slope which is appreciably different from the corresponding narrow-beam case. The lower slope of the initial part of the curve corresponds to the portion of the shield in which the energy spectrum, or quality, of the radiation is being modified by Compton scattering (Fig. 2.7.2). For light-element absorbers the slope of the ultimate straight-line section may differ by as much as 50% from the initial slope, so that a single value of half-value-thickness (HVT), without a further specification of whether it is the first, second etc. HVT, may be misleading. Some authors give values corresponding to the initial slope, while others prefer to deal with the linear portion; care must be taken when using published figures to verify which is meant.

When $E_0 > E_{\min}$, the photons which most easily penetrate the shield will be those which have been degraded to energies in the region of $E_{\min}$. The slope which the attenuation curve finally approaches at great depths is then the value appropriate to photons of energy $E_{\min}$, rather than the photons of the incident energy E_0 . When the incident radiation is not monoenergetic these general trends may be greatly modified by the filtering action of the shield.

It is convenient to describe the difference between the results of measurements in narrow- and broad-beam cases by means of *build-up factors*. A build-up factor always refers to some measurable property of the photon beam (e.g. intensity, number of photons, energy-flux, or biological dose), and is defined as the ratio of this quantity when the effects of all quanta are included to that obtained when only the uncollided flux is considered. For instance, the dose-rate D , in maximum permissible levels, at a distance r from a point source emitting S photons of energy E_0 MeV per second is related to the thickness of shielding t by the equation

$$D = Be^{-\mu_0 t} \frac{S}{4\pi r^2 \xi}, \quad (2.7.1)$$

where B is the dose build-up factor for a point source (Fig. 2.7.3(a)) and ξ is the factor given in Fig. 1.5.1; the factor $1/4\pi r^2$ allows for the reduction in intensity due to the spreading of the photons as they leave the source.

Since the spectrum of γ -ray energies transmitted by a thick shield is quite different from the incident spectrum (Fig. 2.7.2), and since all radiation detectors are energy sensitive, there will be a different build-up factor for each detector. Which is chosen as standard is purely a matter of convenience. The build-up factor for biological dose, the usefulness of which is obvious, is obtained by weighting the contribution of the photons in proportion to σ_a for tissue (equation 2.5.5). The other important build-up factor is the energy factor, which is required for calculations of the heating effect of gamma rays; it is also of interest for the additional reason that many gamma-ray detectors have sensitivities approximately proportional to the quantum energy. Since biological dose per quantum is also approximately proportional to the quantum energy in the practically important region from 0.5 to 5 MeV (Fig. 1.5.1), the dose and energy factors for this range usually do not differ by more than about 20%–30%.

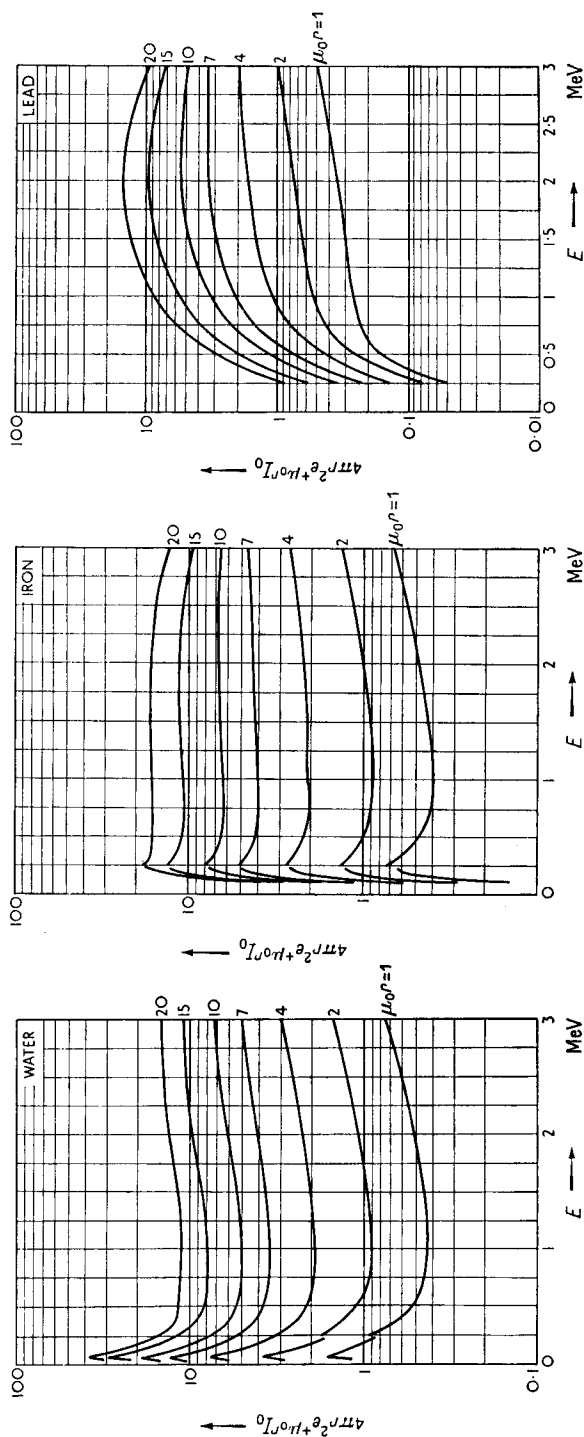


Fig. 2.7.2. Calculated curves illustrating the degradation of energy as photons of initial energy 3 MeV pass through increasing thicknesses of water, iron, and lead. The source is a point isotropic source, emitting one photon per unit time; μ_0 is the linear absorption coefficient for 3 MeV photons, r the barrier thickness, and I_0 the energy flow per unit time per unit area per unit energy interval, integrated over all directions of the scattered quanta. The break in the curve occurs at the lowest energy which can be reached in a single Compton collision. The lower-energy end of the spectrum is cut off by photoelectric absorption, at a much higher energy in lead than in water. (From GOLDSTEIN and WILKINS⁽²⁶⁾.)

Calculation of Build-up Factors

Although the physical laws which govern the interaction of gamma rays with matter are well understood, and the relevant cross-sections are mostly known experimentally with sufficient accuracy for shielding applications, the calculation of build-up factors and other quantities depending on the multiply scattered radiation is still a difficult problem. This is due to the particular mathematical form of the Klein-Nishina equation, and the resultant complexity of the Compton scattering term in the "transport equation." This integro-differential equation—describing the number of gamma rays in a given energy interval with directions lying within a given solid angle which enter and leave a volume element per unit time—proves to be incapable of exact solution, and consequently a number of approximate methods⁽²³⁾ for dealing with the problem have had to be developed. Some of these methods rely on the simplifying effects of approximations to the physical laws, while others use numerical techniques to solve the proper transport equation exactly in certain restricted regions of phase space. The latter approach is usually to be preferred, since the results are more accurate, and less liable to uncertainties as to their validity.

The method which has proved to be the most useful and the only one which will be described in any detail, is the *moment* or *polynomial* method of SPENCER, FANO, GOLDSTEIN, and WILKINS.⁽²³⁻²⁶⁾ It provides a means of calculating the flux of multiply scattered quanta in an infinite medium as a function of both energy and depth of penetration from the source, thus enabling most of the required build-up factors to be determined. The method consists of first expanding the flux in a series of Legendre polynomials in the angular coordinates, and by this means reducing the original equation of three variables (position, energy and direction) to a sequence of coupled integro-differential equations of two variables (position and energy). As a further simplification, these equations are multiplied by powers of x , the spatial variable, and are integrated over all space; in this way one obtains a double sequence of linear integral equations of the Volterra type for the spatial moments of the Legendre coefficients. These equations, which contain only the energy as a continuous variable, are then solved numerically, and various techniques are used to reconstruct the required flux from the moments so obtained.

The method is useful for penetration distances of up to 15 mean free paths and, providing that sufficient machine memory is available, for energy ranges extending from source energies of the order of 10 MeV down to about 50 or 100 keV. In view of the rapid attenuation by the photoelectric effect at lower energies, this range is sufficient for most shielding applications. The errors are greatest in the regions of greatest penetration and lowest energy, but even here the computed energy spectra and build-up factors are thought to be accurate to within 20%. Over most of the range 5% accuracy is claimed. This has been verified in a number of experimental tests,⁽²⁶⁾ covering energies up to 6 MeV and thickness of up to 2 metres in water,^(27, 28) and attenuation by factors of up to 10^5 in iron⁽²⁹⁾ and lead.^(29, 30)

In anticipation of the demand for application of the moment method an extensive series of calculations of energy spectra and build-up factors has been carried out by the American Nuclear Developments Associates Group, using

the SEAC electronic computer. These calculations covered a comprehensive range of materials, source energies, and geometries, so that the majority of shielding problems can be dealt with directly or by suitable interpolation. The build-up factors reproduced here in Fig. 2.7.3 represent no more than a fraction of the total output. The reader wishing for further information is referred to the final report of this work,⁽²⁶⁾ which gives a detailed description of the method and its applications.

Of the other methods of calculating build-up factors for shallow or medium penetrations, several are of limited applicability, and none have been as widely applied as the moment method; they will therefore not be discussed further here. Details will be found in a review article by FANO,⁽⁴²⁾ in the discussions given by GOLDSTEIN and ARONSON,⁽²³⁾ CAVE, CORNER, and LISTON,⁽³²⁾ and PEEBLES,⁽³³⁾ and in various other references in the literature.⁽³⁴⁻³⁹⁾

Build-up factors at great depths. The most serious limitation of the moment method is probably its restriction to penetration distances of the order of 15 mean free paths, corresponding to attenuations of about 10^6 times. Beyond this depth the labour of calculation increases very sharply, and the computation makes excessive demands on machine time. For greater distances SPENCER and FANO⁽³¹⁾ have developed asymptotic methods similar to those used in WICK's⁽⁴¹⁾ treatment of the same problem in neutron transport. It is found that, except for energies very close to that of the source, the energy spectrum of the flux can be expressed in the form

$$V(x) \cdot Y(E, \theta), \quad (2.7.2)$$

where x is the depth of penetration and E and θ denote the energy and direction of the quanta. The function $V(x)$, which describes the spatial variation of the flux, and which can be used to determine the behaviour of most build-up factors of interest, can be expressed in an analytic form. Two cases must be distinguished, depending on the energy variation of the linear absorption coefficient in the shield, and on the energy of the source. Since the cross-section for pair production increases with energy, the total linear absorption coefficient μ has a minimum value at some energy $E_{\min}$, which may lie above or below E_0 , the energy of the primary radiation from the source.

(a) $E_0 < E_{\min}$. In this case the primary quanta will have the greatest ability to penetrate the shield, and

$$V(x) = x^{k-p} e^{-\mu_0 x}, \quad (2.7.3)$$

where μ_0 is the linear absorption coefficient at energy E_0 ,

k is a factor for which values are given in Table 2.7.1 (page 49), and

$p = 0$ for a plane collimated source,

$p = 1$ for a plane isotropic source, and

$p = 2$ for a point isotropic source.

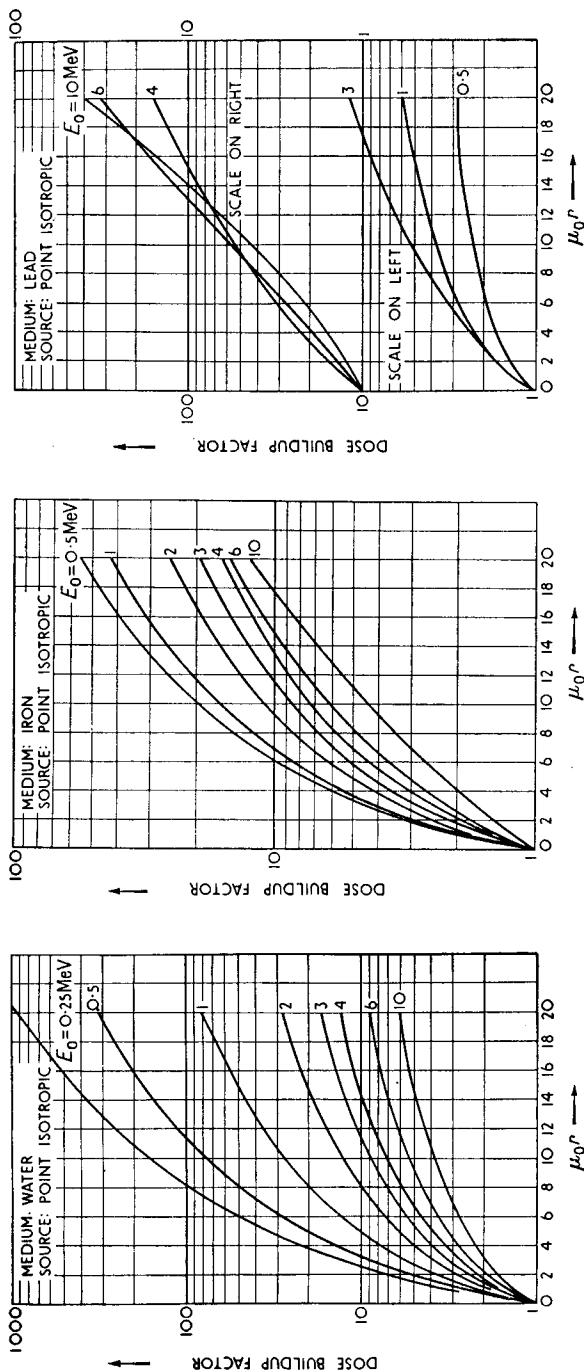


Fig. 2.7.3(a). Dose build-up factors for point isotropic sources as a function of shield thickness r .

Fig. 2.7.3. Build-up factors for plane and point sources in water, iron, and lead, as a function of photon energy and shield thickness; μ_0 is the linear absorption coefficient at the source energy E_0 MeV. Factors for materials of intermediate atomic number can be roughly estimated by interpolation.

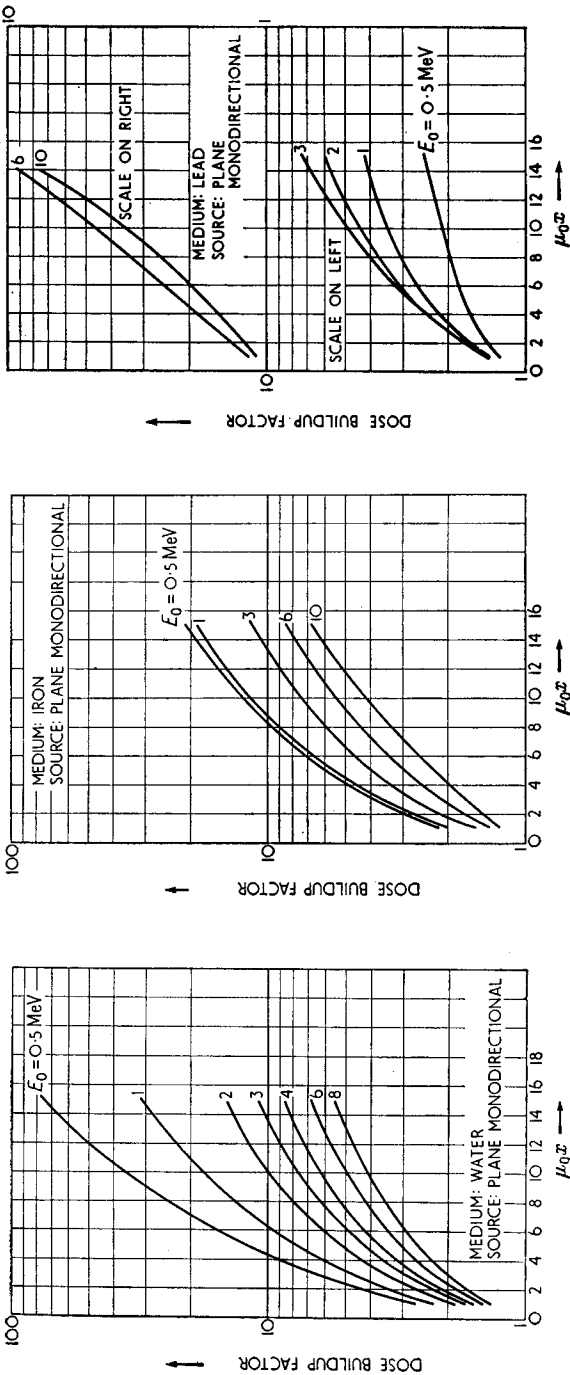


Fig. 2.7.3(b). Dose build-up factors for plane monodirectional sources as a function of shield thickness x .

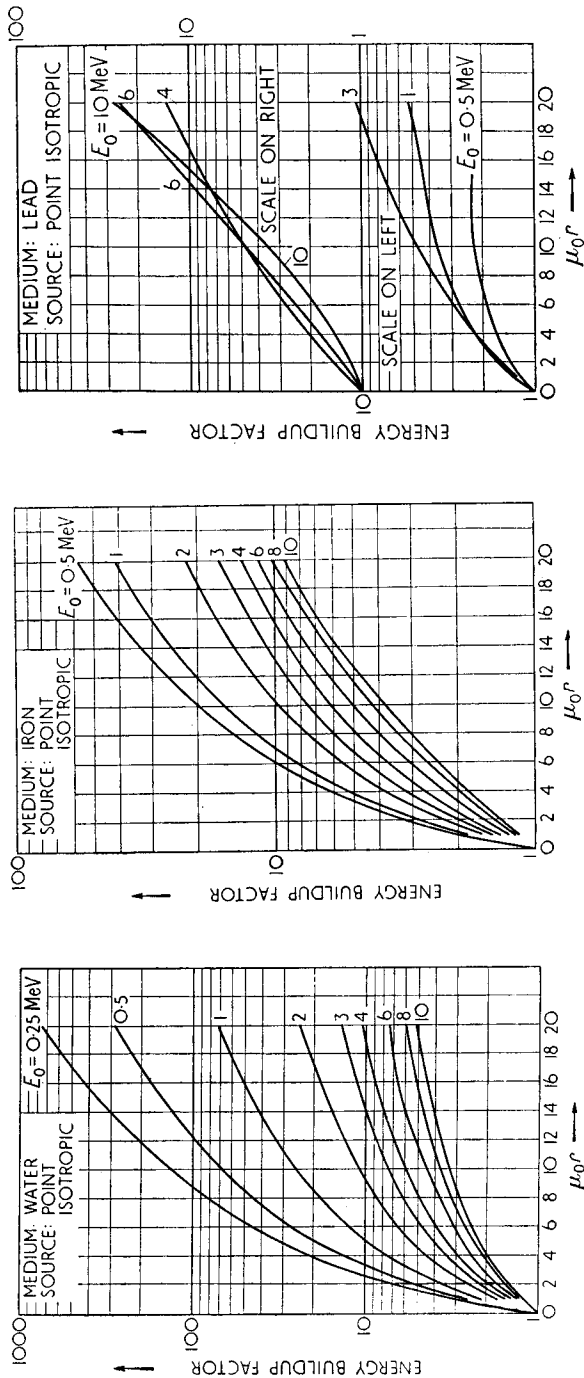


Fig. 2.7.3(c). Energy build-up factors for point isotropic sources as a function of shield thickness r .

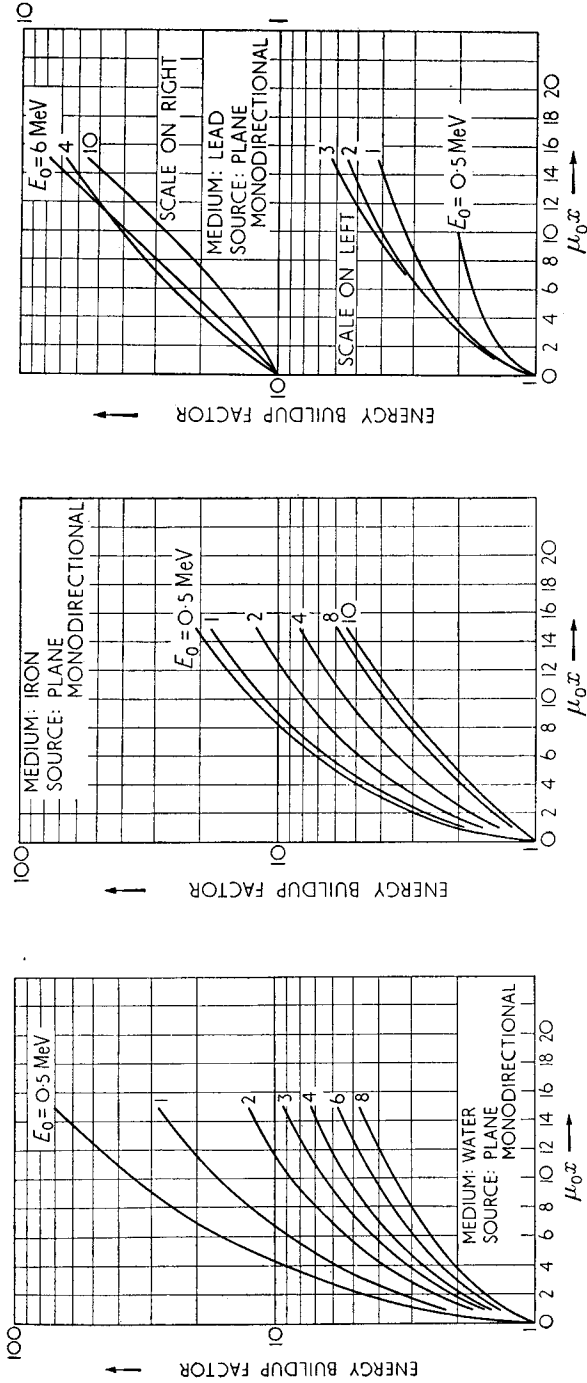


Fig. 2.7.3(d). Energy build-up factors for plane monodirectional sources as a function of shield thickness x .

Table 2.7.1. Values of the Exponent k Appearing in Formula (2.7.3)

Source energy (MeV)	Shielding material					
	H ₂ O	Al	Fe	Sn	Pb	U
10	0.881	1.23				
8	0.900	1.19	6.10			
6	0.920	1.14	2.60			
4	0.980	1.08	1.56	4.39		
3	1.04	1.15	1.36	2.60	4.85	6.45
2	1.17	1.22	1.30	1.54	1.07	0.880
1	1.52	1.52	1.61	1.25	0.680	0.550
0.8	1.64	1.64	1.70	1.25	0.579	0.450
0.6	1.85	1.85	1.78	1.09	0.430	0.340
0.4	2.26	2.22	1.90	0.780	0.260	0.196
0.3	2.64	2.61	1.75	0.530	0.154	0.120
0.2	3.32	2.78	1.28	0.267	0.0725	0.0550
0.15	3.98	2.55	0.840	0.136	0.038	0.027

(From FANO⁽⁴²⁾)

(b) $E_0 > E_{\min}$. Under these circumstances the quanta for which the shield is most transparent are those which have been degraded in energy to the region of $E_{\min}$. In this case

$$V(x) = x^{-(\frac{5}{6}+q)} \exp \{-\mu_{\min}x + H(\mu_{\min}x)^{1/3}\} \quad (2.7.4)$$

where $\mu_{\min}$ is the linear absorption coefficient at energy $E_{\min}$. The constant H is given in Table 2.7.2 for various media of interest, together with the corresponding values of $\mu_{\min}/\rho$, the minimum mass absorption coefficient, and $E_{\min}$. For plane collimated and plane isotropic sources $q = 0$, and for point isotropic sources $q = 1$.

Table 2.7.2. Constants for Use in Asymptotic Formula (2.7.4)

Material	$\chi_{\min} = \mu_{\min}/\rho$ (cm ² /g)	H	$E_{\min}$ (MeV)
Water	0.0167	2.0	45
Aluminium	0.0216	2.1	20
Iron	0.0300	2.8	8
Tin	0.0351	2.6	4.5
Tungsten	0.0391	2.5	3.5
Lead	0.0410	2.3	3.4
Uranium	0.043	2.1	3.4

(From FANO⁽⁴²⁾)

The variation of the build-up factors with distance can be derived from the expressions (2.7.3) and (2.7.4) by dividing by the contribution due to the unscattered radiation in the usual way; but it should be noted that the *absolute* values are not given, since this would also require a knowledge of the function $Y(E, \theta)$ in expression (2.7.2). The constant of proportionality can be found approximately by fitting to the results of the moment method at great depths, but this may not be very reliable since the distances at which the asymptotic theory becomes applicable are not known with certainty. However, the error

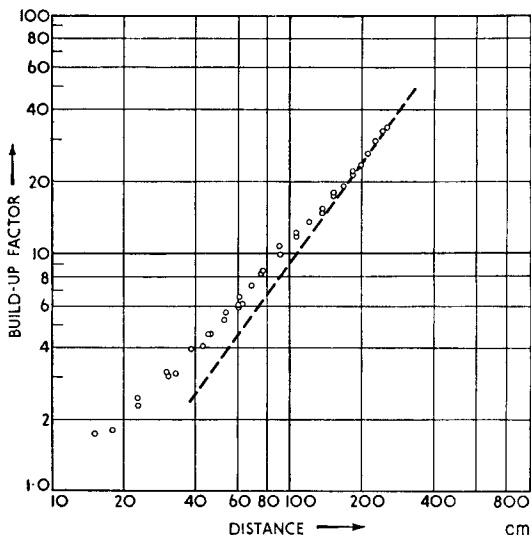


Fig. 2.7.4. Build-up factor for Co^{60} gamma rays in water (WHITE⁽²⁸¹⁾). The dashed line shows the trend (proportional to $(\text{depth})^{1.4}$) predicted for great depths by the theory of FANO, HURWITZ, and SPENCER.

should not be very serious in the context of a shielding calculation. Fig. 2.7.4, showing the attenuation of Co^{60} gamma rays (1.2 MeV) in water, indicates that in this particular case the asymptotic expression begins to become valid at about 13 mean free paths (200 cm). It should be pointed out that the form of the spatial variation predicted by the asymptotic theory is the same for all types of build-up factor.

SPENCER⁽⁴³⁾ has also developed a semi-asymptotic numerical method which enables the absolute value of the build-up factors to be calculated. Not many applications of this method have been published at the time of writing, but agreement has been obtained with the results of the moment method for the case of 10 MeV photons from a plane-collimated source at a depth of 14 mean free paths in lead.

Build-up factor expressed as the sum of two exponentials. The calculation of gamma ray attenuation by thick shields has been simplified still further in an approximate treatment introduced by TAYLOR.⁽²⁶⁾ The method depends on the

fact that the calculated build-up factors B for a point isotropic source can be fitted approximately by functions of the form

$$B(\mu_0 r) = A_1 e^{-a_1 \mu_0 r} + A_2 e^{-a_2 \mu_0 r} \quad (2.7.5)$$

In this expression μ_0 is the attenuation coefficient for the primary gamma rays of energy E_0 ; B is the build-up factor at a depth $\mu_0 r$ mean free paths in the medium; and $A_1 = 1 - A_2$, a_1 , and a_2 are functions of E_0 chosen so that the expression fits the values of B computed by the moment method over a wide range of values of $\mu_0 r$ (1 to 20) and E_0 (0.5 MeV to 10 MeV), with an accuracy of about 5%. The usefulness of the method is due to it being possible to choose A_1 , a_1 , and a_2 for a given material in such a way that they are independent of $\mu_0 r$. Attenuation calculations which can be solved analytically for the case of exponential attenuation can then also be solved when build-up is important by the simple expedient of altering the coefficient of $\mu_0 r$. The functions A_1 , a_1 , and a_2 are given in Table 2.7.3 for both energy and dose build-up in several different media.

Table 2.7.3. *Values of Constants for Equation (2.7.5.)*

Energy Absorption Build-up Factor for Water (Point Isotropic Source)

E_0 (MeV):	1	2	3	4	6	8	10
A_1	13.5	8.1	5.6	4.5	3.4	2.8	2.5
$-a_1$	.100	.068	.059	.0555	.0525	.05	.0473
$+a_2$	.010	.0405	.073	.11	.156	.17	.1719

Energy Build-up Factor for Water (Point Isotropic Source)

E_0 (MeV)	0.7	1	2	3	4
A_1	20	12	6.4	4.9	4
$-a_1$	.115	.095	.067	.059	.050
$+a_2$	-.039	.016	.086	.1082	.1195
Worst error in fit (%)	18.5	15	9.0	6.5	6.0

Dose Build-up Factor for Aluminium (Point Isotropic Source)

E_0 (MeV):	1	2	3	4	6	8	10
A_1	8.0	5.5	4.5	3.8	3.1	2.3	2.25
$-a_1$	0.11	0.082	0.074	0.066	0.064	0.062	0.060
$+a_2$	0.044	0.093	0.116	0.130	0.152	0.150	0.128
error at $\mu_0 r = 10$	8%		4%				

E_0 (MeV):	0.5	1	2	3	4	5.11	6	8	10
A_1	1.65	2.45	2.60	2.15	1.65	1.20	.96	.67	.50
$-a_1$	.032	.045	.071	.097	.123	.152	.175	.204	.214
$+a_2$	.296	.178	.103	.077	.064	.059	.059	.067	.08
Worst error in fit (%)	1.5	3.9	4.9	5.0	3.8	3.1	3.9	5.2	3.3
		(Greatest error 5.2%)							

Energy Build-up Factor for Lead (Point Isotropic Source)

E_0 (MeV):	0.5	1	2	3	4	5.11	6	8	10
A_1	2.20	2.65	2.68	2.18	1.68	1.19	.87	.45	.30
$-a_1$	.013	.038	.05675	.08	.105	.136	.1675	.206	.2211
$+a_2$	.140	.141	.1385	.133	.125	.1135	.0995	.0159	0
Worst error in fit (%)	4.4	4.8	4.3	4.6	5.0	5.3	0.9	2.1	3.1
(Greatest error 5.3%)									

(From TAYLOR⁽²⁶⁾ and LAKEY (unpublished))

Build-up factors in heterogeneous shields. The discussion has so far assumed that the shield is homogeneous and consists of a single element. However, some reactors have heterogeneous shields consisting of alternate layers of a heavy element (to attenuate gamma rays) and water or other hydrogenous material (to attenuate fast neutrons). The moment type of calculation cannot be applied to multilayer shields of this kind, and approximate methods of estimating their performance therefore have to be used. These methods may be illustrated by reference to a shield consisting of two layers, one of heavy and one of light elements. The order in which the layers are arranged has an important bearing on the overall build-up factor. We will first consider the case in which the photons pass through the heavy element after passing through the water. It will be remembered that the large build-up factor in water is due to the probability of capture remaining small until the photons have been greatly degraded in energy. However, these soft radiations are very rapidly attenuated in the succeeding slab of heavy element, so that the effective build-up factor of the composite shield is much less than the product of the build-up factors for the two components taken separately. It will, in fact, be very roughly equal to the build-up factor of a shield composed of a single slab of the heavy element with a thickness (in mean free paths) adjusted to be equal to that of the composite shield.

If the order of the slabs is reversed the build-up in water can make its full contribution to the dose at the surface. It is then best to take as the build-up factor for the whole shield the product of the factors for the two component slabs. The basis for this approximation is that the scattered photons which succeed in penetrating the heavy material will be mainly those which have suffered comparatively small changes in energy and direction, and which will therefore penetrate the water slab with nearly the same ease as the uncollided photons. When the energy of the source photons is well above 3 MeV a somewhat different rule should be used, as the majority of the photons escaping from the heavy-element slab into the water will then have energies around 3 MeV. In this case it would therefore be best to assume that all the radiation entering the water consists of photons of this energy.

Rules of thumb of this kind, based on physical intuition and common sense, can usually be developed to deal with heterogeneous shields. The results are admittedly somewhat rough, but in the context of a shielding calculation the accuracy will usually be acceptable.

Build-up factors in finite shields. In the calculation of build-up factors by the moment method it is assumed that the medium is infinite in extent. The factor applicable to a finite medium must be smaller, since photons escaping from the boundary can no longer contribute to the scattered dose. However, the difference between the two factors is sufficiently small for it to be hardly worthwhile in practice to make any correction to the infinite medium factors. This has been demonstrated quantitatively by BERGER and DOGGETT,⁽⁴⁵⁾ the results of whose calculations are given in the accompanying table.

Table 2.7.4. Comparison of Build-up Factors in Finite and Infinite Shields

The quantity tabulated is $\frac{B(t, t) - 1}{B(t, \infty) - 1}$, where $B(t, t)$ is the build-up factor for transmission through a slab shield of thickness t , and $B(t, \infty)$ is the factor applying at a depth t in the semi-infinite medium. The estimated accuracy is $\sim \pm 2\%$.

Energy (MeV)	Medium	$\mu_0 t$				
		1.0	2.0	4.0	8.0	16.0
0.66	Water	0.663	0.713	0.783	0.785	0.784
1.0		0.720	0.754	0.821	0.828	0.830
4.0		0.885	0.912	0.920	0.926	0.933
1.0	Iron	0.821	0.851	0.888	0.895	0.895
4.0		0.910	0.923	0.936	0.932	0.949
10.0		0.959	0.972	0.974	0.978	0.977
1.0	Tin	0.911	0.924	0.935	0.938	0.946
4.0		0.926	0.955	0.967	0.974	0.978
10.0		0.960	0.962	0.973	0.971	0.969
1.0	Lead	0.951	0.969	0.975	0.979	0.982
4.0		0.977	0.982	0.990	0.992	0.994
10.0		0.990	0.995	0.992	0.994	0.995

Build-up factors for collimated radiation obliquely incident on slab absorbers. Up to this point we have implicitly assumed that the line joining the source to the point at which the build-up factor is required passes more or less along the normal to the surfaces of the slab absorbers which have been considered. However, in practice build-up factors are also frequently needed for the case of oblique incidence shown in Fig. 2.7.6. In this case it would be incorrect to use the factors already given, because the geometry is unusually favourable to the transmission of scattered radiation (see diagram). PEEBLES⁽³³⁾ has made use of the method of successive scatterings to obtain build-up factors for the case of an infinite collimated beam of radiation incident on an infinite parallel-sided plane slab of iron or lead at an angle β (measured from the normal). Fig. 2.7.5 gives his values for the energy build-up factor; as we have already pointed out, the corresponding dose factors would be expected to be similar.

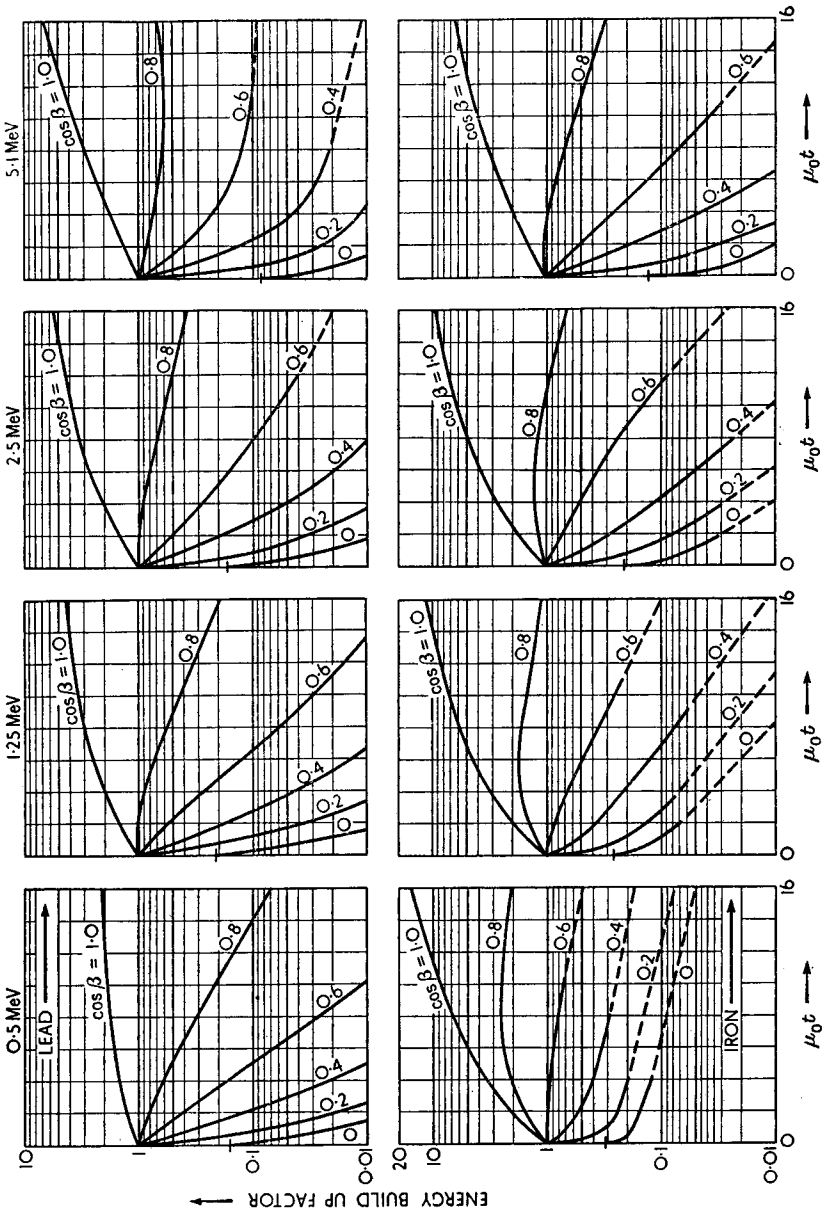


Fig. 2.7.5. Calculated energy build-up factors for oblique incidence (see text). Broken lines indicate that the values are of doubtful accuracy (PEEBLES⁽³³⁾).

The energy build-up factor B_E used in this figure is defined in the following way:

$$B_E = e^{+\mu_0 t} \left(\frac{\text{Total transmitted energy}}{\text{Total incident energy}} \right),$$

where t is the thickness of the slab *measured along the normal*, and μ_0 is the linear absorption coefficient for the incident radiation. Values are given for $\cos \beta = 1, 0.8, 0.6, 0.4, 0.2$, and zero. The last of these cases requires some explanation. The radiation is assumed to be travelling just inside the surface which faces away from the point of interest. If t is allowed to tend to zero, we may imagine that the path of the photon always remains within the surface,

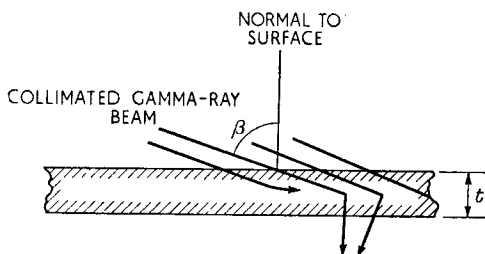


Fig. 2.7.6.

and that its chance of suffering a collision is unaffected by the gradual diminution of the thickness of the slab. It is clear that in these circumstances the limit to which B_E will tend will be different from the limit when $\cos \beta \neq 0$, since the limit of unity which then applies is merely a reflection of the fact that the probability of collision drops to zero for $t = 0$. The limit will in fact be determined by the relative probability for a photon to suffer a scattering, as opposed to an absorbing, collision. If Rayleigh scattering is neglected, it will be given by $\sigma_c/2\sigma_{\text{tot}}$, since (when $t \rightarrow 0$), exactly half of the scattered radiation leaves the slab in one direction, and half in the other.

2.8. BACK-SCATTERING AND AIR SCATTERING OF GAMMA RADIATION

When highly active gamma-emitting materials are being stored or processed, it is not always necessary to surround them completely with shielding. A cheaper and more flexible method, which in many cases is quite good enough, is to build a shielding wall of sufficient height and thickness to protect people working in the shadow it casts, and to eliminate the necessity for shielding above the source by forbidding access to the roof, or to any position from which the source can be viewed directly. However, for gamma ray sources emitting more than about one watt of radiation (about 200 curies of 1 MeV photons), Compton scattering by the air above the source can cause an appreciable gamma ray intensity even in the shadow of the shielding wall. If the active cell is provided with a periscope, the mirror can also be a source of scattered radiation; and so,

of course, can the roof, if there is one. One therefore needs a method of estimating roughly how important this scattered radiation will be. The same kind of problem arises in other branches of work. For instance, the coolant of a gas-cooled power reactor has to be led through the shield in ducts, which are

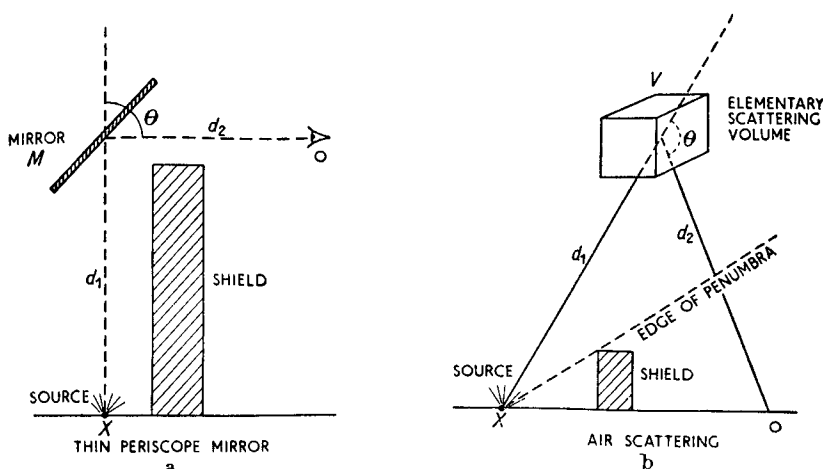


Fig. 2.8.1. Scattering geometries.

necessarily of considerable size if excessive pumping losses are to be avoided. In order to prevent a dangerous leakage of radiation, the duct must be bent at one or more points through a large angle; but although this eliminates the direct escape of radiation, scattered radiation can still reach the outside of the shield by successive reflections at the bends.

Thin Scatterer.

The simplest problem we shall consider is that shown in Fig. 2.8.1(a). A thin mirror M is being used by an observer at O to observe an isotropic source of gamma rays at X . The intensity at O of gamma rays scattered by the mirror is required.

The number of electrons in the mirror is $\approx 6 \times 10^{23} G_m Z_m / A_m$, where G_m , Z_m and A_m are respectively the mass in grams of the mirror, and the atomic number and atomic weight of the material of which it is composed. As the mirror is thin we may justifiably neglect multiple scattering in the mirror itself; and provided the distance XMO is small compared with the mean free path of photons in air ($= \mu^{-1}$, Table 2.5.3) we may also neglect multiple scattering by the air. In order to simplify the algebra we shall further assume that the dimensions of the mirror are small compared with the dimensions d_1 and d_2 . With these approximations the calculation of the required intensity becomes a simple matter. If the source intensity of photons of initial energy E_0 is S_X photons per second, the number of photons scattered per second into unit area at O is

$$\phi_{sc} = S_X \cdot \frac{1}{4\pi d_1^2} \cdot 6 \times 10^{23} \cdot G_m \frac{Z_m}{A_m} C(\theta) \cdot \frac{1}{d_2^2}, \quad (2.8.1)$$

where the electronic Compton differential scattering cross-section per unit solid angle $C(\theta)$, given in Fig. 2.5.2, must be in cm^2 if the distances d_1, d_2 are in cm. The photons scattered in the direction of O have an energy E_1 , which may be determined from equation (2.5.1b) or Table 2.5.1, when the values of E_0 and θ are known. If ξ_1 is the flux in photons $\text{cm}^{-2} \text{sec}^{-1}$ required to give one maximum permissible level of photons of the scattered energy E_1 (as given by Fig. 1.5.1), then the scattered intensity at O , in maximum permissible levels of radiation, is equal to ϕ_{sc}/ξ_1 . Since the mirror is thin the result does not depend on its orientation.

Extended Thin Scatterer

When working with high-intensity sources in an arrangement similar to that shown in Fig. 2.8.1(b), scattering from the air may be important. The geometry of the arrangement usually precludes any simple analytic treatment, but numerical solutions of sufficient accuracy may be obtained by dividing the irradiated space visible to the observer at O into a number of elementary volumes, and computing the contribution from each volume separately. Provided the distance $(XV + VO)$ is much less than one mean free path in air, it is sufficient to consider single scattering only, and to neglect any attenuation of the gamma radiation by the air. The calculation then proceeds as in the previous case, with G_m, Z_m, A_m and θ replaced by the corresponding quantities for each elementary volume of air.

Scattering from a Thick Plane Slab

The assumption that once-scattered photons are the only important contributors to the scattered dose breaks down completely if applied to the radiation scattered back from thick slabs of material. Both the energy spectrum and the angular distribution are found to be much more complicated than the single-scattering approximation would predict, and the once-scattered photons are found to constitute a comparatively small proportion of the total number back-scattered.

If one performs the idealized experiment of allowing a thin collimated pencil of gamma-rays to fall on a scattering slab, and observes the scattered radiation with a highly directional detector, the energy spectrum is found to depend markedly on whether or not the axis of the detector is coplanar with the incident beam. For instance, in the coplanar case and for cobalt 60 incident radiation ($\approx 1.2 \text{ MeV}$), the energy spectrum of the scattered radiation is found to show two peaks. The higher of these can be identified with photons which have undergone only one Compton scattering. The lower peak comes from doubly scattered photons: it is found on detailed examination that although an infinity of pairs of angles can all give the same total deflection, a large proportion of these possible scattering modes cause almost the same loss in energy, so that many of the doubly scattered photons cluster together to form a peak in the energy distribution (Table 2.8.1). When the axes of the incident beam and the detector do not lie in the same plane the peak due to singly scattered photons is for purely geometrical reasons no longer observed. There is still a broad peak at low energies, just above the energy at which the photoelectric effect prevents

further build-up of multiply scattered radiation in the slab. The distribution is, however, more or less isotropic, and evidently comes from photons which have been scattered so many times that they have "forgotten" their initial direction.

Table 2.8.1.

Most probable energy (E_{ds}) of photons scattered through an angle θ as a result of two collisions, compared with the energy (E_{ss}) of photons scattered through the same angle as a result of a single collision. Initial photon energy (Co^{60} radiation) ≈ 1.2 MeV. (After HAYWARD and HUBBELL⁽⁴⁴¹⁾)

θ (degrees)	E_{ss} (keV)	E_{ds} (keV)
50	667	122
70	479	122
90	362	132
120	268	155
150	225	190

Because of these complications, an experimental study of back-scattered radiation either has to be extremely elaborate—involving difficult measurements of the energies and directions of all the scattered photons—or else confined to a simple measurement of the integrated back-scattered dose. In practice it has been found more fruitful to attack the problem with the aid of the Monte Carlo technique. This consists, in essence, of performing a theoretical experiment with a large number of incident photons. Tables of random numbers are used to introduce correctly the element of probability, and to decide when a photon suffers a collision, in which direction it is scattered, what its subsequent history might be, and, finally, whether it recrosses the boundary of the medium before it is absorbed. The cross-sections of the individual processes are known with sufficient accuracy, and the calculation can be adjusted to give each process its correct weight. The result then truly represents a possible series of photon life histories, and only needs to be extended to sufficient cases to give a satisfactory statistical sample. The usefulness of the technique has been greatly extended by the advent of high-speed electronic computers, which allow a great many more cases to be considered than would be possible with hand-computing methods. Unless supplemented by analytical methods, the technique is not particularly suitable for calculation of the attenuation of thick shields, since the sampling accuracy then depends on the number of photons which succeeds in passing through a barrier; as the shield becomes thicker the number of case-histories which would need to be considered for a reasonable accuracy rapidly becomes prohibitively large. The method is, however, much better suited to the study of back-scattering, since the chance of reflection (sometimes called the albedo) is relatively high (often of the order of 10%), and small sample errors can be achieved without an excessive use of machine time.

Table 2.8.2 gives the results of a systematic study of back-scattering by BERGER

and DOGGETT,⁽⁴⁵⁾ using the SEAC computer programmed to carry out a semi-analytic Monte Carlo calculation. The quantity tabulated is the energy reflection build-up factor, which is defined as follows: consider an isotropic detector placed in the path of a broad beam of radiation, and denote the energy flow through the detector by F_0 . Then place a barrier of thickness x immediately behind the detector. This increases the energy flow through the detector to F . Then the build-up factor $B(0, x)$ is defined as being equal to F/F_0 . The energy factor would be expected to be not greatly different from the dose factor. The second part of the table shows how the the energy reflection build-up factor grows as the thickness of the scattering slab is increased. A slab two mean free paths in thickness behaves, as far as backscattering of radiation is concerned, as an effectively infinite slab. What little experimental work exists appears to bear out the results of BERGER and DOGGETT's calculations, and the approximate theoretical treatment of CORNER and LISTON⁽⁴⁶⁾ also gives results of the same order of magnitude.

Table 2.8.2.

(A) Energy reflection build-up factors for broad beams of mono-directional radiation incident on barriers of effectively infinite thickness at an angle θ (from the normal).

Energy (MeV)	Water			Iron		Tin		Lead	
	$\theta \rightarrow$	0°	60°	0°	60°	0°	60°	0°	60°
0.4		1.125	1.278						
0.66		1.100	1.189						
1.0		1.081	1.153	1.061	1.142	1.022	1.086	1.009	1.042
4.0		1.012	1.035	1.011		1.005		1.001	
10.0		1.002	1.007						

(B) Dependence of reflection on barrier thickness. The quantity $[B(0, x) - 1] \div [B(0, \infty) - 1]$ is listed; it measures the amount of radiation reflected from a finite barrier in terms of the radiation reflected from a barrier of infinite thickness (i.e. $x \rightarrow \infty$).

Material	Energy (MeV)	θ	Thickness of barrier in mean free paths = $\mu_0 x$		
			0.5	1	2
water	0.66	0°	0.65	0.88	0.99
water	0.66	60°	0.81	0.96	1.00
iron	1.0	0°	0.79	0.93	1.00
iron	1.0	60°	0.89	0.98	1.00
tin	1.0	0°	0.95	0.99	1.00
lead	1.0	0°	0.975	1.00	1.00

(From BERGER and DOGGETT⁽⁴⁵⁾)

2.9. GAMMA RADIATION PRODUCED IN NUCLEAR REACTORS

A fission reactor is a copious source of gamma radiation, which is an unavoidable by-product of most of the reactions involved. Gamma rays are emitted both at the moment of fission, and later when the fission neutrons are captured or scattered inelastically in either the core or the shield. They continue to be produced in important amounts by the slow decay of the radioactive fission products

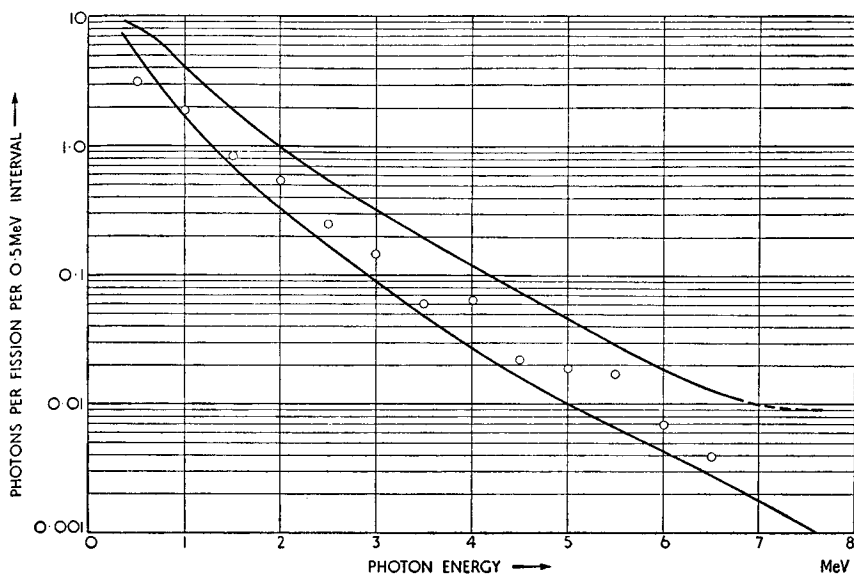


Fig. 2.9.1. Gamma rays emitted in the fission of U^{235} . The circles are the experimental points of GAMBLE.⁽⁴⁹⁾ The solid lines define the upper and lower limits of error in KIRKBRIDE's measurement.⁽⁵⁰⁾ Measurements by PELLE⁽¹⁰²⁾ over the range 0.5 to 2.3 MeV agree closely with GAMBLE's data.

for many years after the neutron chain-reaction has ceased. Further, materials which have been exposed to bombardment by neutrons are usually radioactive, and, like the fission products, continue to give off radiation after the reactor has been shut down. Of these various sources of gamma radiation, only fission and fission product decay will be discussed in the two following sections; the remainder will be dealt with in Chapters 3 and 6.

Gamma Rays Emitted During Fission

The term "fission gamma rays" usually means those photons which are emitted within a time of the order of one microsecond after the emission of the fission fragments. Early measurements^(47,48) showed that for U^{235} the total fission gamma energy is of the order of 5 MeV. Somewhat higher figures have been obtained in more refined experiments by GAMBLE,^(49,101) KIRKBRIDE⁽⁵⁰⁾ and PELLE,⁽¹⁰²⁾ who used sodium iodide crystal spectrometers to observe the gamma rays from thermal-neutron-induced fissions. The pulse height

distributions obtained by GAMBLE and KIRKBRIDE were similar, but the results have been interpreted somewhat differently in the two cases. The interpretation of measurements on such a continuous spectrum with a single sodium iodide crystal is not very straightforward, because a photon of a given energy can produce in the detector a wide range of pulse sizes, depending on whether the whole of the energy is absorbed photoelectrically or by pair production in the crystal, or whether a smaller (and variable) amount is absorbed by Compton scattering. An additional complication is that the division between the three processes is energy dependent. GAMBLE gives the total gamma energy per fission as about 7.5 MeV, and states that on the average 7 photons are emitted per fission with energies between 0.25 and 7 MeV; KIRKBRIDE finds that the total gamma ray energy per fission is approximately 12 MeV. Both authors agree as to the shape of the spectral distribution of the prompt photons, except at the lowest energies (Fig. 2.9.1). PELLE investigated the range 0.5 to 2.3 MeV with an improved type of spectrometer, which gave unambiguous results, in excellent agreement with those of GAMBLE.

KIRKBRIDE also investigated the fission gamma spectra for thermal-neutron-induced fission of U^{233} and Pu^{239} , and found results identical with those for U^{235} . No results have yet been published on gamma spectra in fissions induced by neutrons of greater than thermal energies; but in view of the statistical nature of the fission process, involving very many possible modes of decay, it would be extremely surprising if the shape of the spectrum were at all sensitive to neutron energy. One might plausibly expect that the changes in the spectrum would be unimportant as long as the neutron energy was appreciably less than 1 MeV.

2.10 GAMMA RAYS FROM FISSION PRODUCTS

The fission of a heavy element can take place in a great variety of different ways, each of which results in the formation of a pair of unstable fission fragments. It is found experimentally that for fission induced by thermal neutrons the symmetric mode of fission into two equal fragments is much less probable than asymmetric fission into fragments of mass numbers around 96 and 136. The probability that a fragment with a particular mass number is formed in fission—a quantity known as the fission yield—is shown in Fig. 2.10.1; since two fragments are formed per fission, the area under the curve is 200%. It will be noticed that the curves for thermal fission of U^{235} and Pu^{239} , and for fast fission of U^{238} , are not identical. These differences, and also the predominance of asymmetric fission, are probably connected with the shell structure of the nucleus.⁽⁵¹⁾

Stable nuclei of medium atomic weight have a smaller ratio of neutrons to protons than the heavy fissile elements. It follows that the newly formed fission fragments will have too many neutrons for stability (even after emission of the prompt neutrons) and must therefore decay by one or more β decay processes, which are in many cases followed by the emission of hard gamma radiation, before a stable nucleus is formed. In the course of these decay chains the mass number of the decaying nucleus remains constant, but its atomic

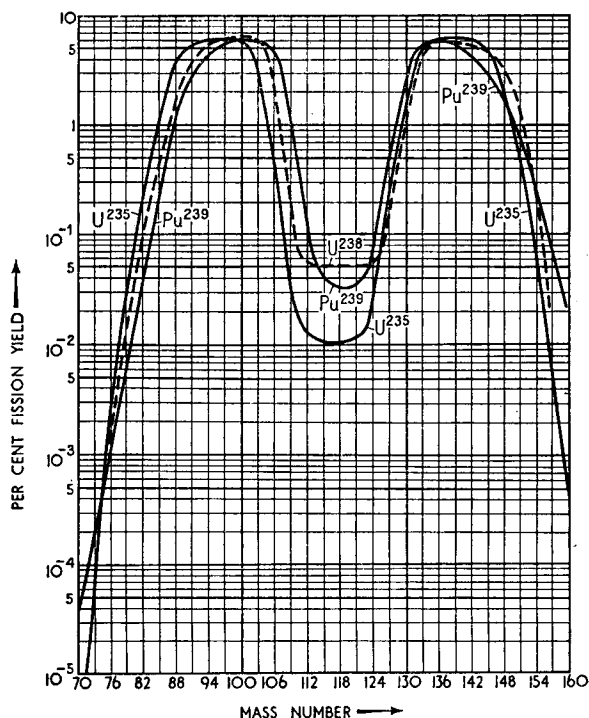
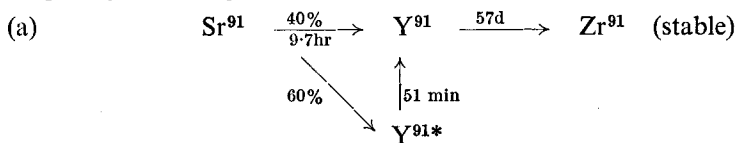


Fig. 2.10.1. Mass yield curves for thermal neutron fission of U^{235} , U^{238} , and Pu^{239} , and fast neutron fission of U^{238} (from CORYELL and SUGARMAN⁽⁵²⁾). The fission yields shown are approximate; actual values are scattered above and below the values given by these smooth curves.⁽⁵³⁾

number increases by one unit at each step. For instance, two of the most frequently occurring chains are the following;



Certain exceptional fission product nuclei occasionally decrease their neutron-proton ratio by the emission of delayed neutrons, immediately following beta-emission. This property, which is such an important factor in reactor control, need not concern us here, but will be considered later in Section 3.9.

Thanks to a prodigious amount of radiochemical research, tabulations^(52,53) of the details of these decay chains, and of the radiations emitted by all except the shortest-lived nuclei, are now available. From these tables estimates can be made of the total beta and gamma energy radiated by the fission products, and of the distribution of quantum energies of the gamma rays. These quantities

are functions both of the time of decay and of the time of irradiation in the pile: if a uranium sample is irradiated for a period T_0 when the pile is operating at a given power Π_0 , the initial activity of a fission product species formed directly in fission, and of fission yield y_i and half-life t_i , will be proportional to $\Pi_0 y_i [1 - e^{-0.693 T_0 / t_i}]$. Thus after a short irradiation there will be a greater proportion of short-lived activities present in the mixed-fission products than after a long irradiation. Since in general the longer-lived fission products tend to emit less penetrating radiation than those of smaller half-life, a group of mixed-fission products formed during a short irradiation will require more shielding than a group emitting the same total gamma ray energy, but formed in the course of a long irradiation.

Total Energy Radiated by the Mixed Fission Products formed in the Irradiation of U^{235}

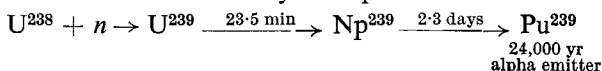
By considering the mixed fission products as a statistical assembly, WAY and WIGNER⁽⁵⁴⁾ have derived an approximate formula for the rate of release of energy Π_{FP} at a time T seconds after the shut-down of a reactor, in terms of the operating power Π_0 and the irradiation time T_0 seconds. This is of the form:

$$\frac{\Pi_{FP}}{\Pi_0} \propto T^{-c} - (T + T_0)^{-c} \quad (2.10.1)$$

where the exponent c is approximately equal to 0.2. An expression of this type fits the experimental data satisfactorily, provided enough different types of nuclei are decaying at any given time for statistical arguments to be valid. This is not the case at very long decay times, and may also cease to be true at very short times after shutdown. The following modified formula, due to UNTERMYER and WELLS,⁽⁵⁶⁾ fits the observed decay of the heat production after shut-down of a reactor fuelled with natural uranium over the entire experimental range, from $T = 1$ to $T = 10^8$ seconds:

$$\frac{\Pi_{\text{tot}}}{\Pi_0} = 0.1 \{ (T + 10)^{-0.2} - 0.87(T + 2 \times 10^7)^{-0.2} \} - 0.1 \{ (T + T_0 + 10)^{-0.2} - 0.87(T + T_0 + 2 \times 10^7)^{-0.2} \} \quad (2.10.2)$$

In this formula Π_{tot} is the total observed rate of heat production, at time T seconds after shutdown. It excludes the unobservable energy carried off from beta decay processes by neutrinos (Section 2.11). It does however include, in addition to the fission product decay energy, a further contribution from the decay of uranium 239 and neptunium 239. This arises because in a reactor fuelled with natural or near-natural uranium some of the neutrons are absorbed in U^{238} , and form U^{239} which then decays to Np^{239} and Pu^{239} :



The heating due to fission products alone may be obtained by subtracting the following quantities from formula (2.10.2): (i) Heat produced in U^{239} decay:

$$\frac{\Pi_U}{\Pi_0} = 0.0025 \{ \exp(-T/2040) - \exp(-(T + T_0)/2040) \} \quad (2.10.3a)$$

(ii) Heat produced in Np^{239} decay:

$$\frac{\Pi_{\text{Np}}}{\Pi_0} = 0.0013 \{ \exp(-T/290,000) - \exp(-(T + T_0)/290,000) \} \quad (2.10.3b)$$

Expressions (2.10.3) imply that the conversion factor of the reactor was about 0.8, which is typical of large natural uranium reactors designed for maximum neutron economy.

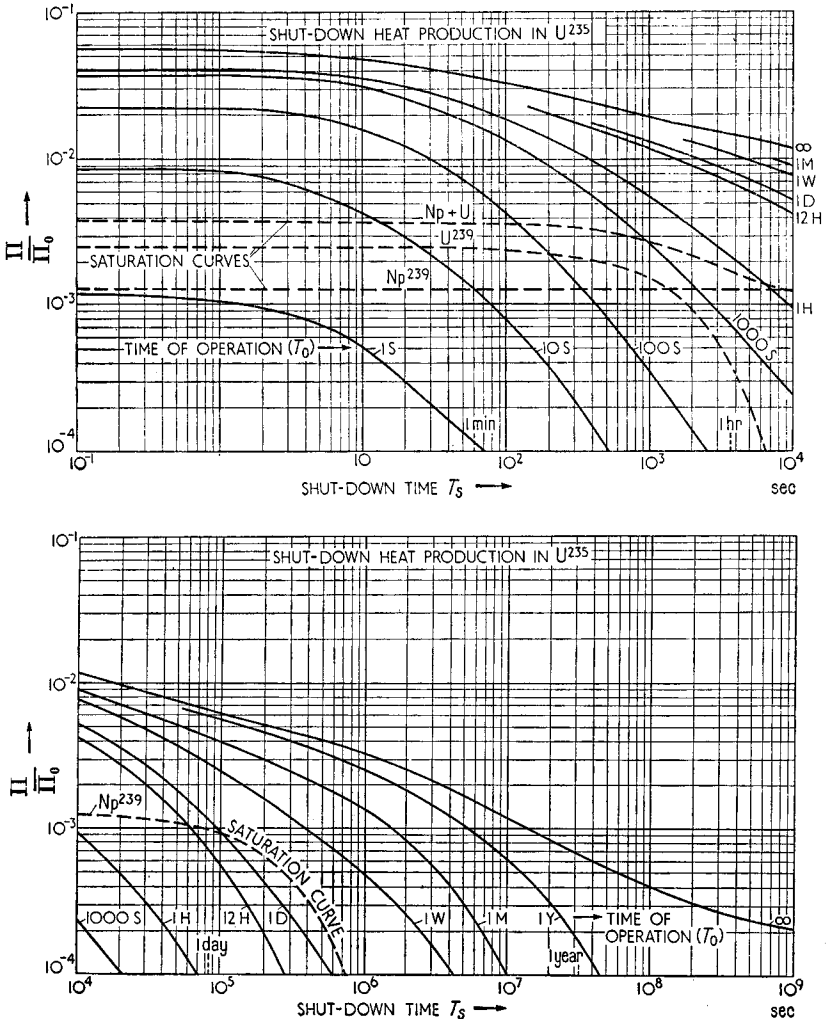


Fig. 2.10.2. Shut-down heat production Π due to fission products formed from U^{235} . Also shown are curves for heating from decay of U^{239} and Np^{239} , when the reactor conversion factor ~ 0.8 . The heat release in the sample during irradiation = Π_0 (UTERMYER and WEILLS⁽⁵⁶⁾).

Curves of the observable energy released by the decay of fission products are given for various times of operation and decay in Fig. 2.10.2. The same figure also shows the saturation contribution from U^{239} and Np^{239} , after the reactor has operated for a period long compared with 2.3 days, the half-life of Np^{239} . The accuracy of the fission product curves is probably within $\pm 50\%$ for decay times from 1 to 100 seconds, $\pm 30\%$ from 10^2 to 10^4 seconds, $\pm 10\%$ from 10^4 to 10^6 seconds, and $\pm 50\%$ from 10^6 to 10^8 seconds. At times shorter than 1 second the error may be large. In using the figure it is useful to remember that a reactor power of 1 megawatt corresponds to a release of energy at a rate of 6.25×10^{18} MeV per second.

The observable energy radiated by fission products was found by BORST to be divided between gamma and beta radiations as follows:

$$\frac{\text{Gamma energy}}{\text{Total observable energy}} = \begin{array}{l} 0.65 \text{ at 20 minutes after a short irradiation,} \\ 0.60 \text{ after 3 days,} \\ 0.50 \text{ from 50 to 100 days after a short irradiation.} \end{array}$$

By combining this observation with the data of Fig. 2.10.2 an estimate may be made of the total energy emitted as gamma radiation. Roughly, 6 MeV per fission of gamma energy are emitted by fission products;⁽⁵⁵⁾ and, if the neutrino energy is neglected, beta decay contributes a further 7 MeV.

Fission Product Gamma Ray Spectrum

Calculations of the effect of fission product gamma rays through large thicknesses of shielding require a knowledge of the gamma ray spectrum (i.e. the distribution in energy of the photons), as well as a knowledge of the total energy carried off as gamma radiation. Since each type of nucleus has its own particular mode of decay, and emits its own characteristic gamma radiations, the problem of determining the spectrum is essentially that of computing the activities of the different fission product species. Provided one is not interested in times of irradiation and decay much shorter than one day, the disintegration schemes of the important gamma-emitting fission products are well known, and a reliable spectrum can be obtained by summation.

When the activities of only a few gamma-emitting fission products are of interest (which might be the case in some stages of a chemical separation plant) they can be conveniently computed from tables prepared by HOWLETT *et al.*⁽⁵⁷⁾ The tables, which are for thermal fission of U^{235} , and times of irradiation and decay greater than 1 day, make use of the following formalism. The function $N_i(t)$ is defined as the total number of disintegrations per fission of the i th fission product that will occur after time t . The fissions are, for the moment, assumed to have occurred in a burst at time zero, the total number of fissions included in the burst being F . The activity of the i th fission product after a decay interval T must then be

$$A_i(T) = -F \left(\frac{dN_i}{dt} \right)_{t=T}$$

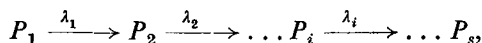
Now suppose that the F fissions, instead of occurring in a very short burst, take

place uniformly throughout an irradiation period T_0 . The number of fissions during a small time interval dt is then $dt \cdot (F/T_0)$. At time T after shut-down the fission products will have decayed for varying intervals between T and $(T + T_0)$, so that the resulting activity is

$$\begin{aligned} A_i(T_0, T) &= - \int_T^{(T_0+T)} \frac{dN_i(t)}{dt} \cdot \frac{F}{T_0} dt \\ &= (F/T_0) \cdot \{N_i(T) - N_i(T_0 + T)\} \end{aligned} \quad (2.10.4)$$

Thus in order to evaluate $A_i(T_0, T)$ is it sufficient to know the function $N_i(t)$.

Consider the general case of a chain of fission products resulting from an instantaneous burst of fissions at time zero:



where λ_i (the *decay constant*) is the reciprocal of the mean life of the i th fission product and P_i denotes both the nature of this fission product and its abundance at time t after fission. The following equations hold:

$$\begin{aligned} dP_1/dt &= -\lambda_1 P_1 \\ dP_2/dt &= \lambda_1 P_1 - \lambda_2 P_2 \\ dP_i/dt &= \lambda_{i-1} \cdot P_{i-1} - \lambda_i P_i \\ dP_s/dt &= \lambda_{s-1} \cdot P_{s-1} \end{aligned} \quad (2.10.5)$$

The disintegration rate per fission of any fission product P_i at time t after fission is $\lambda_i \cdot P_i(t)$, so that the total number of disintegrations that will occur after time T is

$$N_i(T) = \int_T^\infty \lambda_i \cdot P_i(t) \cdot dt \quad (2.10.6)$$

Using the relations (2.10.5), equation (2.10.6) becomes

$$N_i(T) = \sum_{r=1}^i P_r(T), \text{ since } P_i(\infty) = 0. \quad (2.10.7)$$

Fig. 2.10.3 shows the values of $P_i(t)$ as calculated by HOWLETT *et al.* for those gamma-emitting fission products which are (a) sufficiently abundant (b) sufficiently long-lived and (c) emit sufficiently hard gammas to be important contributors to the gross fission product radiation after one day's decay. The solid lines in this figure are for nuclides formed directly in fission, for which $N_i = P_i$. The broken lines are intended to draw attention to the fact that they refer to daughter products, for which N_i must be calculated with the aid of equation (2.10.7). A key to the figure is given in Table 2.10.1 (p. 69). This table includes the values of fission yields assumed for these computations, which refer to thermal fission in U^{235} . For thermal fission of Pu^{239} , and for fast fission of U^{238} or Th^{232} , adjustments in the values of $P_i(t)$ can be made with the aid of Fig. 2.10.1; the greatest variations occur around mass number 109.

In order to avoid overcrowding Fig. 2.10.3 two approximations have been introduced: (a) nuclides formed by the decay of short-lived precursors have been treated as though they were formed directly in fission; (b) where a daughter-product is short-lived and emits a gamma ray, this gamma ray has been treated

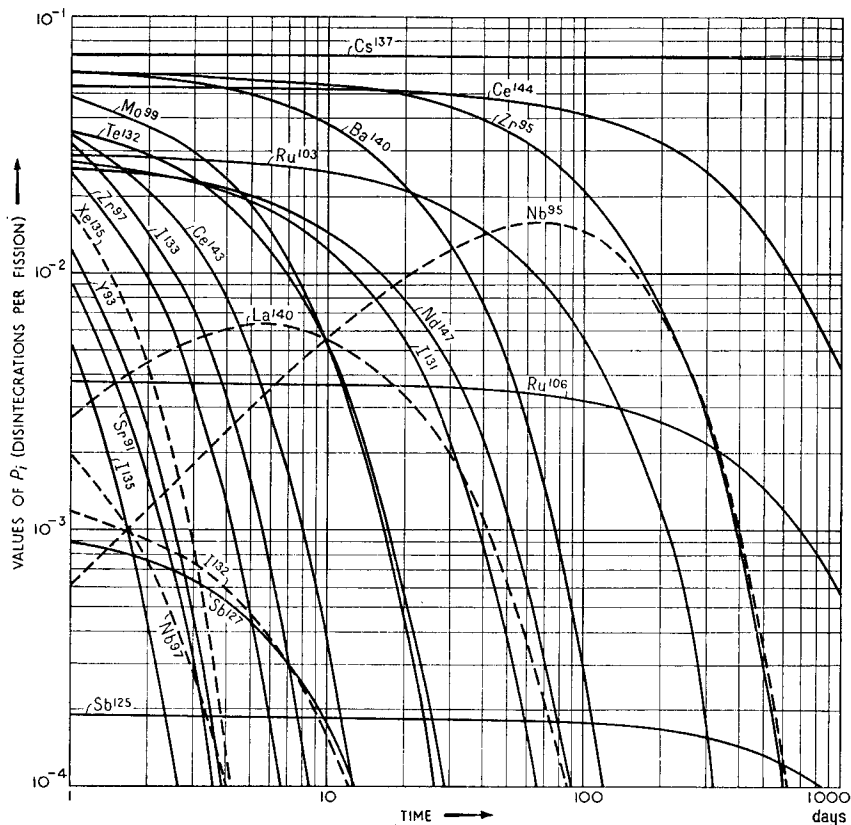


Fig. 2.10.3. Curves for computing activities of the important gamma-emitting fission products; see text (HOWLETT, JOSEPHS, RENNIE and STORY⁽⁸⁷⁾).

as though it were emitted by the parent. A reference is given in the table where this has been done.

The values of P_i given in Fig. 2.10.3 are simply the numbers of product nuclei per fission at time t . If the activity is calculated using equations (2.10.4) and (2.10.7), and T_0 is in days, then $A_i(T_0, T)$ will be in disintegrations per day. In a sample which has been given a total irradiation of M megawatt days per ton of uranium, the total number of fissions that have taken place is approximately $2.7 \times 10^{15} \cdot M$ fissions per gram.

The data given in Fig. 2.10.3 are strictly true only when the yields have not been modified by appreciable neutron capture in the fission products subsequent to their formation. The effect of neutron capture will be small for most of the

Table 2.10.1. *The More Important Gamma-Emitting Fission Products:
Data Assumed in Preparation of Fig. 2.10.3.*

Isotope	Half-life	Fission yield (%)	Gamma rays (MeV) and abundance	Remarks
Sr ⁹¹	9.7 h	5.0	1.3 (40%) 0.6 (40%)	40 % of disintegrations lead to Y ^{91*} (51 min) which is responsible for 0.6 MeV gamma ray
Y ⁹³	10 h	6.3	0.7	
Zr ⁹⁵	65 d	6.0	0.71 (98%) 0.22 (2%)	0.22 MeV gamma due to Nb ^{95*} (90 hour) daughter
Nb ⁹⁵ (= Cb ⁹⁵)	35 d	6.0	0.77	Daughter of Zr ⁹⁵
Zr ⁹⁷	17 h	6.5	0.75	Gamma due to short-lived daughter (60 sec Nb ⁹⁷)
Nb ⁹⁷	75 min	6.5	0.66	Daughter of Zr ⁹⁷
Mo ⁹⁹	67 h	6.2	0.76 (~13%) 0.14 (~100%)	
Ru ¹⁰³	42 d	2.85	0.49 (94%)	
Ru ¹⁰⁶	1 yr	0.38	0.51 (17%) 0.73 (17%) 1.25 (1%) 2.90 (2%)	Gamma due to short-lived daughter Rh ¹⁰⁶
Sb ¹²⁵	2.7 yr	0.02	0.62 (33%) 0.44 (50%)	Daughter of short-lived parent
Sb ¹²⁷	93 h	0.1	0.72 (16%)	
I ¹³¹	7.8 d	3.0	0.64 (15%) 0.36 (~80%)	Curve in Fig. 2.10.3 assumes all I ¹³¹ formed directly in fission; since in fact 20 % comes from 30 h Te, curve somewhat overestimates the Iodine activity for decay times less than 4 days
Te ¹³²	77 h	4.4	0.22	Parent of I ¹³²
I ¹³²	2.4 h	4.4	2.0 (~2%) 0.6 (~50%) 1.4 (~50%)	Daughter of Te ¹³²
I ¹³³	22 h	6.7	0.53 (94%) 0.85 (5%) 1.4 (1%)	
I ¹³⁵	6.7 h	5.9	1.4 (~50%) 0.52 (25%)	0.52 MeV gamma due to daughter (13 min Xe)
Xe ¹³⁵	9.1 h	5.9	0.25	Daughter of I ¹³⁵ . Curve in Fig. 2.10.3 neglects an additional direct yield of Xe ¹³⁵ of 0.29 %
Cs ¹³⁷	37 yr	7.0	0.67 (100%)	Gamma is from 2.6 min Ba ¹³⁷ daughter
Ba ¹⁴⁰	12.8 d	6.5	0.54 (<40%)	Parent of La ¹⁴⁰
La ¹⁴⁰	40 h	6.5	1.6 (~90%) 2.5 (3%) 0.8 (~50%) plus softer gammas	
Ce ¹⁴³	33 h	5.7	0.2 (?) 0.9 (?)	0.18 MeV of gamma energy per disintegration
Ce ¹⁴⁴	282 d	5.4		Followed by 17.5 min Pr which may emit 2.6 MeV gamma (2%), 2.2 MeV gamma (2%), 1.1 MeV gamma (5%), and softer radiations
Nd ¹⁴⁷	11 d	2.7	0.58 (~10%) 0.93 (~4%)	

Data from NBS circular 499, and supplements 1-3.

No account is taken in the above table of radiation of energy less than 200 keV.

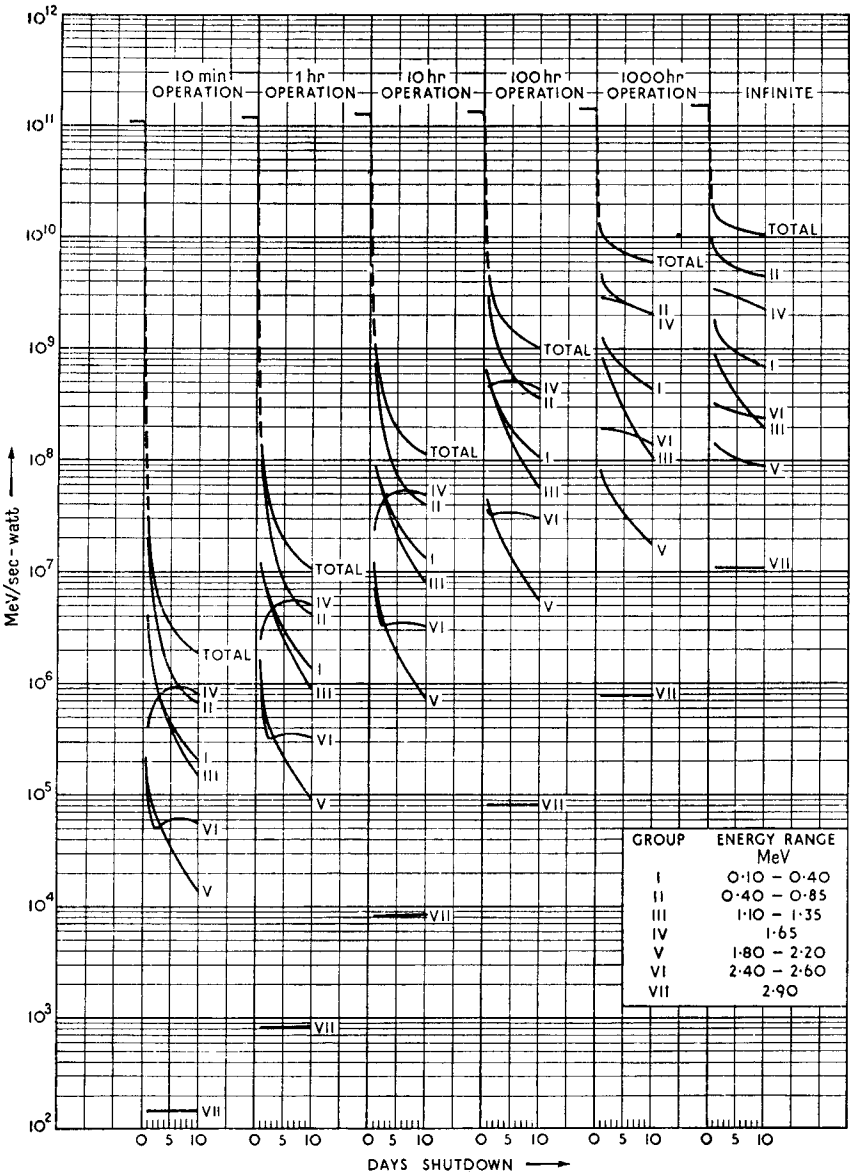


Fig. 2.10.4. Gamma ray energy emitted from a sample of uranium at times up to 10 days after irradiation in a reactor for various times; the energy is expressed in MeV per second-watt of heat generated in the sample during irradiation. Curves are given for seven groups of gamma ray energies (MOTEFF⁽⁵⁸⁾).

isotopes listed in Table 2.10.1, with the exception of the well-known "poison" Xe^{135} , which in a high-flux reactor is rapidly converted to Xe^{136} , which is stable.

The spectrum of gamma rays emitted from mixed fission products could be determined by a straightforward but somewhat lengthy calculation from the data already given. However, the necessity for such tedious computations has been largely eliminated by the work of MOTEFF.⁽⁵⁸⁾ Basing his calculations on data which are practically identical with those given in Table 2.10.1, he obtains the number of photons emitted per unit time in each of the seven energy groups shown in Fig. 2.10.4, for periods of irradiation from 10 minutes to infinity, and for periods of decay from 1 day to 10 days.

It is difficult to extend calculations of the gamma ray spectrum from mixed fission products to short decay periods ($\ll 1$ day) because, in the case of many short-lived isotopes, there is very little definite information as to what gamma rays are emitted. Some of the short-lived decays are very energetic and may well emit penetrating gamma radiation. It is in fact known from an experiment of BERNSTEIN *et al.*⁽⁵⁹⁾ that one or two gamma rays of energy greater than 2.2 MeV are released per fission of U^{235} from fission products of half-life greater than 1 second. Such penetrating gamma rays may be important contributors to the dose outside a gamma shield, even if the fission yield of the nuclide concerned is small.

When a nuclear reactor is operating, the total amount of energy going into fission product gamma rays is comparable with that liberated as prompt fission gamma rays and capture gamma rays, but the latter tend to be more penetrating, so that the fission product gamma rays do not usually need to be considered when the shield is being designed. An obvious exception to this general rule is the type of homogeneous reactor in which the fuel can be made to circulate outside the main reactor shield. Although lack of detailed knowledge of the gamma rays emitted by short-lived fission products makes it difficult to calculate the shielding required for such a design with any precision, a safe upper limit can sometimes be determined. The most favourable case is when a lead or barytes concrete shield is to be used, since these materials show a minimum attenuation for gamma rays of about 3 MeV. The data given above enable one to make rough estimates of the total energy radiated as gamma rays, for times after fission greater than 1 second. If the assumption is made that the whole of this energy is emitted in the form of 3 MeV quanta, the calculated shield thickness cannot fail to err on the safe side. It is more difficult to arrive at a reasonable design criterion for gamma shields made of light elements, which do not show a minimum attenuation in the range of gamma energies which it is reasonable to assume in such a calculation.

2.11. SHIELDING OF ELECTRONS OF ENERGIES LESS THAN A FEW MILLION ELECTRON-VOLTS

(a) Range-energy Relation

Electrons of energy less than a few MeV lose energy when passing through matter principally as a result of ionizing interactions with the orbital electrons

of the absorber. It has already been pointed out in connection with the Compton effect that equal masses of all materials except hydrogen contain very nearly the same number of orbital electrons; consequently, the stopping of electrons of these energies also depends essentially on the mass per unit area of the absorber. Although the range of an electron of a given energy measured along its path in the absorber is a more or less fixed quantity, the path is itself extremely tortuous. As a result, the crow-flight distance travelled before the electron is

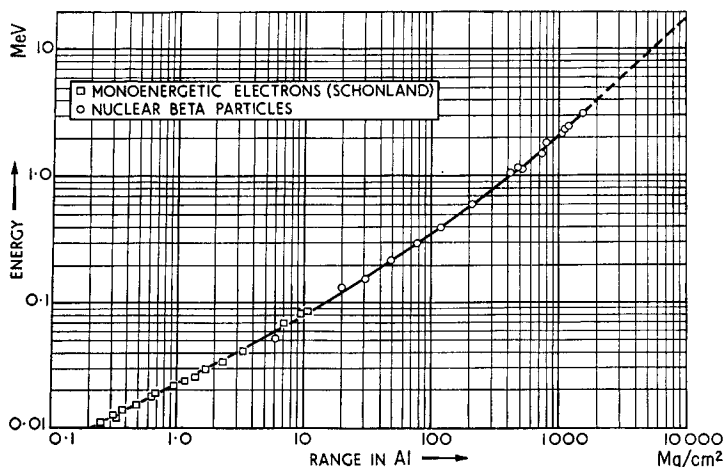


Fig. 2.11.1. Maximum range of electrons in aluminium as a function of their energy; for the continuous energy-spectrum emitted by radioactive materials the energy referred to is the maximum energy (from GLENDENIN⁽⁶⁰⁾).

brought to rest varies within very wide limits, a phenomenon which is described as straggling. The greatest depth of penetration observed is referred to as the *maximum range*; owing to straggling it is not a very well defined quantity, and different methods of estimation give results differing by one or two per cent. For mono-energetic beta particles of energy E MeV ($0.7 < E < 3$) the observed maximum range in aluminium, R g cm⁻² (Fig. 2.11.1), can be conveniently expressed by the well-known Feather rule:

$$R = 0.542E - 0.133 \quad (2.11.1a)$$

The values for the constants in this equation, which are those proposed by GLENDENIN and CORYELL,⁽⁶⁰⁾ differ slightly from those originally suggested by FEATHER. At energies very much below 0.5 MeV the accuracy of this expression is poor; a better representation of the low-energy experimental data is given by FLAMMERSFELD's formula, which is satisfactory over the range 50 keV–3 MeV:

$$E = 1.92(R^2 + 0.22R)^{1/2} \quad (2.11.1b)$$

A useful rough rule of thumb is that the range in g cm⁻² is half the energy in MeV. In other elements the ranges (in g cm⁻²) of electrons with energies around 1 MeV are of the same order as the range in aluminium.

The electrons emitted by radioactive nuclei ("beta rays") are not mono-energetic, but have continuous energy spectra of the kind shown in Fig. 2.11.2. This continuous energy distribution of particles formed in transitions between one nucleus and another, both of well-defined energy, appears to imply a breakdown of the law of conservation of energy. Rather than accept such a conclusion, physicists have preferred to adopt the neutrino hypothesis, originally proposed by PAULI, and elaborated by FERMI. According to this hypothesis, the neutrino is an uncharged particle of spin $\frac{1}{2}\hbar$ and zero or small mass (certainly less than 0.01 times the electron mass), which is emitted in the beta-decay process. It is

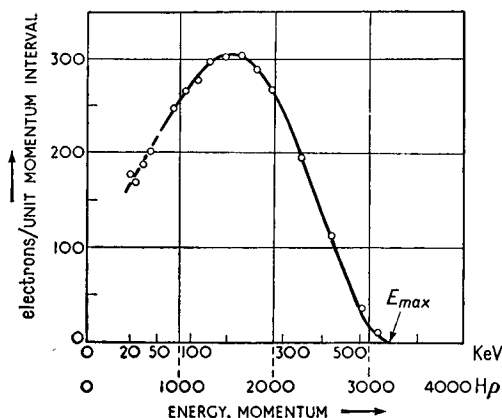


Fig. 2.11.2. Continuous energy distribution of negative beta particles emitted by a radioactive source (Cu^{64}) (after WU and ALBERT⁽¹⁰³⁾).

capable of carrying away energy, but interacts with matter so weakly that its existence can be inferred only from indirect evidence. The variable energy of the decay electrons can be quantitatively explained in terms of a partition of the available energy between the electron and the neutrino. The energy given to the electrons is sometimes described as the observable energy, to distinguish it from the energy carried away by the neutrinos, which is to all intents and purposes irrecoverable.

The great majority of beta spectra follow very closely the energy distribution derived by FERMI on the basis of the neutrino hypothesis. The probability is vanishingly small for the whole of the available disintegration energy to be given to the electron, which would then have the *end-point energy*, E_{max} . The exact shape of the spectrum is a function of the atomic number of the nucleus and of the end-point energy; but a feature common to all is a peaked distribution, with a most probable energy of the order of $E_{\text{max}}/3$. The Fermi distribution function is given convenient numerical form for a number of isotopes by BLEULER and ZUNTI.⁽⁶¹⁾ A helpful introductory discussion of the Fermi distribution law is given by WHITEHOUSE and PUTMAN.⁽⁶²⁾

In determining the maximum range of electrons from a continuous Fermi distribution with the aid of equations (2.11.1) it is, of course, necessary to use

the value of the end-point energy, $E_{\max}$. The number of electrons penetrating to depths smaller than the maximum range falls off rapidly as the depth increases, partly owing to the effect of straggling, and partly also due to the progressive filtering out of the lower-energy electrons from the Fermi spectrum. It is found experimentally that the number of electrons decreases approximately exponentially with increasing absorber thickness until an attenuation by a factor of about 20 has been reached, after which the absorption becomes more rapid. The thickness of absorber a necessary to reduce the number of electrons by half (the *half-thickness*) is given approximately⁽⁶²⁾ by the empirical formula $a = 0.095(Z/A)E_{\max}^{3/2} \text{ g cm}^{-2}$, which is sufficiently reliable for most purposes when applied to all elements from hydrogen to copper; absorption curves are given for various energies in ref. 61. The existence of the quasi-exponential attenuation law must be fortuitous, because the fraction of electrons transmitted depends in a complicated manner on so many factors: the source energy-distribution, the variation of range with energy, multiple scattering, and the atomic number of the absorber.

(b) *Back-scattering of Electrons*

When a stream of electrons strikes the surface bounding a dense medium, a considerable fraction will be scattered back and will eventually re-emerge from the surface. This fraction is found to depend only slightly upon the electron energy, provided it is greater than about 0.6 MeV, but is very appreciably affected by the atomic number of the medium. This is because the elastic nuclear scattering cross-section increases as the square of the atomic number, while the competing process of inelastic electron-electron scattering, by which the electrons lose energy, depends simply on the number of orbital electrons, that is, on the first power of the atomic number.

Among the most useful experimental results are those obtained with electrostatic generators. TRUMP and VAN DE GRAAFF⁽⁶³⁾ allowed a collimated beam of monoenergetic electrons to fall normally on a surface forming one electrode of an ionization chamber, and in this way were able to measure the total number of electrons scattered back into the whole of the 2π solid angle. They found that in addition to the energetic electrons which were scattered back, there were also appreciable numbers of very slow electrons, with energies to the order of 20 eV. These have not been counted in the graph showing their results (Fig. 2.11.3) on account of their extremely small penetrating power.

TRUMP and VAN DE GRAAFF's results would lead one to expect that at higher energies the reflection coefficient, for a given value of Z , would become more or less independent of energy; and above about 0.6 MeV this is borne out by measurements with radioactive beta-emitters.⁽⁶⁴⁻⁶⁾ Such measurements are usually directed towards the determination of back-scattering correction factors f_B for use in absolute beta counting, and consequently use an isotropic source in contact with a scatterer, instead of the collimated arrangement of TRUMP and VAN DE GRAAFF. The experimental results are therefore not directly comparable, though the absolute values prove to be not greatly different for the two geometries. The data assembled by BURTT⁽⁶⁴⁾ for isotropic sources of negative electrons in contact with plane back-scatterers are shown in Figs.

2.11.4 and 2.11.5. These results, like those in Fig. 2.11.3, do not include back-scattered electrons of very low energy. The mass per unit area of back-scatterer

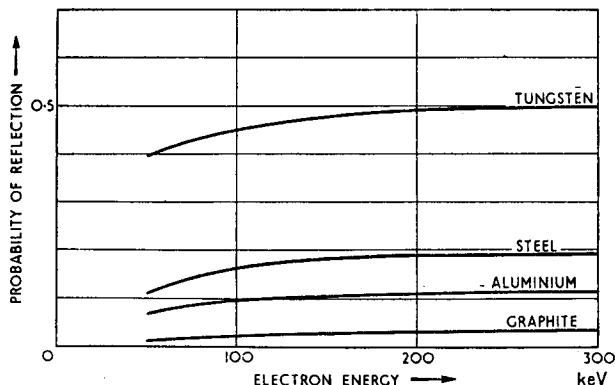


Fig. 2.11.3. Probability of back-scattering of electron normally incident on surface (see text).

M_{sat} required for saturation back-scattering is found to be very nearly independent of the atomic number of the scatterer, but depends on the end-point energy of the beta emitter according to the following empirical relationship, which is due

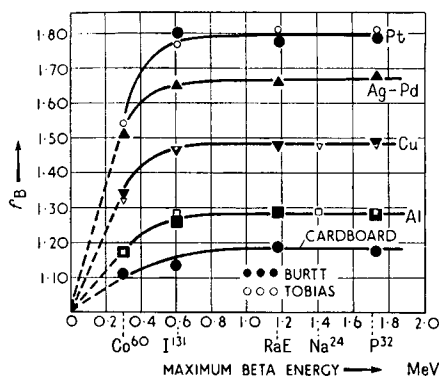


Fig. 2.11.4. Saturation back-scattering as a function of maximum beta energy for several backing materials in contact with an isotropic source of beta particles (BURTT⁽⁶⁴⁾).

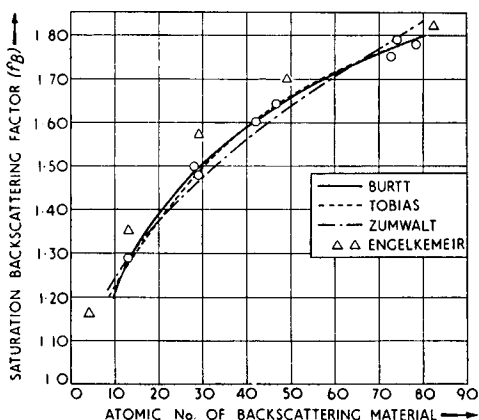


Fig. 2.11.5. Saturation back-scattering factor as a function of atomic number of backing material (BURTT⁽⁶⁴⁾).

to YAFFE and JUSTUS:⁽⁶⁶⁾ $M_{\text{sat}} = 0.116E_{\text{max}}^{2/3}$, where M_{sat} is in grams per cm^2 ($E_{\text{max}} < 3 \text{ MeV}$).

Positrons (from Na^{22} ; end-point 0.58 MeV) have been shown by SELIGER⁽⁶⁷⁾ to be back-scattered to a somewhat smaller extent than negative electrons (from P^{32} ; end-point 1.72 MeV). This result is to be expected, on account of the finite

probability for the annihilation of a positron while it is still moving with a considerable fraction of its original energy (see Section 2.4).

YAFFE and JUSTUS⁽⁶⁶⁾ and SELIGER⁽⁶⁷⁾ have also shown that the back-scattering of electrons is highly anisotropic, being concentrated in the direction of the normal to the scattering surface.

(c) *X- and Gamma-ray Production during the Stopping of Electrons*

Before an electron is brought to rest in a thick target it may suffer a number of collisions with nuclei, in each of which it would experience an acceleration deflecting it from its original path. Classical physics asserts that when a charged particle is accelerated it necessarily emits electromagnetic radiation. A quantum-mechanical treatment would describe the event less deterministically in terms of a certain probability of the electron making a transition from one state to another with the emission of a photon; but from either point of view one would expect some part of the energy of electrons stopped in a target to be radiated as X- or gamma radiation. This radiation is in fact observed experimentally—the X-ray tube has made it a matter of common experience. It is known as *bremsstrahlung*, i.e. braking radiation.

According to the BETHE-HEITLER theory the average intensity radiated in a collision between an electron and a nucleus is proportional to the square of the atomic number of the nucleus (equation 2.12.1). However, this dependence on Z is observed experimentally only for very thin targets. For thick targets, in which the electron is completely stopped, the total energy radiated is proportional approximately to the first power of Z (Fig. 2.11.7). This may be understood when it is remembered that at electron energies of a few MeV and below the range of an electron is controlled almost entirely by ionizing collisions with the orbital electrons of the target material. The number of nuclei encountered by an electron before it is completely stopped is therefore roughly proportional to Z^{-1} , so that the bremsstrahlung intensity in *thick* targets will be proportional to the first, and not the second, power of the atomic number. An application of the BETHE-HEITLER theory by C-S WU⁽⁶⁸⁾ shows that when electrons of kinetic energy E MeV are stopped in a thick target, the average amount of energy radiated as bremsstrahlung is equal to ωE , where ω is given by the approximate expression:

$$\omega = \frac{1.98 \times 10^{-4} [1.96E + 2]}{[1 + 0.35 \log_{10} (82/Z)]} \cdot Z \quad (2.11.2)$$

The *angular distribution of thick-target bremsstrahlung* does not depend in any simple fashion upon either the energy of the bombarding electron or upon the atomic number of the absorber. The experimental results of BUECHNER *et al.*,⁽⁶⁹⁾ for targets slightly thicker than the maximum range of the electrons, are given in Fig. 2.11.6. Although the dependence of intensity on atomic number in any given direction is complicated, the integrated intensity (Fig. 2.11.7) shows the simple, approximately linear, variation with Z required by equation (2.11.2).

The *spectral distribution of thick-target bremsstrahlung* is roughly independent of the atomic number of the absorber and of the energy of the electron. By an

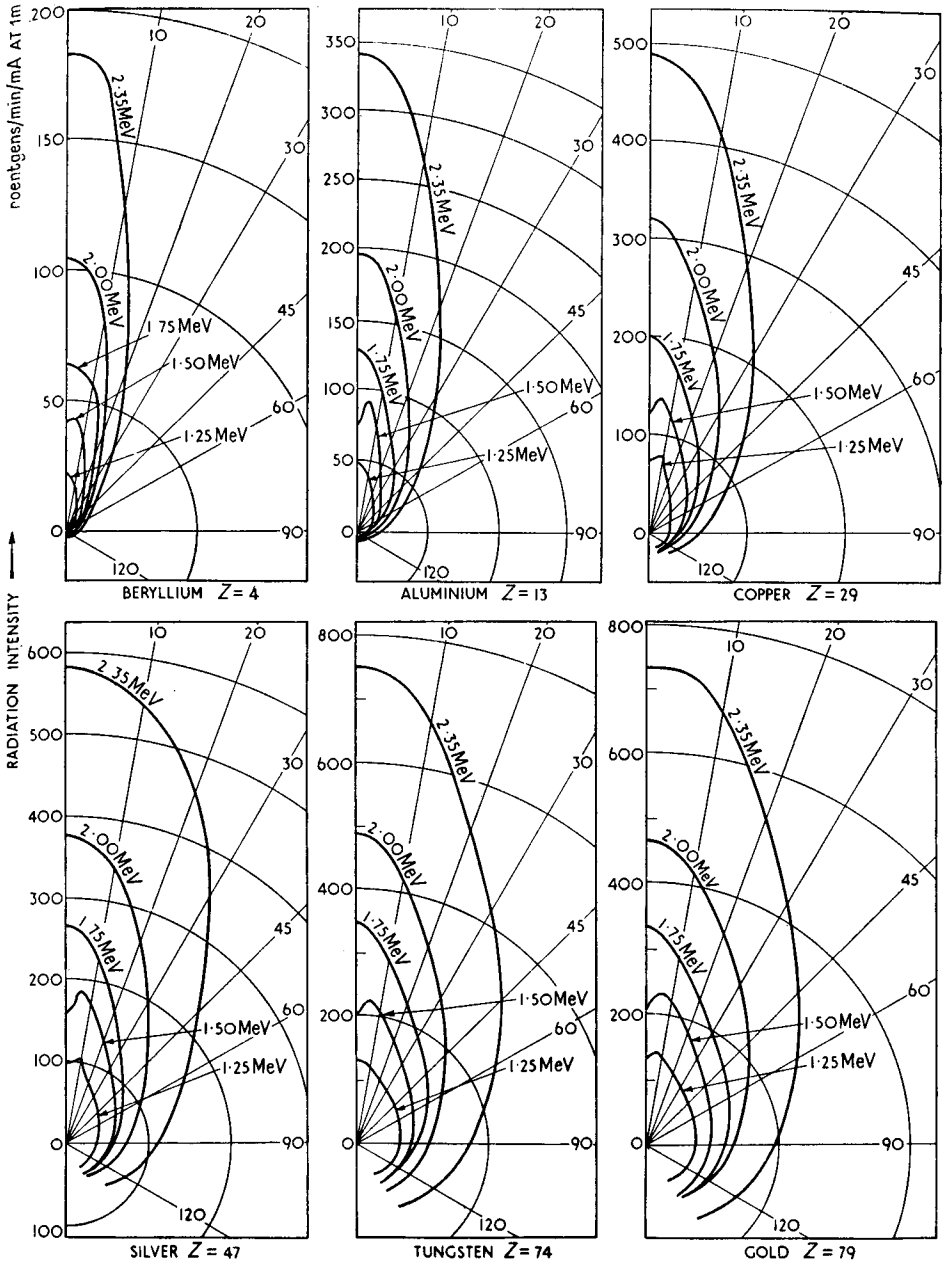


Fig. 2.11.6. Intensity and angular distribution of bremsstrahlung produced by stopping monoenergetic electrons in targets slightly thicker than the maximum range of the electrons (BUECHNER *et al.*⁽⁶⁹⁾).

integration of an expression given by HEITLER, WYARD⁽⁷⁰⁾ finds that the energy radiated per unit energy interval $I(h\nu)$ is given approximately by the formula

$$I(h\nu) = \text{constant} \left\{ 4 \left(1 - \frac{h\nu}{E} \right) - 3 \frac{h\nu}{E} \ln \left(\frac{E}{h\nu} \right) \right\} \quad (2.11.3)$$

where E is the kinetic energy of the monoenergetic electron, and $h\nu$ the (variable) quantum energy. The energy distribution given by this formula, normalized

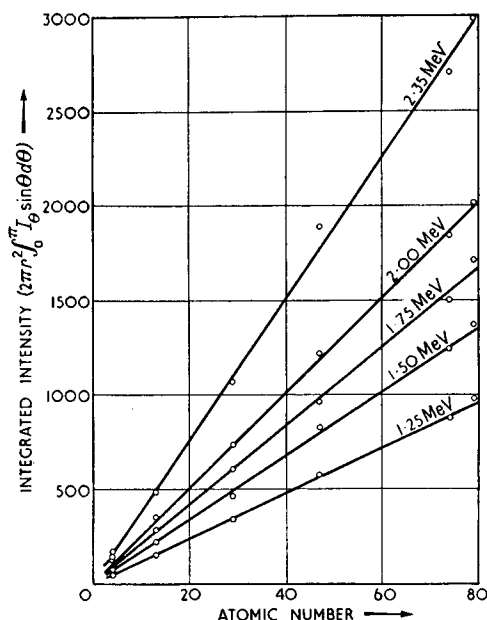


Fig. 2.11.7. Integrated intensity of thick-target bremsstrahlung, showing linear dependence on atomic number. (BUECHNER *et al.*⁽⁶⁹⁾.)

to unit area under the curve, is shown in Fig. 2.11.8 (Curve *A*). Curve *B* in the same figure is the corresponding function for the case when the electrons are not monoenergetic, but have the Fermi distribution characteristic of a beta-emitter. Some experimental results obtained with a tungsten target are given in Fig. 2.11.9. The cut-off at low energies is due to the reabsorption of the softer radiations by the target material.

Superimposed on the continuous energy distribution of the bremsstrahlung are the $K, L, \dots$ peaks of characteristic radiation caused by the ejection of electrons from the inner shells of target atoms during ionizing collisions with the fast incident electrons (cf. fluorescent radiation following photoelectric absorption, Section 2.3). Although these peaks are of importance in X-ray technology, their existence can safely be neglected in the context of electron shielding, for the obvious choice of a light element as a beta shield (to reduce bremsstrahlung production to a minimum) automatically means that the characteristic radiations are of low energy, and therefore easily absorbed.

(d) *Inner Bremsstrahlung*

In addition to the bremsstrahlung resulting from the stopping of beta rays in an absorber (which of course includes the source material), a further small

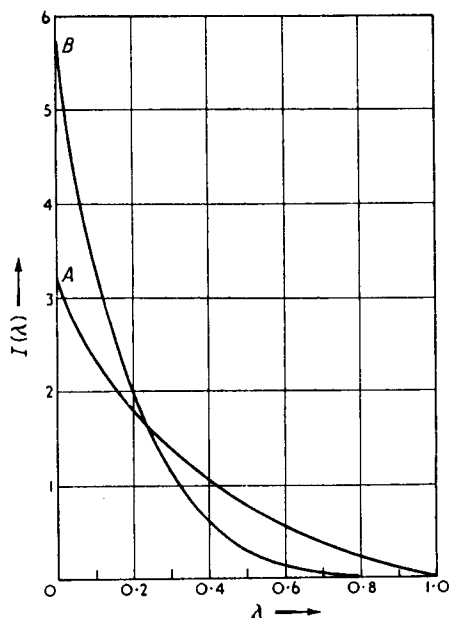


Fig. 2.11.8. Calculated intensity distribution of bremsstrahlung resulting from the complete stopping of a single electron.

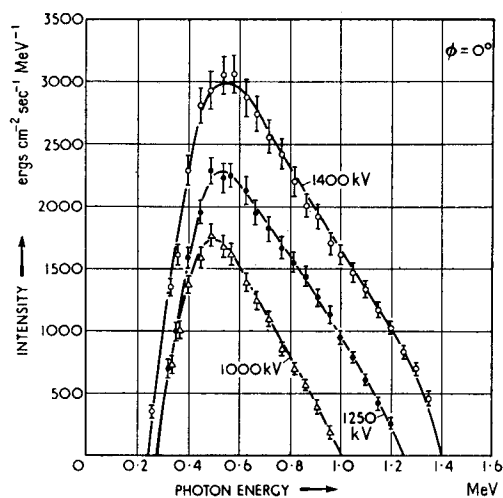
Curve A: Monoenergetic electrons of energy E_0 ; $\lambda = h\nu/E_0$.

Curve B: Beta-ray spectrum of maximum energy $E_{\max}$; $\lambda = h\nu/E_{\max}$.

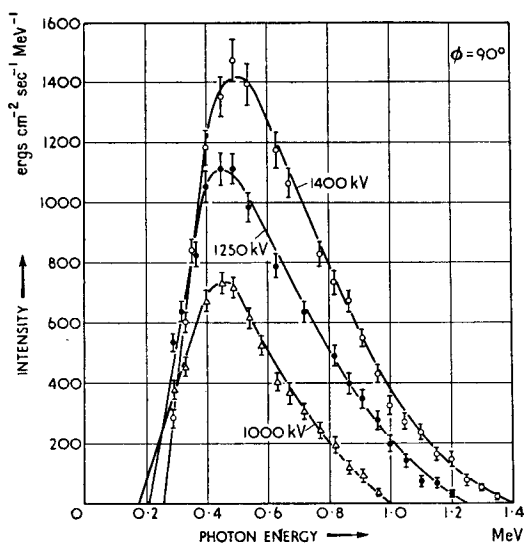
The area under both curves is normalized to unity (after WYARD⁽⁷⁰⁾).

amount of gamma radiation is emitted as an intrinsic part of any “pure” beta decay process. This second type of radiation, which has its origin in the nucleus, is described as *inner* bremsstrahlung, to distinguish it from the extra-nuclear, *outer*, or *external* bremsstrahlung. Its existence is due to the fact that the beta process involves the acceleration of an electron. Although this is a description in classical terms, a detailed quantum-mechanical treatment confirms that the radiation must exist. It is important to realize that it has no connection with the gamma radiation which frequently *follows* a beta decay process, and which is due to the daughter product being left in an excited state by the beta transition. Unlike the inner bremsstrahlung, the latter type of gamma radiation does not have a continuous energy distribution, but consists of a few discrete gamma-ray lines.

BLOCH,^(72a) and KNIPP and UHLENBECK,^(72b) have shown that the fraction of the total energy radiated as inner bremsstrahlung depends only on the energy of the emitted electron; it increases monotonically with electron energy, and



(a)



(b)

Fig. 2.11.9. Bremsstrahlung spectra resulting from the stopping of a collimated beam of electrons in a tungsten target of thickness slightly greater than the maximum range of the electrons. (a) Forward direction ($\phi = 0^\circ$); (b) at right angles to direction of beam ($\phi = 90^\circ$). Beam current reaching target = 0.5 mA. Distance of point of observation from target = 1 metre (MILLER *et al.*⁽⁷¹⁾).

reaches a value of about $\frac{1}{2}\%$ for the very energetic B^{12} decay (end-point 13.4 MeV). Some experimental determinations of the intensity of inner bremsstrahlung are given in Table 2.11.1.

Table 2.11.1. Observed Intensities of Inner (IB) and External (EB) Bremsstrahlung

Isotope	Type	Target	Max. beta energy (MeV)	$\frac{\text{Total photon energy}}{\text{Max. beta energy}}$ (per cent)	Ref.
S^{35}	IB	—	0.168	0.007	74
RaE	IB	—	1.17	0.037	75
Y^{91}	IB	—	1.5	0.064	75
P^{32}	IB	—	1.72	0.07	75
				0.085	76
P^{32}	EB	Al	1.72	0.26	76
P^{32}	EB	Sn	1.72	1.4	76

A similar weak continuous high-energy gamma-ray spectrum accompanies K electron capture (p. 144). The number of gamma quanta per K electron captured is expected to be $\sim(\alpha/12\pi)(W/mc^2)^2$, where W is the energy available for the capture process, and $\alpha(\approx 1/137)$ is the fine structure constant.⁽⁷³⁾

The *spectral distribution* of the inner bremsstrahlung covers the complete range of possible photon energies, from zero to the beta end-point energy $E_{\max}$, but the majority of the energy is concentrated in the softer radiations: for high-energy electrons, photon energies less than $\frac{1}{4}E_{\max}$ account for more than half of the energy radiated.

The conversion of some of the disintegration energy into penetrating gamma radiation by the two processes of inner and outer bremsstrahlung limits the amount of pure beta emitter that can be despatched by normal freight services without danger to unexposed photographic plates in transit in the same vehicle. For instance, as we shall see later, the American rules governing the shipment of radioisotopes would require special arrangements to be made for the carriage of more than about a millicurie of P^{32} , unless some gamma-ray shielding were provided.

2.12. GAMMA RADIATION (BREMSSTRAHLUNG) PRODUCED DURING THE STOPPING OF HIGH-ENERGY ELECTRONS

The rate of energy-loss by ionization in condensed media is practically constant for electrons of all energies greater than about 2 MeV. In addition, a high-energy electron loses energy by the emission of bremsstrahlung at a rate which increases with increasing energy. The fractional rate of energy loss (i.e. $U^{-1} dU/dx$) increases asymptotically to a maximum value. This maximum is considerably greater for materials of high atomic number than for light elements. The

rapid dependence on Z may be understood classically, since the intensity of the emitted radiation should be proportional to the square of the acceleration experienced in a collision; this in turn is proportional to Z^2 . The same consideration shows that the contribution to bremsstrahlung from collisions with orbital electrons should be relatively unimportant; it may be taken into account by replacing Z^2 in the theoretical expressions by $Z(Z + g)$, where g is a number of the order of, but less than, unity.^(14,78)

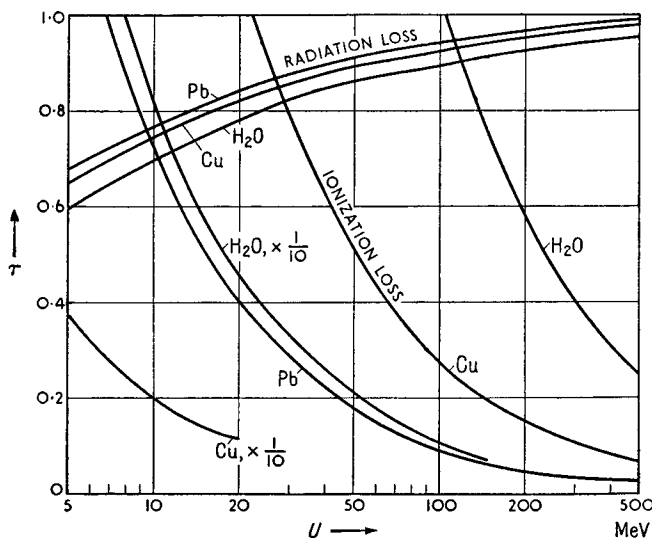


Fig. 2.12.1. Fractional energy-loss τ per radiation length by ionization and radiation, as a function of U , the total energy (i.e. rest and kinetic) of an electron passing through lead, copper and water (LAWSON⁽⁸⁰⁾).

For high-energy electrons of *total* energy $U = mc^2 + E_0$, where $U \gg 137mc^2Z^{-1/3}$, the average loss of energy by radiation per gram per cm^2 , $(dU/dx)_{\text{rad}}$, is given by⁽⁷⁸⁾

$$-\left(\frac{dU}{dx}\right)_{\text{rad}} = 4\alpha \frac{N}{A} Z^2 r_0^2 U [\ln(183Z^{-1/3}) + \frac{1}{18}] \quad (2.12.1)$$

where α is the fine structure constant $= 1/137$, N is Avogadro's number (6.02×10^{23}), A is the atomic weight and Z the atomic number of the target material, and r_0 is the classical radius of the electron (2.82×10^{-13} cm). It is convenient to introduce a quantity referred to as a *radiation length*, X_0 gcm^{-2} , and defined by the equation

$$1/X_0 = 4\alpha(N/A)Z^2 r_0^2 \ln(183Z^{-1/3}). \quad (2.12.2)$$

Equation (2.12.1) then becomes

$$-X_0(dU/dx)_{\text{rad}} = (1 + b)U, \quad \text{where } b \approx 0.0135. \quad (2.13.3)$$

This expression shows that high-energy electrons lose by radiation approximately $1/e$ of their energy when traversing one radiation length of absorber. For values of U less than those for which equation (2.13.3) applies, the fractional energy-loss per radiation-length, $\tau_{\text{rad}} = (X_0(dU/dx)_{\text{rad}}U^{-1})$, is smaller than the limiting high-energy value; but from Fig. 2.12.1 there is no rapid variation of τ_{rad} with either U or Z . In contrast, the competing process of energy-loss by ionization (collision loss) also shown in the same figure, gives a fractional energy-loss per radiation length $(X_0(dU/dx)_{\text{ion}}U^{-1})$ which is a rapidly varying function of U . The ionization loss becomes unimportant in comparison with the radiation loss at high energies. The critical energy E_c MeV at which the two types of loss are equal in magnitude is given, according to BETHE and HEITLER,⁽¹⁴⁾ by the approximate relation

$$E_c \approx 800/Z \tag{2.12.4}$$

while the ratio of the radiation and ionization losses is

$$\frac{(dU/dx)_{\text{rad}}}{(dU/dx)_{\text{ion}}} \approx UZ/800 \tag{2.12.5}$$

where the electron energy U is in MeV.

Table 2.12.1 gives values of X_0 and E_c for a number of elements. For a mixture of elements the radiation length is given by

$$(1/X_0)_{\text{mixt}} = \Sigma(p_i/X_i) \tag{2.12.6}$$

where p_i is the fraction by weight of the i th element.

Table 2.12.1. *Radiation Lengths and Critical Energies*

Substance	Z	A	X_0 (g cm ⁻²)	E_c (MeV)
Hydrogen	1	1	58	400
Helium	2	4	85	260
Carbon	6	12	42.5	120
Nitrogen	7	14	38	100
Oxygen	8	16	34.2	90
Aluminium	13	27	23.9	57
Argon	18	39.9	19.4	43
Iron	26	55.8	13.8	31
Copper	29	63.6	12.8	28
Lead	82	207.2	5.8	11
Air			36.5	110
Water			35.9	120

The values given in the last column are approximate.

Energy Distribution of Bremsstrahlung

The differential radiation probability per unit radiation length, $\phi(U,v) dv$ is defined⁽⁷⁸⁾ as the probability that a photon having an energy W lying in the range $W = vU$ to $(W + dW) = (v + dv)U$ will be emitted during the passage of the electron through one radiation length of absorber. Then the function

$v\phi(U, v) = \Phi(U, v)$ given in Fig. 2.12.2 for air and lead describes how the energy carried off is distributed between photons having different values of v . The advantage of using Φ rather than ϕ is that the former remains finite at $v = 0$.

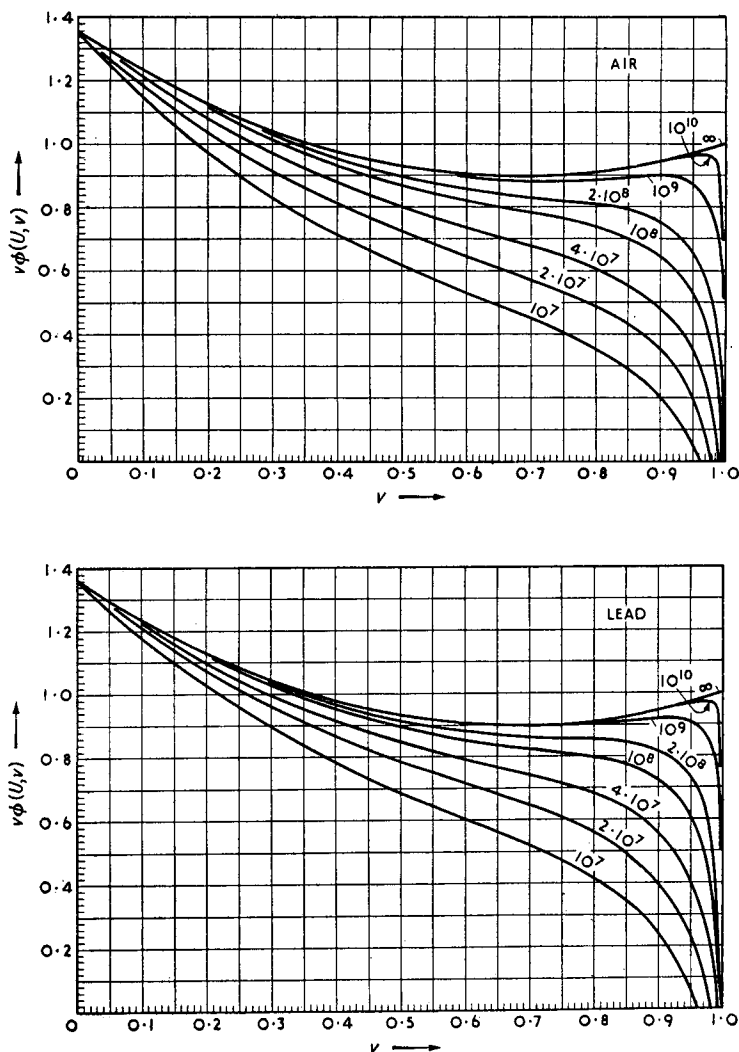


Fig. 2.12.2. Differential radiation probability per radiation length for electrons of various energies (a) for air, (b) for lead. The numbers attached to the curves indicate the energy U of the primary electron in eV. The functions v and ϕ are defined in the text (ROSSI and GREISEN⁽⁷⁸⁾).

It will be seen that there is a large probability that an electron will lose a considerable fraction of its energy in a single collision, a phenomenon which is largely responsible for the straggling of high-energy electrons.

Angular Distribution of Bremsstrahlung

As with other phenomena that occur at relativistic velocities, the photons emitted by an electron during a collision with a nucleus are predominantly emitted in the forward direction. The majority lie within a cone of semi-angle $\theta = mc^2/U$. The exact angular distribution of the radiation emitted by a collimated beam of electrons is of importance in the shielding of high-energy electron accelerators, since a given amount of energy emitted as bremsstrahlung will be more difficult to shield the more it is concentrated in the forward direction. The calculation of this angular distribution can be either very difficult, or comparatively straightforward, depending on the target thickness, which we shall denote by the symbol t (radiation lengths). We shall make use of the treatment of the problem given by LAWSON⁽⁸⁰⁾ and shall consider four ranges of t values.

Case I, t very small ($\ll 5 \times 10^{-4}$). The electrons will not have lost their initial collimation by multiple scattering, and the angular distribution of the emitted radiation will be determined solely by the intrinsic distribution of the bremsstrahlung process. According to SCHIFF⁽⁸¹⁾ (and with the notation used above), the cross-section per atom $\sigma(vU, \theta) U dv d\theta$ for the emission of a photon with energy between vU and $(v + dv)U$ at an angle between θ and $(\theta + d\theta)$ is given by

$$\sigma(vU, \theta) U dv d\theta = \frac{4Z^2 r_0^2}{137} \frac{dv}{v} \theta d\theta \frac{U^2}{\eta^2} A$$

$$\text{where } A = \left\{ \frac{\eta^4}{(\eta^2 + U^2 \theta^2)^2} [(v^2 - 2v + 2) \ln M(\theta) - (2 - v)^2] \right. \\ \left. + \frac{\eta^6 U^2 \theta^2}{(\eta^2 + U^2 \theta^2)^4} [(1 - v)(16 - 4 \ln M(\theta))] \right\}$$

$$\text{and } \frac{1}{M(\theta)} = \left[\frac{\eta v}{2U(1 - v)} \right]^2 + \left[\frac{\eta^2 Z^{1/3}}{111(\eta^2 + U^2 \theta^2)} \right]^2 \quad (2.12.7)$$

where for compactness the symbol η has been used in place of mc^2 for the rest energy of the electron. This formula reduces to a simple expression for the cross-section per unit solid angle at $\theta = 0$. The spectrum is not strongly dependent on angle, especially for large v , and the calculated energy distribution of the photons emitted in the forward direction (Fig. 2.12.3) is therefore not greatly different from the overall spectrum. The form of the latter may be obtained by integration of equation (2.12.7):

$$U dv \int_0^\infty \sigma(vU, \theta) d\theta = \frac{2Z^2 r_0^2}{137} \frac{dv}{v} \left\{ \left(\frac{4}{3} - \frac{4}{3} v + v^2 \right) \left(1 + \ln M(0) - \frac{2}{b} \tan^{-1} b \right) \right. \\ \left. + (1 - v) \left[\frac{2}{b^2} \ln(1 + b^2) + \frac{4}{3b^3} (2 - b^2) \tan^{-1} b - \frac{8}{3b^2} + \frac{2}{9} \right] \right\}$$

$$\text{where } b = 2U(1 - v)Z^{1/3}/111 v mc^2. \quad (2.12.8)$$

This equation, which gives results which are not significantly different from those of ROSSI and GREISEN (Fig. 2.12.2), has the useful feature of being in closed form.

Case II. Somewhat thicker targets ($t \sim 5 \times 10^{-4}$). Multiple scattering of the electrons has now become a factor contributing to the breadth of the X-ray beam; but the calculation becomes extremely difficult when the spread due to multiple scattering is of the same order as the intrinsic spread. The root mean square angle of multiple scattering after traversing a target of thickness t radiation lengths has been shown by ROSSI and GREISEN⁽⁷⁸⁾ to be $\approx (440t)^{1/2}/U$ (U in MeV); this value has to be compared with mc^2/U which is the order of magnitude of the intrinsic spread of the emitted radiation. Case I corresponds to $mc^2/U \gg (440t)^{1/2}/U$, or $t \ll (5 \times 10^{-4})$ radiation lengths, Case III to $1 \gg t \gg (5 \times 10^{-4})$, and Case II the difficult intermediate condition.

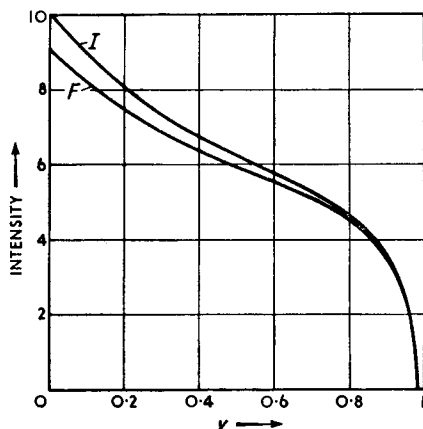


Fig. 2.12.3. Forward and integrated spectra for $U = 24.5 mc^2$ and $Z = 78$. The ordinates are computed from those parts of equations (2.12.7) and (2.12.8) which are inside the curly brackets. (LAWSON⁽⁸⁰⁾).

Case III. Moderately thick targets. For still thicker targets, in which the electrons have not on the average lost an appreciable fraction of their original energy, but have been multiply-scattered to a considerable degree, the angular distribution of the bremsstrahlung is determined almost entirely by the angular distribution of the electrons. It has been shown by LAWSON⁽⁸⁰⁾ that in this case the fraction of the electron energy emitted per unit solid angle in the forward direction $R(0)$ (see Fig. 2.12.5) is given by

$$R(0) = \frac{U^2}{440} \frac{\tau_{\text{rad}}}{\pi} \ln(950t) \quad (2.12.9)$$

where τ_{rad} is the fractional loss of energy by radiation per radiation length, given in Fig. 2.12.1. At values of $\theta \neq 0$, $R(\theta)$ is given to a satisfactory degree of accuracy by

$$R(\theta) = \frac{\tau U^2}{440\pi} [E_1(U^2\theta^2/440t) - E_1(U^2\theta^2/\gamma m^2 c^4)] \quad (2.12.10)$$

E_1 being the exponential integral shown graphically in Fig. 5.1.1 and $\gamma = 1.78$ being EULER'S constant. Some numerical values of $R(\theta)$ are shown in Fig. 2.12.4.

For Case III the spectrum does not depend on the direction of emission, and will therefore have the same shape as the integrated spectrum given in Fig. 2.12.2. To obtain the absolute numbers of quanta emitted per unit solid angle the data of Fig. 2.12.2 must be multiplied by a factor equal to $R(\theta)/\tau$. Although the formulae given above are not entirely in agreement with experiment at large angles⁽⁷⁷⁾ they are sufficiently accurate for most purposes, and are considerably simpler to apply than some other formulae that have been developed.

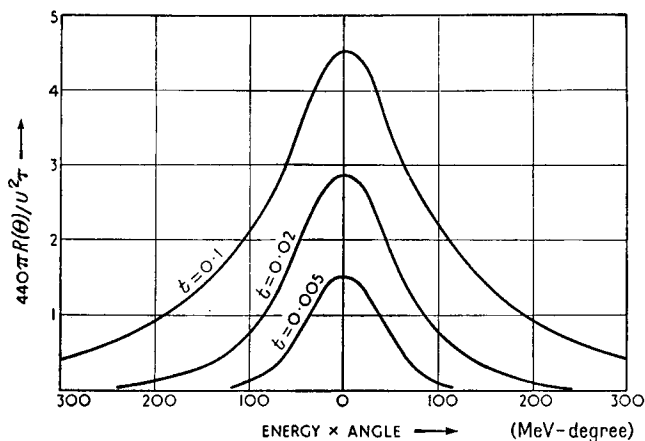


Fig. 2.12.4. Angular distribution of radiation for various target thicknesses. See text for definition of $R(\theta)$, U and τ . The target thickness t is measured in radiation lengths (LAWSON⁽⁸⁰⁾).

Case IV. Very thick targets. The cases already discussed arise in practice with betatrons and other cyclic accelerators. The electrons spiral towards the target until they just begin to graze it; they then emit radiation, change their energy abruptly, depart from their correct orbit, and are lost in the walls of the "donut." Thus whatever the physical shape of the target, a considerable amount of radiation from betatrons and synchrotrons will have thin-target characteristics. In contrast, linear accelerators usually send a collimated beam normally into the target, and in this case the electrons are frequently stopped altogether. For low U and low Z , when ionization is the most important source of energy-loss in the target, it would be comparatively straightforward to compute the intensities and spectra by integrating over the electron energy distribution in the target; the effect of the slowing down of electrons would then be to diminish the proportion of high-energy photons in comparison with those of lower energy.

BETHE and HEITLER⁽¹⁴⁾ have given a simplified treatment of the case when radiation losses are the important factor, but an accurate calculation is difficult. However, the forward intensity may be estimated with an accuracy sufficient for most purposes by taking advantage of the logarithmic dependence of intensity on target thickness. If t is first put equal to 0.1, and then to 1.0, two values of

$R(0)$ (equation 2.12.9) will be obtained, differing by 50%. These two values would be expected to bracket the true value. This has been verified experimentally for a copper target at 7.3 MeV.⁽⁸²⁾

Shield Thickness Required for Electron Accelerators

Provided the intensity, angular distribution, and energy distribution of the gamma rays produced at the target of an electron accelerator are known, the shielding required can be roughly estimated using (a) the narrow-beam absorption coefficients given in Fig. 2.6.4, (b) the build-up factor data discussed in Section 2.7, and (c) the factor given in Fig. 1.5.1 for converting photon flux to biological dose. Of these, (c) is at best an approximation at these high energies, for reasons which will be discussed in the next section, while (b) involves an

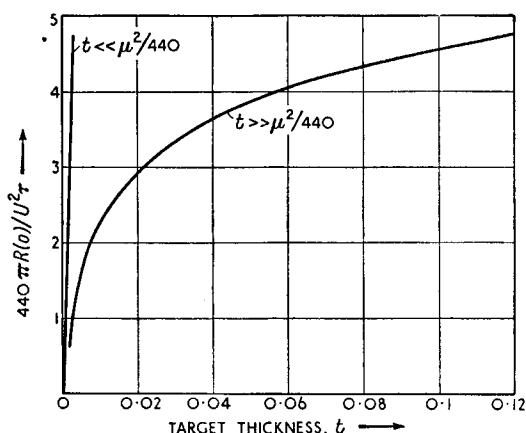


Fig. 2.12.5. Forward intensity as a function of target thickness. See text for definition of $R(0)$, U and τ ; μ represents the electron rest energy (0.51 MeV). The target thickness t is measured in radiation lengths (LAWSON⁽⁸⁰⁾).

extrapolation of the existing data. Moreover, this procedure neglects the fact that high-energy photons can produce high-energy Compton recoils, which in turn can give rise to further bremsstrahlung. An exact calculation would involve an elaborate cascade theory treatment. In these circumstances it is simpler to rely whenever possible on experimental results obtained with actual shields.

Some measurements^(83,84) of the attenuation of bremsstrahlung by ordinary concrete, for various electron energies, are given in Figs. 2.12.6 and 2.12.7. They show that by the time an electron energy of 38 MeV has been reached, the attenuation per unit length has become almost independent of energy. This is to be expected, since the medium-heavy elements of which concrete is largely composed have a minimum value for the total gamma ray cross-section in this energy region (Fig. 2.6.4). The build-up factor was found to be no more than 2 even for an attenuation of 10^5 . This low value, which is characteristic of the very high-energy range, is due to two main causes: as the energy goes up the pair production absorption process becomes more important in comparison with

Compton scattering (Fig. 2.5.3): while at the same time Compton scattering it elf removes a greater fraction of the photon energy per collision and therefore behaves more like an absorption process (Table 2.5.2).

The comparatively small extent to which the collimation of the gamma ray beam is destroyed by multiple scattering in the shield is a particularly noticeable

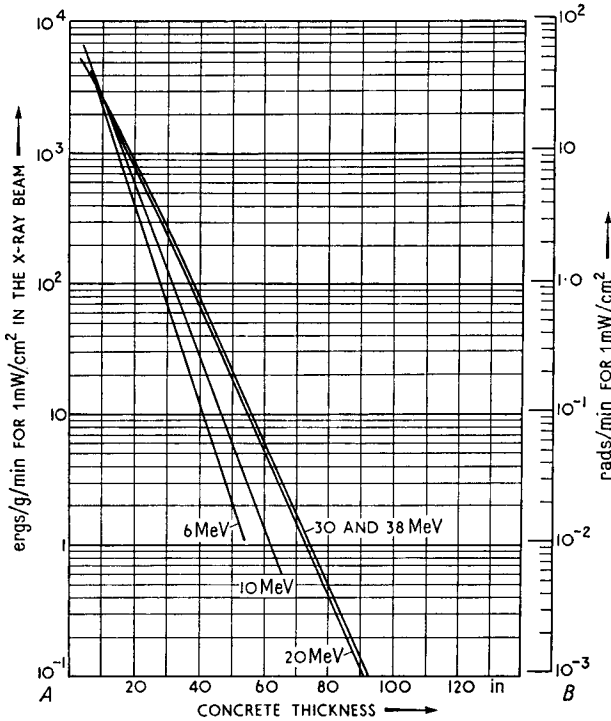


Fig. 2.12.6. Attenuation by a concrete shield of biological dose from bremsstrahlung produced by a betatron operating at energies of 6, 10, 20, 30, 38 MeV.

Density of concrete: 147 lb/ft³.

Scale A: Absorbed energy in engs/g/min behind various thicknesses of concrete, plotted for a beam intensity of 1 milliwatt/cm² at the same position. The energy absorbed is calculated on the assumption that 93 engs/g = 1 e.s.u./cm³ of tissue.

Scale B: the dose received in rads for the same conditions as scale A (from N.B.S. Handbook 55).

feature of Fig. 2.12.7, and one which emphasizes the predominantly forward nature of high-energy gamma ray scattering. It is clear that for thin gamma targets it would be wasteful to give as much shielding in the sideways direction as in the forward direction. However, with experimental accelerators one cannot be sure that work will always be confined to the well-collimated forward-directed beams given by thin targets; and most designers prefer to buy flexibility at the cost of some extra sideways shielding. It must also be borne in mind

that, where the machine is to be used for neutron production, it is possible that the neutrons, and not the gamma rays, may in some cases be the limiting factor as far as shielding is concerned. This is particularly true when the source employs photoneutron production from heavy elements, since thick targets of these elements are at once copious sources of neutrons (Section 3.11) and efficient gamma ray absorbers.

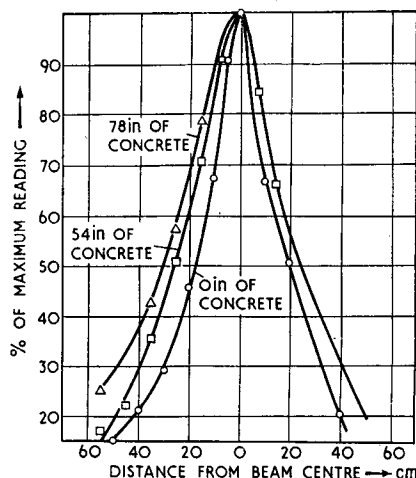


Fig. 2.12.7. Distribution of dose behind concrete barriers placed in gamma-ray beam of a betatron operating at 38 MeV. Target-to-detector distance was 611 cm (from KIRN and KENNEDY⁽⁸³⁾).

2.13. MEASUREMENT OF HIGH-ENERGY GAMMA-RAY INTENSITY

At low energies the unit of quantity of X- or gamma radiation which is normally employed is the röntgen; but, as stated briefly in Chapter 1, above about 3 MeV its use is not recommended. The reasons why this limitation should be necessary will become clear if we consider how a measurement in röntgens would be made. Since the definition refers to a quantity of ionization liberated in a given mass of air, the simplest type of instrument to use would be an air ionization chamber. The definition presupposes that it is possible to measure *all* the "associated corpuscular emission" liberated in a given volume. This is clearly quite simple as long as the Compton recoil electrons, or the photoelectrons, have ranges small compared with the dimensions of the ionization chamber used for the measurement; but at higher photon energies, when their range becomes of the same order as the chamber dimensions, or exceeds them, it seems at first sight as though it would be difficult to make a measurement that fulfils the requirements of the definition sufficiently rigorously. Not only would the chamber fail to record all the associated corpuscular emission from the sample of air in the chamber, but an increasing fraction of the observed ionization would be the result of the interaction of the radiation with the wall of the chamber. Table 2.13.1 shows that wall effects become important, even in quite large air chambers, at comparatively low quantum energies.

The effect of the chamber wall can be eliminated by choosing as the wall some material that has as nearly as possible the same radiation absorbing properties, weight for weight, as air. This is not a difficult choice to make, since in light

Table 2.13.1 *Maximum Range of Compton Recoil Electrons*

Quantum energy (MeV)	Maximum energy of Compton recoil electron (MeV)	Maximum range of recoil electron in air at S.T.P. (cm)
0.1	0.028	1.1
0.2	0.088	9
0.5	0.331	68
1.0	0.798	220
4.0	3.78	1400
10.0	9.77	3800
20.0	19.8	9000

elements the Compton process is the most important method by which gamma rays of a wide range of energies (0.1 to 10 MeV) cause ionization. This process involves collisions with electrons, and its probability therefore depends simply on the electronic density of the material, which in all light elements other than hydrogen is close to 3×10^{23} electrons per gram. An "air-walled" chamber is therefore made of some light element (usually graphite), and the wall thickness is chosen to satisfy two conditions: (a) it must not be so thick that it appreciably attenuates the gamma-ray beam, (b) it must be thicker than the range of the most energetic recoil electrons which can be produced by the gamma rays being studied. When both of these conditions are fulfilled, a state of electronic quasi-equilibrium exists in the chamber wall. Let us assume further that (c) the air cavity of the chamber is small compared with the range of the secondary electrons in air, and that (d) the gamma-ray absorbing properties of the graphite air-wall can be regarded as sufficiently close to the properties of solid air of the same electron density. Under these four conditions the ionization energy produced in the cavity is independent of the wall thickness, and the number of recoil electrons entering the cavity from the walls exactly balances the energy carried away into the walls by electrons produced in the air cavity. The chamber therefore effectively measures the whole of the associated corpuscular emission, as the definition requires. The energy absorbed locally per unit mass in the wall L is then related to the number of ion pairs per unit mass of gas in the cavity J by the BRAGG-GRAY relationship:

$$L = JW_s, \quad (2.13.1)$$

where W is the average energy expended by the ionizing particles per ion pair formed in the gas, and s is the ratio of the mass stopping power of the wall material to that of the gas for electrons. Thus, provided electronic equilibrium has been established, a measurement of the ionization in the cavity also serves the useful purpose of being an indirect measurement of the rate of energy deposition in the wall.

The quantity that determines to what extent one can achieve in practice the state of electronic equilibrium required by the definition of the röntgen is the factor by which the gamma rays are attenuated in traversing a wall thickness equal to the range of the most energetic Compton recoil electrons. When this factor departs appreciably from unity it becomes difficult to obtain even an approximate measurement of the quantity of radiation in röntgens. It will be seen from Fig. 2.13.1 that this is the case at energies above a few MeV.

A further complication is that at high energies a portion of the gamma radiation absorbed in the wall of the ionization chamber can be reconverted to electromagnetic radiation, either as bremsstrahlung resulting from the slowing

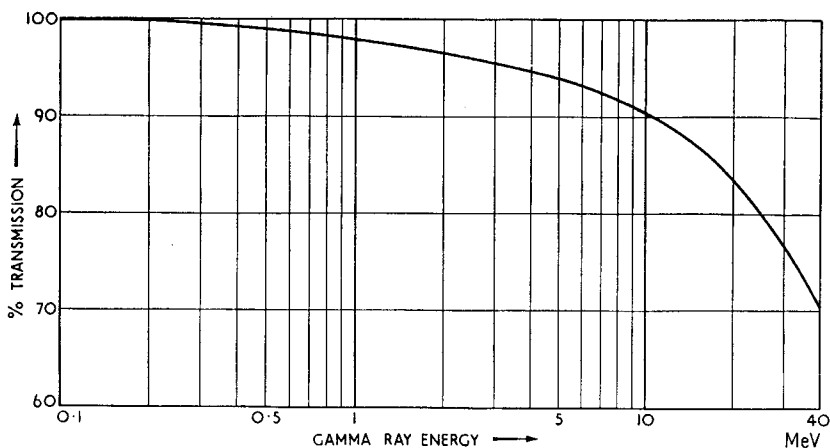


Fig. 2.13.1. Transmission of gamma rays through a wall of graphite of thickness equal to the maximum range of Compton recoil electrons.

down of energetic recoil electrons, or from annihilation or fluorescent radiation following absorption. The definition of the röntgen gives no guidance as to whether this portion of the gamma radiation should be counted or ignored.

As a result of these practical difficulties the International Commission on Radiological Units does not recommend the use of the röntgen as a unit at gamma ray energies above 3 MeV (for this reason the high-energy portion of Fig. 1.5.1. is shown as a broken line). At higher energies the radiation intensity should be measured in photons $\text{cm}^{-2} \text{sec}^{-1}$, or in watts cm^{-2} . These are quantities which are capable of direct measurement, either calorimetrically or by means of an ion chamber whose efficiency is calculable.

Intensity determination by calorimetry⁽⁸⁵⁻⁸⁸⁾ is a method which has been in use since the early days of the subject,⁽⁸⁹⁻⁹²⁾ but which has since been largely superseded by the more convenient methods based on ionization. A block of some dense material, preferably shaped so that the absorption of the beam is as high as possible, and the loss by scattering as low as possible, is placed in an evacuated vessel in the flux of radiation whose intensity is to be determined. The rise in temperature of the material is measured by means of a thermocouple or thermistor (i.e. a thermally sensitive resistor), and compared with the change in

temperature of a standard sample, mounted in a similar arrangement but out of the beam. The apparatus used by LAUGHLIN and BEATTIE^(85,86) for measuring the energy flux from a 22.5 MeV betatron is shown in Fig. 2.13.2; an accuracy of about 5% was attainable.

Ionization chamber of calculable efficiency. If a cavity chamber of the type shown schematically in Fig. 2.13.3 is exposed to a beam of high-energy radiation in a site remote from objects which can scatter radiation, the ion current will depend in a perfectly definite way on the wall thickness, the energy flux, and the

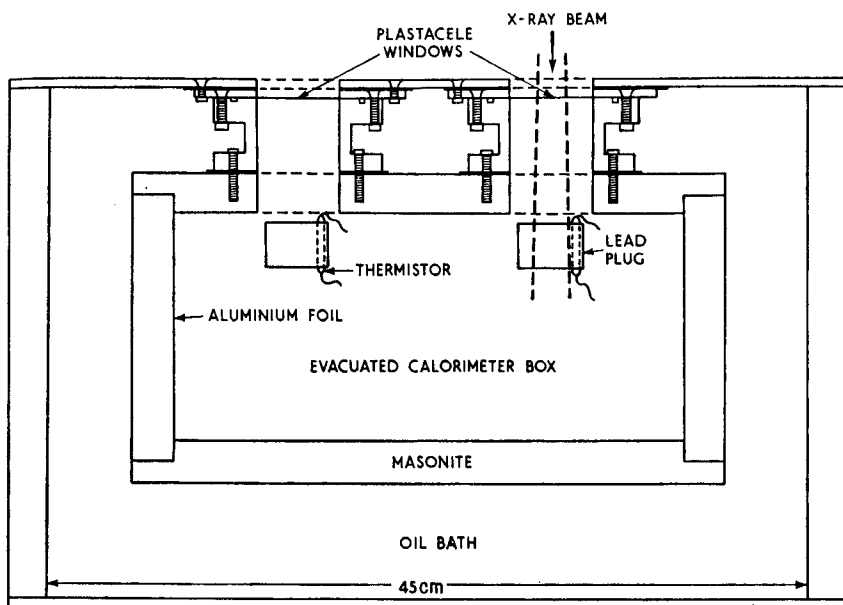


Fig. 2.13.2. Calorimetric determination of gamma ray intensity (LAUGHLIN and BEATTIE^(85, 86)).

quantum energy. It is possible to calculate the efficiency of such a chamber to an accuracy of about 5%, provided certain simplifying conditions are satisfied: the chamber should be made of a light element, and the radiation should be of not too high an energy, so that the recoil electrons lose energy mainly by ionization. Several authors have discussed chambers of this kind, with walls of aluminium⁽⁹³⁻⁹⁵⁾ or graphite.⁽⁹⁶⁻⁹⁸⁾ When differences in wall material are taken into account, the calculations agree within the expected accuracy. Such chambers may therefore be regarded as secondary standards suitable for use up to quantum energies of about 25 MeV. Above this energy the radiation loss in the walls can no longer be neglected, and in addition the range of the secondary electrons becomes very large. Both of these factors complicate the calculation, and make it difficult to obtain results of high accuracy.

For radiation of a given quantum energy the efficiency of the chamber illustrated in Fig. 2.13.3 varies with the wall thickness in the manner shown in Fig. 2.13.4. As the thickness of the aluminium wall is increased from zero the

ion current per watt of radiation rises at first linearly; but at a thickness somewhat less than the range of the most energetic secondary electrons the curve flattens off and passes through a maximum. Thereafter, the efficiency decreases

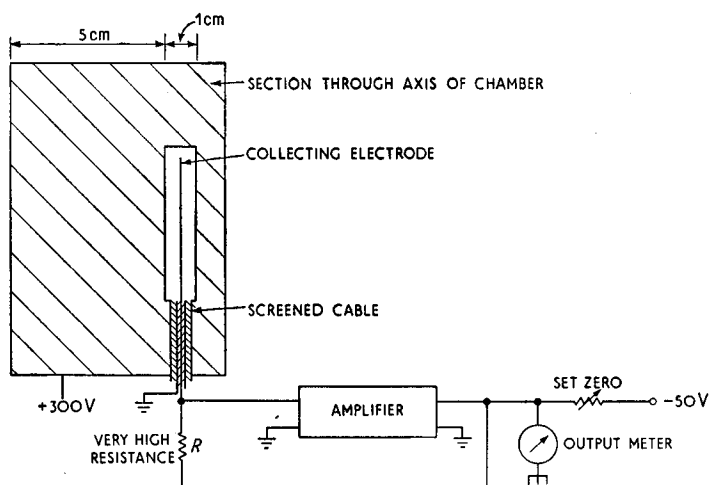


Fig. 2.13.3. Schematic diagram of the aluminium-walled ionization chamber to which the results of Fig. 2.13.4 refer.

monotonically with increasing thickness due to the absorption of the incident gamma radiation by the wall. It is typical of the behaviour of high-energy gamma radiation that the maximum ionization density occurs at an appreciable depth;⁽⁹⁹⁾

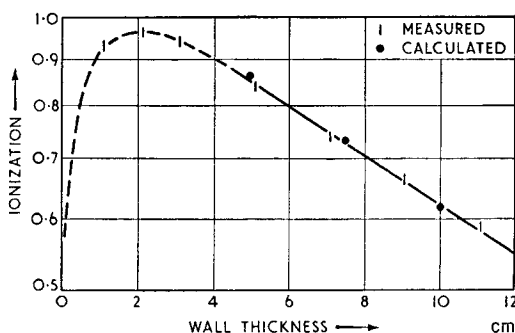


Fig. 2.13.4. Curve of ionization current against wall thickness obtained with 23 MeV bremsstrahlung from a tungsten target. Theoretical points normalized to give the best fit are also shown (FLOWERS *et al.*⁽⁹⁹⁾).

this is the fundamental reason why radiation therapy for deep-seated tumours has necessitated the use by hospitals of betatrons for the production of these radiations.

It is not always convenient to use the substandard type of ionization chamber referred to above, and a number of other arrangements based on the readily available commercial thimble chambers have been widely used (see reference 84

for summary and bibliography). A typical arrangement, the one actually used in obtaining the results of Fig. 2.12.6, consists of a block of lucite of side 11.5 cm, in which a 25 r Victoreen chamber is centrally situated. It is hardly possible to calculate the efficiency of this type of detector, but experimentally⁽¹⁰⁰⁾ it is found that below 25 MeV the response to radiations of different energies is not very different from that of an aluminium-walled chamber. Above this energy the range of the secondary electrons is greater than the distance of the 25 r chamber from the surface of the lucite cube, the effect being that the efficiency no longer increases as the energy is raised, but tends to a nearly constant value (Fig. 2.13.5).

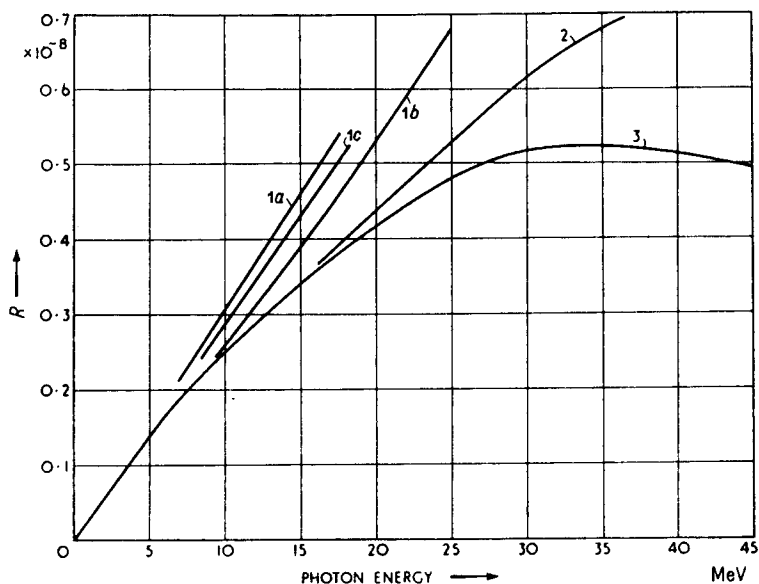


Fig. 2.13.5. Sensitivity of various types of gamma ray detectors (cavity ionization chambers) as a function of photon energy.

R = e.s.u. of ions of either sign per cm^2 of air per photon per cm^2 .

1. Aluminium-wall substandard

(a) SPICER⁽⁹⁵⁾ (b) FLOWERS *et al.*⁽⁹⁴⁾ (5 cm wall)

(c) FOWLER *et al.*⁽⁹⁸⁾ (2.5 cm wall).

2. Graphite-wall substandard (LAX⁽⁹⁶⁾).

3. 25 r thimble chamber in centre of 11.5 cm lucite cube (MCELHINNEY and SIEWERS⁽¹⁰⁰⁾).

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CHAPTER THREE

SHIELDING AGAINST NEUTRONS: NEUTRON PHYSICS

3.1. GENERAL PRINCIPLES

NOTHING comparable with the orderliness of gamma-ray attenuation theory yet exists for neutrons. The much greater complexity of the physical processes, and the difficulties of measuring many of the cross-sections, have combined to divert attention from the analytical approach in favour of more empirical methods. However, shields are so rarely identical that the designer still needs a sufficiently detailed knowledge of nuclear physics and attenuation theory to enable him to make intelligent estimates of the effect of changes in geometry and constitution. When accurate predictions are more than usually difficult to make, it is almost always possible to choose some method of calculation which cannot fail to err on the side of safety. As a result the shield is of course heavier than it need be, and up to the present, when the demand has been for speed in reactor design, this penalty has usually been acceptable. For any mobile power plant, however, it is very necessary to eliminate surplus weight, and then the only reliable test of the shield design is by direct measurements on a full-sized model.

A better appreciation of the difficulties of an analytical approach is given by listing the data which would be handled in an ideal theory. Neutrons may be either captured or scattered by nuclei, with differing probabilities for each constituent of the shield. Scattering processes may be either elastic or inelastic, and they may be isotropic or asymmetrical. Moreover, the majority of neutron cross-sections vary rapidly with neutron energy, so that, as the neutron loses energy by being repeatedly scattered in the shield, the relative proportions of capture and scattering do not remain constant. The variation of cross-section with neutron energy is often violent and erratic, with marked resonance peaks (e.g. Fig. 3.3.2). Finally, when the neutron is eventually captured, the resulting nucleus emits capture gamma rays, which make their own contribution to the biological dose at the surface of the shield. This is a formidable list of variables.

Instead of attempting to calculate the overall effect of the shield by piecing together all the individual processes, it is easier to make integral measurements on actual shields with source neutrons of the correct energy. Ideally one would like to be able to measure, at every point in the shield, the density and angular distribution of neutrons of all energies, and the secondary gamma ray flux and its energy distribution. In practice one has to be satisfied with considerably less information, notably because it is difficult to make measurements with neutrons in the energy range between 1 keV and 100 keV. For neutrons of energy greater than 100 keV (a category which includes the majority of the neutrons formed in the fission process) a number of experimental methods are available, and many of the cross-sections are known with sufficient accuracy.

Below 1 keV there is another region in which measurements are comparatively straightforward. The amount of information available increases rapidly as we go still further down the energy scale, until, for thermal neutrons (page 105) of energy $\sim 1/40$ eV, there are very few properties which have not been measured. Fortunately, as will be explained later, the more easily measurable ranges at the top and bottom of the energy scale are those which are of the greatest assistance in elucidating the behaviour of the shield.

The fact that the properties of neutrons, as measured by their cross-sections, are functions of the neutron velocity, has led to the adoption of the loose but descriptive terminology summarized in Table 3.1.1.

Table 3.1.1. Summary of Terms describing Neutron Energies

Term	Derivation	Range of energies covered	
Thermal	Neutrons in thermal equilibrium with surroundings	Most probable energy at 20°C	0.025 eV
		Maxwellian distribution at 20°C extends to about	0.1 eV
Epithermal	Neutrons of energy greater than thermal	Energies above about	0.2 eV
Cadmium	Neutrons which are strongly absorbed by cadmium	Energies below about	0.4 eV
Epicadmium	Neutrons which are not strongly absorbed by cadmium	Energies above about	0.6 eV
Slow	-----	Usually means less than	1-10 eV
Resonance	In pile neutron physics usually refers to neutrons which are strongly captured in the resonances of U^{238} , or of a few commonly used detectors, e.g. In, Au	Occasionally ⁽¹⁾ means less than	1 keV
		A broad band of energies extending from about to about	1 eV 300 eV
Intermediate	Neutrons of energies between slow and fast	From a few hundred eV to about 0.5 MeV	
Fast	—	Neutrons of energy greater than about	0.5 MeV
Ultra-fast	—	Neutrons of energy greater than about	20 MeV
Pile	Neutrons of all energies present in a nuclear reactor	About 0.001 eV to	~ 15 MeV
Fission	Neutrons formed directly in fission	About 100 keV to	~ 15 MeV
		Most probable energy	0.8 MeV
		Average energy	2 MeV

The neutrons produced in nuclear reactions, against which the shield has to be effective, usually have energies of the order of 10^5 eV or more. This is almost always true whether the neutron source is a reacting pile, an accelerating machine,

or some natural source such as a mixture of radium and beryllium. In neutron-shielding problems one is therefore almost invariably dealing with source neutrons that are described as "fast".

A neutron shield makes use of the following properties of neutrons:

- (i) Only resonance and slow neutrons are easily captured. Fast neutrons must therefore first be slowed down, or moderated, before they can be efficiently absorbed. The only way to do this is to cause them to lose energy in a large number of scattering collisions, by placing a sufficient thickness of material in their path.
- (ii) (a) Fast neutrons above about 0.5 MeV may lose energy by inelastic scattering, leaving the scattering nucleus in an excited state; the excitation energy is subsequently emitted as gamma radiation.
 (b) They also lose energy by elastic scattering, and this is essentially the only process by which neutrons of intermediate energies (from ~ 30 eV to ~ 0.5 MeV) can be moderated. The energy lost by the neutron appears as translational energy transferred to the scattering nucleus, subject to the usual laws of conservation of energy and momentum.
 (c) Below about 30 eV in both solids and liquids another type of inelastic scattering occurs, in which energy is transferred between neutrons and molecular or crystal lattice vibrations. This energy is subsequently evolved as heat.
 (d) A fourth possibility arises because a neutron of energy E_n eV behaves in some respects as though it were a wave of wavelength $2.86 \times 10^{-9} E_n^{-1/2}$ cm. When the wavelength of the neutron is comparable with the size of the nucleus, which is the case for fast neutrons, the neutron may be diffracted; physically, the result is that the neutron is deviated from its path without loss of energy. The process is referred to as diffraction scattering, or shadow scattering, by analogy with the diffraction of light by a small opaque body.
 (e) A number of other types of reaction can occur between fast neutrons and nuclei: the neutron may be captured, it may eject a second neutron from the struck nucleus (the $(n,2n)$ reaction)[†] or it may induce fission in some heavy elements. For shielding materials the probability of these reactions is small compared with the probability of scattering, in the energy range of interest to reactor technology.

Although this chapter is concerned mainly with the range of neutron energies found in nuclear reactors (0 to 20 MeV) it should be mentioned that at greater energies many additional reactions are possible. An ultra-fast neutron may eject several neutrons or protons from a struck

[†] The essential features of a nuclear reaction are conveniently summarized by writing the conventional symbols for the particles and nuclei taking part in the following order:

Target nucleus (Incoming particle, Outgoing particle) Product nucleus.

When the target nucleus is unspecified the type of reaction is defined by the portion inside the brackets. Thus we may describe neutron inelastic scattering as an (n, n') reaction; an $(n, 2n)$ reaction is one in which a neutron ejects a second neutron from a target nucleus, and an (n, α) reaction is one in which α -emission follows neutron absorption. If the only outgoing particle is a photon it is indicated by the symbol γ ; if the photon is accompanied by another particle, the symbol γ is sometimes omitted from the second half of the bracket.

nucleus in a single encounter, and at high enough energies the nucleus may be almost completely disrupted (spallation reaction).

- (iii) The average loss of energy in an elastic collision is greatest when a neutron is scattered by a hydrogen nucleus, and least for the heaviest nuclei (see Chapter 4). This follows from the ordinary laws of mechanics. It is found that the average logarithmic energy loss suffered by a neutron of initial energy E_{n0} in a collision with a nucleus of atomic weight A is⁽³⁾

$$\left[\ln \left(\frac{E_{n0}}{E_{n0} - \Delta E} \right) \right]_{\text{Average}} = 1 + \frac{(A-1)^2}{2A} \ln \left(\frac{A-1}{A+1} \right) \approx \frac{2}{A + \frac{2}{3}} \text{ for } A > 2 \quad (3.1.1)$$

- (iv) The probability of energy-loss by inelastic scattering is greater the heavier is the scattering nucleus, and it is greater also for fast neutrons than for slower neutrons. There is a threshold value of the neutron energy (different for different scattering nuclei) below which no inelastic scattering takes place. Generally speaking, inelastic scattering is unimportant for neutrons below about 0.5 MeV.
- (v) Neutrons will be slowed down until they are eventually in thermal equilibrium with their surroundings (provided, of course, that they continue to escape capture). As thermal neutrons they will diffuse through the medium until they are captured, or escape from its boundaries. Some of the fast neutrons also will leak past the boundaries of the shield. Although some leakage is unavoidable, the essential purpose of the design is to reduce this leakage to an acceptably low level on the open face of the shield.
- (vi) Capture cross-sections for thermal neutrons vary over a range of 10^9 to 1, so that by suitable choice of materials it is easy to arrange for their rapid absorption.
- (vii) The net result of these processes is that in a typical reactor shield the neutron density falls off exponentially with increasing distance from the core. However
- (viii) almost invariably penetrating "capture gamma rays" are emitted after neutron capture, so that a shielding problem still persists even after the disappearance of the neutron. In addition
- (ix) neutron capture causes most materials to become radioactive.

To make an efficient shield for the biologically dangerous fast neutrons generated by a nuclear reactor one therefore uses a mixture which contains

(a) *heavy elements*, which are needed in any case for gamma shielding, and which slow down the fast neutrons by inelastic scattering,

(b) *hydrogen*, to slow down the fast- and intermediate-energy neutrons by elastic scattering,

(c) *material with a sufficiently high thermal neutron capture cross-section* in order to capture the neutrons efficiently after they have been slowed down. If possible, this material should not emit a very penetrating gamma ray after neutron capture; lithium and boron best fulfil both these requirements, but for structural reasons it is not always possible to include them.

In addition,

(d) *the mixture should be of high density* so as to absorb the gamma rays

formed in fission, neutron capture, and neutron inelastic scattering, and also to make the shield compact.

In spite of the complexity of the processes involved, and of the far from complete knowledge of the basic cross-sections, it is still possible to formulate an approximate theory for a fast neutron shield, by taking advantage of the following facts. In most shielding materials the average distance a neutron travels between scattering collisions decreases rapidly as its energy decreases. The spatial distribution of neutrons of low and intermediate energies is therefore hardly affected by the unknown intermediate cross-sections, but is governed principally by the exponential attenuation of the fastest neutrons; so, too, is the thermal neutron source strength, or the number of neutrons becoming thermal per unit time at any given point. The majority of neutron captures take place in or close to the thermal energy region. Since thermal neutrons behave as a fairly homogeneous group, and since their source distribution is calculable and the necessary capture cross-sections are known, the distribution of neutron "deaths" can also be determined. Thus a knowledge of the cross-sections at the two extremes of the energy range is sufficient to enable an approximate description of the neutron attenuation process to be given; such an interpretation of measurements on actual pile shields will be given in Chapter 4.

Some familiarity with the possible interactions between neutrons and nuclei is needed before these elementary considerations can be made more quantitative. A summary of the relevant properties of neutrons will therefore be given, before the main topic is taken up once more in the next chapter. Much of the information given in this summary cannot at present be incorporated into any theory of neutron attenuation, but a broad acquaintance with the physical facts is nevertheless desirable if one is to avoid the error of using approximate theories in cases where the approximations may be unjustified.

3.2. THE NEUTRON

The neutron has a mass of 1.00898 atomic mass units, which is slightly greater than the proton mass of 1.00759 a.m.u. As its name implies, it is uncharged and is therefore unaffected by the electric fields of the atomic electrons or of the nucleus. Consequently it can diffuse through matter unhindered until it encounters a nucleus, with which it can react by virtue of the existence of short-range forces that are nonelectric and specifically nuclear in character. Owing to the relatively small size of the nucleus ($\sim 10^{-12}$ cm) in comparison with interatomic dimensions ($\sim 10^{-8}$ cm) the mean free path for such encounters may be several centimetres, even in solids. The great penetrating power of neutrons, due to their lack of charge, was the property which originally led to their discovery.

Apart from the small mass difference, and the difference in charge, neutrons and protons have several characteristics in common. They are both constituents of nuclei. The short-range nuclear forces appear to act similarly on both of them. (The so-called charge independence hypothesis assumes that the forces in the two cases are actually identical.) Nuclei containing certain "magic" numbers of either protons or neutrons are endowed with special stability, suggesting that the preferred groupings of both particles in the nucleus are identical. Because

of these and other similarities, neutrons and protons are described collectively by the term *nucleons*.

Like other elementary particles, the neutron behaves as though it possesses both wave-like and particle-like properties. The neutron is associated with a de Broglie wave,[†] the square of whose amplitude is everywhere proportional to the neutron density at the same point, and whose de Broglie wavelength λ is a function of the neutron energy:

$$\lambda = h/m_n v = 2.86 \times 10^{-9} [E_n \text{ (in eV)}]^{-1/2} \text{ cm} \quad (3.2.1)$$

where h is Planck's constant (6.624×10^{-27} erg-sec) and m_n, v are the mass and velocity of the neutron respectively. The quantity $\lambda = \lambda/2\pi$ may be used as a convenient measure of the effective physical extension (radius) of the neutron. Thus, as we shall see later (p. 115) the cross-section for an interaction (compound nucleus formation) between a nucleus of radius R , and a fast neutron characterized by a particular value of λ , is $\sim \pi(R + \lambda)^2$, which is the value it would have if the neutron were a material particle of radius λ .

A further property of the neutron which is also common to the other elementary particles is its intrinsic angular momentum, or *spin*. This has a magnitude of $\hbar/2$, where $\hbar = h/2\pi$. Spin is a property which must be taken into account in theoretical expressions for slow neutron cross-sections by the inclusion of a statistical weight factor, which measures the probability of the correct lining up of the angular momenta of the neutron and the target nucleus. Experimentally determined cross-sections will of course automatically include the correct spin weighting, and shielding calculations based on experimental values therefore have no need to take any further account of spin.

Neutron Cross-section and Flux

The probability of any neutron-induced event which is independent of any other event can be described in terms of a cross-section. Cross-sections can be assigned for scattering, capture, fission, etc., and will in general be different for different neutron energies and for different target nuclei; they are conventionally measured in barns (10^{-24} cm²), and are numerically equal to the geometrical cross-sections which individual nuclei would need to possess in order to give the correct reaction rate when bombarded by neutrons of the appropriate energy but of zero extension. The total atomic cross-section σ_t is the sum of the partial cross-sections: $\sigma_t = \sigma_{\text{capture}} + \sigma_{\text{scattering}} + \dots$. The product of an atomic (or "microscopic") cross-section and the number of atoms per unit volume is described as a *macroscopic* cross-section; it is usually denoted by a capital sigma, Σ , with the appropriate suffix, and is measured in units such as cm² per cm³ or, more concisely, in cm⁻¹.

Consider a thin slab of area S , containing N_s nuclei, and assume that a beam of neutrons, of density n neutrons per cm³ and velocity v cm/sec, is normally incident on the slab. The slab is supposed to be "thin"; that is, it does not appreciably attenuate the beam. Let the cross-section per nucleus for a particular

[†] A short bibliography of books dealing with the wave nature of matter and other fundamental concepts of nuclear physics will be found at the end of the chapter.

reaction (which, in order to fix our ideas, can be taken as capture) be σ . Then the chance of a neutron being captured during its passage through the slab is clearly $N_s\sigma/S$, and the total number of captures per unit time C is

$$C = nvS \cdot N_s\sigma/S = nvN_s\sigma \quad (3.2.2.)$$

The scalar quantity nv is known as the *flux* of neutrons of velocity v . The number of captures C , being independent of S , is unaffected by the shape of the slab (so long as it is thin); C is also unaffected by the degree of collimation of the incident neutrons, since nuclei behave as though they are spherically symmetrical.

Some processes, those which involve the co-operative action of many nuclei, by their very nature cannot be described solely in terms of cross-section. Thus for example the diffraction of neutrons by a crystal is a function not only of the nuclear cross-sections, but also of the dimensions and degree of organization of the crystal lattice.

Thermal Neutrons

Neutrons diffusing in the interior of a sufficiently large medium in which the capture is slight will eventually reach a transient state of approximate thermal equilibrium with their surroundings. The neutron velocities are then distributed according to the well-known Maxwell law (Fig. 3.2.1):

$$dn = \frac{4n}{v_0^3\sqrt{\pi}} v^2 e^{-(v/v_0)^2} dv \quad (3.2.3)$$

In this expression v_0 is the most probable velocity, n the total density of neutrons of all velocities, and dn the number per unit volume with velocities between v and $v + dv$. The neutrons have three degrees of freedom; therefore, by the ordinary laws of thermodynamics, their average energy must be equal to $3kT/2$ (k is Boltzmann's constant, T the absolute temperature). If m_n is the mass of the neutron, and $\bar{v}^2$ the mean square velocity, the average energy is found from equation (3.2.3) to be equal to

$$\bar{E}_n = \frac{1}{2}m_n\bar{v}^2 = \frac{1}{2}m_n(3v_0^2/2) = 3kT/2 \quad (3.2.4)$$

The most probable velocity v_0 is therefore equal to $(2kT/m_n)^{1/2}$. At a temperature of 20.0°C $v_0 = 2176$ m/sec, and the most probable energy is 0.02475 eV. In standard tables neutron capture cross-sections are usually quoted for mono-energetic neutrons of velocity 2200 m/sec, or 0.0253 eV.

Two further relationships are easily proved by manipulation of equation (3.2.3) with the aid of the integrals given in the footnote.† The most

† Integrals of the type

$$I_q = \int_0^\infty v^q e^{-\lambda v^2} dv,$$

where q is zero or a positive integer, are well known from their frequent occurrence in the kinetic theory of gases. They obey the relationships of the type

$$\begin{aligned} I_2 &= -\frac{dI_0}{d\lambda}, & I_3 &= -\frac{dI_1}{d\lambda}, \\ I_4 &= +\frac{d^2I_0}{d\lambda^2}, & I_5 &= +\frac{d^2I_1}{d\lambda^2}. \end{aligned}$$

(continued overleaf)

probable flux is found to occur at the root mean square velocity, or an energy of $\frac{3}{2}m_n v_0^2$. Secondly, if the cross-section of a particular material varies as $1/v$ (which is very often the case in the thermal region, as will be explained later), the average capture mean free path $\bar{\lambda}_c$ for thermal neutrons is $\frac{2}{\sqrt{\pi}}$, or 1.128

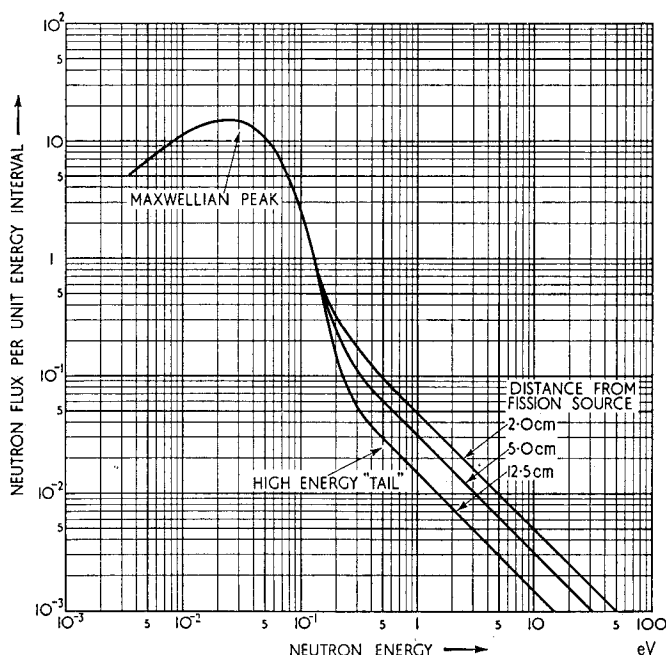


Fig. 3.2.1. Energy distribution of low-energy neutrons at various distances from a fission source in water at 18°C.

The curves are normalized to unit total flux under the Maxwellian.

Above 0.5 eV the spectrum is proportional to E^{-1} (POOLE⁽⁴⁾).

times the mean free path at the most probable velocity, $\lambda_c(v_0)$. In calculating the diffusion length for thermal neutrons with the aid of equation (4.2.10), one must use $\bar{\lambda}_c$, and not $\lambda_c(v_0)$.

Thermal neutrons are never produced directly in nuclear reactions, but can only be obtained by slowing down fast neutrons. If the neutron source emits

The first of the two fundamental integrals, I_0 , is the Gauss probability integral, and is equal to $\frac{1}{2}\sqrt{\frac{\pi}{\lambda}}$; the other, I_1 , can be evaluated by elementary methods, and is equal to $\frac{1}{2\lambda}$.

By using the relationships given above, it is easily shown that $I_2 = \frac{1}{4}\sqrt{\frac{\pi}{\lambda^3}}$, $I_3 = \frac{1}{2\lambda^2}$ and $I_4 = \frac{3}{8}\sqrt{\frac{\pi}{\lambda^5}}$.

neutrons continuously there will be at every point in the medium an equilibrium mixture of fast, intermediate, and thermal neutrons. A pure thermal spectrum is therefore never obtained. For experimental purposes it is possible to reduce the "high energy tail" (Fig. 3.2.1) to negligible intensity as compared with the thermal peak by using large blocks of a material of very low thermal capture cross-section and good slowing-down properties—as in the special graphite thermal columns provided in some research reactors. In neutron shielding, however, rather the opposite situation occurs, because it is one's aim to eliminate the neutrons by capture as soon as they have been slowed down. It is, in fact, usually preferable to capture them as near to the source as possible, instead of waiting until nearly all are thermalized, because the rest of the shield helps to attenuate the capture gammas which result (Section 3.5). Thus the thermal spectrum is always accompanied by a large proportion of higher-energy neutrons. Nevertheless, as already explained, the majority of neutrons entering the shield are first approximately thermalized and then captured; this is the explanation of the special attention paid to thermal neutrons in shielding theory. It is fortunate that their behaviour when diffusing is easily calculated and measured. The relative contributions of thermal and epithermal neutrons to any particular process (e.g. capture) can be very simply measured by the cadmium difference method mentioned on page 131.

3.3. THE NUCLEUS

Short-range Nuclear Forces and Nuclear Binding Energies†

To the external world a nucleus of atomic weight A and atomic number Z behaves as though it were a collection of protons and $(A-Z)$ neutrons, constrained within a sphere of fairly well-defined radius $R = 1.45A^{1/3} \times 10^{-13}$ cm. Inside the nucleus the constituent nucleons may be thought of as being in constant agitation, but prevented from leaving the nucleus by strong, short-range attractive forces. Forces of this kind must be postulated to explain why the nucleus is not immediately blown apart by the electrostatic repulsion of the protons. The attractive forces cease to be effective outside the distance R , and the only influence of the nucleus is then on account of the Coulomb forces due to its total electric charge. These properties of the nucleus would be roughly represented by a model consisting of a number of sticky balls in contact, some of the balls being electrically charged. A more generally useful description, however, is in terms of potential energy.

Let us consider what happens when one neutron is removed from an isolated stable nucleus and taken away to infinity. Since the nucleus is stable, this process requires the expenditure of work. The neutron is unaffected by the electric charge of the nucleus, so that the only forces against which the work

† The standard introductory text on nuclear theory is the book by BLATT and WEISSKOPF, *Theoretical Nuclear Physics*, published by Wiley, New York. The more experimental aspects of the subject are dealt with in the excellent monograph by B. T. FELD, "The Neutron," in Vol. 2 of *Experimental Nuclear Physics*, also published by Wiley. The authors wish to acknowledge the invaluable help they received from both these books when preparing this chapter.

must be done are the nuclear forces, which do not extend beyond the distance R . Thus, by the time the neutron has passed outwards through the nuclear surface, the neutron-nucleus system will have acquired an amount of potential energy equal to the separation (or binding) energy of the neutron, S_n . If the potential energy of the system is plotted against the separation of the two components (Fig. 3.3.1) the process may be pictured as the lifting of the neutron out of a

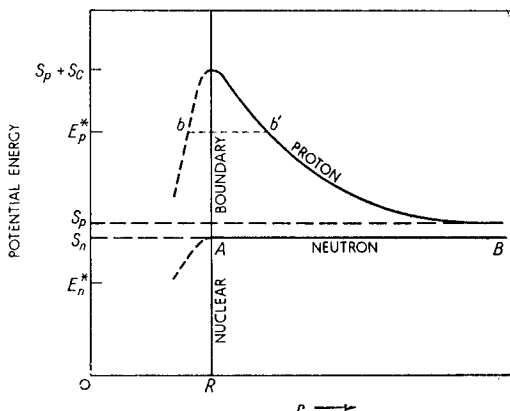


Fig. 3.3.1. Potential energy diagram for the separation of a neutron or proton from a stable nucleus.

The potential energy of the nucleus-nucleon system is shown as a function of the distance r of the nucleon from the centre of the nucleus (see text).

“potential well” of depth equal to the separation energy. For the rest of its journey to infinity the neutron has no further interaction with the nucleus, and the portion of the curve AB is therefore flat. When the neutron is returned to the nucleus the energy of the system then exceeds the energy of the original stable nucleus (in the so-called ground state) by an amount equal to S_n plus any kinetic energy it possessed before re-entering the nucleus. The excess energy is described as “excitation energy.” Nuclei in highly excited states are unstable, and rapidly get rid of their excess energy by emission of gamma rays or material particles (see below).

For a large number of medium and heavy nuclei the separation energy S_p of a proton which has been removed to infinity is much the same as that for a neutron: $S_n \approx S_p \approx 8$ MeV. However, the shape of the potential energy curve is quite different for a proton, owing to the electrostatic repulsion of the nucleus. Let us assume that a proton has been detached in some way from a stable nucleus and has been removed to infinity, where it is at rest. If we now attempt to return it to the nucleus, it is necessary to do work against the electric forces. By the time the proton has reached the nuclear surface the total energy acquired by the system from the work done on it amounts to

$$S_C \approx \frac{(Z-1)e^2}{R}$$

where the suffix denotes the contribution on account of the Coulomb forces and e is the charge on the proton and $(Z - 1) \cdot e$ the charge on the residual nucleus. Once the proton has passed inside the nuclear surface it comes within range of the attractive nuclear forces, and the potential energy of the system falls again. The hump in the potential energy curve is colloquially described as the Coulomb barrier.

Now let us consider under what circumstances a particle can escape from the nucleus, assuming that it has been given an amount of energy E_n^* or E_p^* by some unspecified process. In Fig. 3.3.1 we have shown as an example the case of a neutron with excitation energy $E_n^* < S_n$; the neutron clearly cannot escape from the nucleus. According to classical physics the same would apply to a proton of excitation energy $E_p^* < (S_p + S_C)$. However, wave-mechanical considerations show that as long as $E_p^* > S_p$ there is a nonzero probability of the particle leaking out through the potential barrier; this probability is smaller the greater is the barrier height $[S_p + S_C - E_p^*]$, and the thicker is the barrier bb' at the energy E_p^* .

The relationship $S_n \approx S_p$, which is a reasonable approximation for the medium-heavy elements, does not in general hold for the lighter elements. For small values of Z both S_n and S_p show large fluctuations. The values for any given light nucleus can be found directly from the isotopic and nucleonic masses with the aid of the Einstein relation $E = mc^2$. Expressed in the practical units usually employed in nuclear physics, this relationship asserts that 1 atomic unit of mass is equivalent to an energy of 931 MeV. The separation energy in MeV of a neutron in the nucleus (A, Z) is therefore

$$S_n = 931 (M_{A-1, Z} + m_n - M_{A, Z}) \quad (3.3.1)$$

Note that the masses M to be used in this expression can be either the atomic or the nuclear masses, since the same number of electrons is attached to both the (A, Z) and the $(A-1, Z)$ nuclei, and their masses consequently cancel out. For protons, however, we must modify the expression if we wish to continue to use atomic masses, since the removal of a proton from a nucleus also involves the escape of an orbital electron:

$$S_p = 931 (M_{A-1, Z-1} + m_{\text{proton}} + m_{\text{electron}} - M_{A, Z}) \quad (3.3.2)$$

The quantity $m_{\text{proton}} + m_{\text{electron}} = 1.00814$ a.m.u. is of course just the mass of the hydrogen atom.

For values of A less than about 30 the nuclear masses are known with sufficient accuracy for the binding energies to be determinable directly from the above equations.⁽⁵⁾ For heavier elements the precision of the mass measurements is inadequate, and the binding energies have to be inferred from measurements of the threshold energies of reactions such as neutron or proton production under bombardment with energetic gamma rays (see Table 3.3.1).

As we have already mentioned, the binding energy (of a neutron already in the nucleus), shows considerable fluctuations in light elements, and ranges from 1.665 MeV in Be^9 to 19 MeV in C^{12} . As we go to heavier nuclei the fluctuations rapidly become smaller; rough average values for the neutron binding energies are ≈ 8 MeV for medium-weight nuclei ($A \approx 30$ to 150) and ≈ 6 MeV for the heaviest elements ($A > 210$). Nuclear structure gives special stability to nucleon

pairs, so that the neutron binding energy is somewhat greater if the number of neutrons $N = A - Z$ is even than if it is odd. In the same way nuclei containing certain "magic" numbers of neutrons—2, 8, 20, 28, 50, 82, 126—are specially stable, and this is reflected in unusually large values of the neutron binding energy. The most stable of all nuclei are those which are "doubly magic"—that is, magic for both protons and neutrons. The nuclei of He^4 , O^{16} , Ca^{40} , and Pb^{208} are familiar examples.

Values of the binding energies of a neutron in a number of stable nuclei are given in Table 3.3.1. The same table also gives the binding energy released when a neutron is added to an already stable nucleus.

Table 3.3.1 Neutron Binding Energies for the Commoner Elements

Isotope	Binding energy of neutron in nucleus (MeV)	Binding energy released when neutron is added to nucleus (MeV)	Isotope	Binding energy of neutron in nucleus (MeV)	Binding energy released when neutron is added to nucleus (MeV)
H 1	—	2.227	Fe 54	13.8	9.298
D 2	2.227	6.251	56	11.15	7.639
Li 6	5.37	—	57	7.639	10.16
7	7.15	—	Co 59	10.25	7.486
Be 9	1.665	6.816	Ni 58	11.7	8.997
B 10	8.55	—	60	—	8.532
11	11.50	—	Cu 63	10.65	7.914
C 12	19.0	4.949	65	9.97	7.634
N 14	10.7	10.832	Zn 64	11.72	7.876 (?)
O 16	15.8	—	66	11.15	—
F 19	10.40	6.63	67	7.00	9.52
Na 23	12.05	6.96	68	10.15 (?)	—
Mg 24	16.4	7.334	70	9.20	—
25	7.334	11.15	Zr 90	12.30	—
26	11.15	6.440	91	7.20	8.66
Al 27	≈ 14	7.724	Nb 93	8.70	(7.19)
Si 28	16.9	8.468	Mo 92	13.17	—
29	8.468	10.601	Mo 95	—	9.15
P 31	12.27	7.94	97	7.10	—
S 32	15.0	8.64	Cd 113	6.48	9.046
34	10.85	—	Ba 137	—	9.23
Cl 35	—	8.57	W 182	—	(6.182)
37	9.95 (?)	—	183	6.25 (?)	7.42 (?)
K 39	13.2	7.789	186	—	7.1 (?)
41	—	7.34	Pb 206	8.25	6.734
Ca 40	15.8	—	207	6.91	7.380
Ti 46	13.3	—	208	7.38	—
Cr 50	13.4	—	Bi 209	7.28	4.170
52	11.80	7.929	Th 232	6.34	—
53	7.75	9.716	U 238	5.88	—
Mn 55	10.07	7.261			

The numbers given in the left-hand column are the (γ, n) thresholds; $(n, 2n)$ thresholds are $(A + 1)/A$ times larger. The values given in the table are taken from references 5, 6, 7, and 30a.

The compound nucleus. One of the most fruitful conceptions in nuclear physics has been that of the compound nucleus. This model, which was originally proposed by BOHR, supposes that immediately a nucleon enters a nucleus it reacts very strongly with all the other constituents, so that in a very short time its energy is distributed throughout the nucleus. The time taken for this to happen is pictured as being not very much longer than the time taken for a nucleon to cross the nucleus ($\sim 10^{-21}$ sec). The original ingoing particle, having lost most of the energy it gained on falling into the nuclear potential well, is then trapped, and no particle can be emitted by the nucleus until a random series of collisions once more endows one with sufficient energy to escape. If, however, the nucleus sheds part of its excess energy in some other way before a particle is emitted—for instance, by emitting a high-energy capture gamma ray, a process which takes about 10^{-14} or 10^{-15} sec—then the 7 MeV or so which are necessary for escape can never be concentrated on a single particle. A new nucleus of mass $(A + 1)$ has then been formed, and the remaining stages of de-excitation must proceed by photon emission.

For many purposes it is sufficient to describe the reaction between a nucleon and a nucleus in terms of this “strong interaction” model. However, this cannot be a complete description, for if the nucleus were simply a collection of nucleons which were constantly interacting strongly it would be difficult to explain the abundant evidence for some kind of nuclear structure. An additional reason for believing that the strong interaction model is not a completely adequate description is the success of the optical (or “cloudy crystal ball”) model of the nucleus^(8,9) in explaining the observed variation of total cross-section with energy and atomic weight, particularly for neutrons in the energy region from about 1 to 5 MeV. In this model an incoming neutron is assumed to be capable of travelling a certain distance ($\sim 2 \times 10^{-12}$ cm) inside the nucleus before the two interact to form a compound nucleus. This distance is about equal to the diameter of the largest nucleus. The model is therefore intermediate between the strong interaction model and the independent particle approach which seems to be indicated by the observed magic number effects.

The original strong interaction model is now known to be an over-simplification, but as it provides an easily understood qualitative explanation for many nuclear reactions within the limited energy range (0 to 8 MeV) which is of principal interest to us, we shall continue to make frequent use of it in the course of this chapter.

Excited states of nuclei. As we shall see later, there is completely convincing experimental and theoretical evidence that certain values of the excitation energy are very strongly preferred. These values are known as *levels*, and the ensemble of levels for a given nucleus as its level structure. The energy differences show marked and apparently random fluctuations; but provided one discusses the average level spacing there are a number of generalizations which can be made.

The levels tend to be distributed as follows: the first few levels above the ground state are widely spaced, but as the excitation energy goes up they get steadily closer together, until finally, at very high energies, they overlap completely. This same general type of pattern is found for both heavy and light nuclei, but the scale changes very considerably in going from the lightest to the heaviest

elements. The pattern for a fairly light element—sodium—is shown in Fig. 3.5.4. For the lightest elements the first excited levels are at a few MeV above the ground states; in the heaviest elements the corresponding figure is only ~ 40 keV. At excitations ~ 7 MeV (corresponding to the neutron binding energy) the levels are spaced by several hundred keV in the lightest elements, and by only about an electron-volt in the heaviest. Provided the excitation energy E^* is greater than a few MeV, the general variation with E^* of the average level spacing D for nuclei of odd atomic weight is given very roughly by the semi-empirical formula⁽¹⁾

$$D^{-1} = C \exp(4aE^*)^{1/2} \quad (3.3.3)$$

where a , C are constants, typical values of which are given in the table. The information regarding nuclei of even atomic weight is still too scanty for generalizations to be made.

Table 3.3.2. Constants for Formula 3.3.3

Atomic weight	a (MeV ⁻¹)	C (MeV ⁻¹)
27	0.45	0.5
63	2	0.3
115	8	0.02
181	10	0.01
231	12	0.005

The only important exceptions to the general rule of decreasing level spacing with increasing atomic weight are the magic nuclei (see page 110), which have widely spaced levels such as are normally found only in light elements. This property causes their capture gamma ray spectra to be unusually simple (Table 3.5.1).

De-excitation of excited nuclei. Excited states are unstable, and in general the nucleus rapidly reverts to the ground state by the emission of energy in the form of gamma radiation, or by the emission of a particle (proton, neutron, alpha particle, etc.). These are all competing processes, to each of which can be assigned a probability of $1/\tau_i$ per unit time. The quantity τ_i is then the mean lifetime for a particular process (all the other competing processes being considered as temporarily abolished); that is, it is the average time required for the nucleons in their random motion to arrange themselves in a configuration suitable for de-excitation by the i th process. Clearly the average lifetime of the nucleus when all the r possible processes are operative is $\tau = (\sum_{i=1}^r (1/\tau_i))^{-1}$ and the relative probability (or branching ratio) of the i th process is τ/τ_i .

Usually, these probabilities are expressed not in terms of τ_i , but in terms of Γ_i , the “partial width of the level for the i th process.” The term “width” arises from the Uncertainty Principle, which implies that the energy spread of a nuclear energy level is inversely proportional to its lifetime: $\Gamma = \sum_i (\Gamma_i) \approx \hbar/2\pi\tau$ ($\hbar$ being Planck’s constant). If Γ is expressed in the usual practical units of

electron-volts, and τ is in seconds, then $\Gamma\tau \approx 7 \times 10^{-16}$. It is clear from the definition that the total and partial widths are related to the corresponding cross-sections by relationships of the type: $\Gamma_n/\Gamma = \sigma_{\text{scattering}}/\sigma_{\text{total}}$. According to the simple strong interaction model the partial width Γ_i for the emission of a particle i should depend on the excitation energy of the compound nucleus, the kinetic energy of the particle after it has left the nucleus E_i , and the charge on the particle. Provided the energy of the outgoing particle is not too high, the value of Γ_i would be expected to follow the trend

$$\Gamma_i = \text{Constant} \times (\text{level spacing of the excited compound nucleus}) \times (\text{penetrability factor}) \times E_i^{1/2} \quad (3.3.4)$$

where the penetrability factor measures the probability of the particle leaking through the potential energy barrier (p. 109).

Equation (3.3.4) is based on a very crude treatment, and can only be expected to hold on the average. Wide fluctuations may be anticipated, because the width for an individual process depends in fact on the detailed features of the states of both the compound nucleus and the residual nucleus. Indeed, the measured values of Γ_n , the widths for neutron emission (scattering) by individual levels, show that experimentally $\Gamma_n E_n^{-1/2} D^{-1}$ is far from constant, but varies erratically from level to level. However the *mean* value of $(\Gamma_n/\sqrt{E_n} \times \text{level spacing})$, averaged over many adjacent levels of the compound nucleus, behaves more systematically.

In contrast with the very large fluctuations observed in Γ_n it is found that the width for gamma emission following neutron capture Γ_γ is relatively insensitive to the nature of the compound nucleus. This is because in most cases the gamma rays lead, as a first step in the de-excitation of the compound nucleus, to any of a number of intermediate levels: the observed widths Γ_γ are summed in the measuring process over all these levels, and statistical fluctuations are greatly reduced by this summation. Further details of the radiative capture process are given in Section 3.5.

Formation of compound nucleus by neutron absorption. No mention has so far been made of the factors affecting the formation of the compound nucleus. Of these the most striking is the dependence on the energy of the ingoing particle: marked *resonances* in the reaction cross-section are observed, when the particle energy is such that the energy of excitation of the resulting compound nucleus corresponds exactly to one of its possible excited states (Fig. 3.3.2). In the neighbourhood of a resonance, and when only one level is close enough in energy to be important (i.e. for medium-weight nuclei and low-energy incident neutrons) the cross-section for the formation of a compound nucleus by neutron absorption, $\sigma_{\text{cpd}}(n)$, is given by the Breit-Wigner single-level formula:

$$\sigma_{\text{cpd}}(n) = \pi \lambda^2 g \frac{\Gamma \Gamma_n}{(E - E_r)^2 + \Gamma^2/4} \quad (3.3.5)$$

In this formula Γ is the total width of the excited level, Γ_n the partial width for neutron emission, and $(E - E_r)$ is the difference between the kinetic energy of the ingoing neutron and the energy it would have at exact resonance. The

factor g is the *statistical weight factor*, which measures the probability of the nuclear and neutron spins being properly aligned; it is a number lying between about 0.5 and unity.

For high-energy neutrons, when the levels are too close for the single-level formula to apply, it is still possible to describe $\sigma_{\text{epd}}(n)$ in terms of a suitable average of the effects of the separate levels. For our purposes it is sufficient to

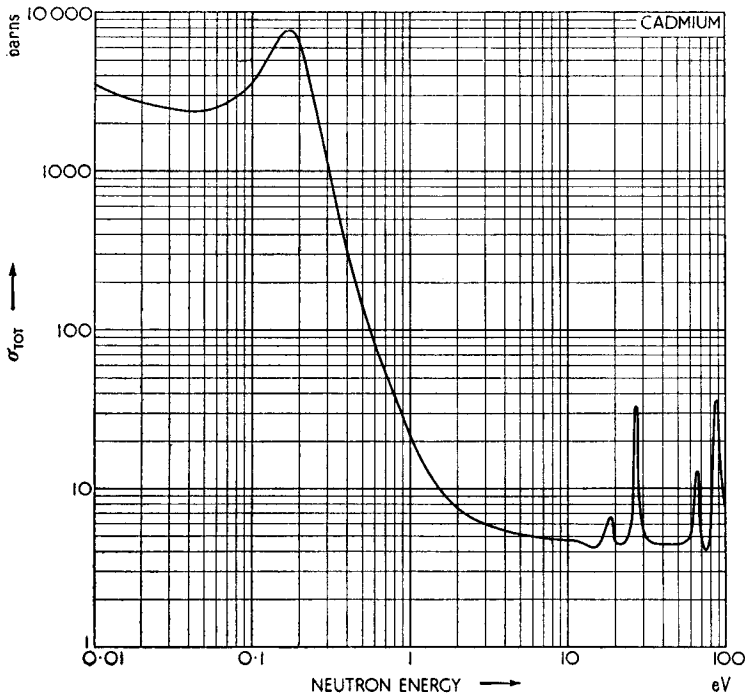


Fig. 3.3.2. Total cross-section of cadmium for low-energy neutrons. The capture resonance at 0.18 eV overlaps completely the thermal Maxwellian spectrum (Fig. 3.2.1); this coincidence forms the basis of the cadmium difference method of measuring epi-thermal fluxes (see pages 131 and 332).

make use of the approximate theory, developed by WEISSKOPF⁽²⁾ and his collaborators on the basis of the strong interaction model, which shows that the value of $\sigma_{\text{epd}}(n)$, averaged over many resonances, is given for a target nucleus of radius R by

$$\sigma_{\text{epd}}(n) \approx \pi(R + \lambda)^2 \cdot \xi \quad (3.3.6)$$

As already mentioned, the quantity λ is a measure of the apparent extent of the neutron, the factor $\pi(R + \lambda)^2$ being the value the reaction cross-section would have if the neutron behaved as a sphere of radius λ . The quantity ξ , sometimes known as the *sticking probability*, is the probability that a neutron will penetrate into the nucleus. It is a function of the change of wavelength a neutron experiences on crossing the nuclear surface, due to its increased kinetic energy inside the nuclear potential well. If we use the symbol k to mean λ^{-1} (where λ is the

value for the neutron outside the nucleus), and K for the corresponding value inside the nucleus ($K \sim \sqrt{10^{26} + k^2} \text{ cm}^{-1}$), then $\xi = 4kK/(k + K)^2$, and therefore

$$\sigma_{\text{epd}}(n) \approx \pi(R + \lambda)^2 \cdot \frac{4kK}{(k + K)^2} \quad (3.3.7)$$

For high enough energies λ/R is small, $k \approx K$, ξ approaches unity, and $\sigma_{\text{epd}}(n)$ approaches $\pi(R + \lambda)^2 \approx \pi R^2$. The cross-section for potential scattering (see page 116) also tends to a value of πR^2 at high energies, so that the total cross-section tends to a value $\approx 2\pi R^2$.

The highly excited compound nucleus loses energy within a very short time of its formation (10^{-14} sec or less) by the emission of a particle, which may be a neutron, proton, alpha particle, or a photon. There is no general rule which gives precise quantitative information as to how a particular level of a given compound nucleus will decay. However, if one limits oneself to averages taken over many levels, there are certain well-marked regularities, which will be discussed in Sections 3.4 *et seq.*

The optical model and deviations from the $\pi(R + \lambda)^2$ law of compound nucleus formation for fast neutrons. The statistical theory which will be extensively used in the summary of neutron reactions which follows is derived from the strong-interaction model. As we have seen, it predicts a monotonic decrease with energy for the cross-section for compound nucleus formation, provided values averaged over many resonances are considered. A similar result is obtained for the scattering, and hence also the total, cross-sections (Section 3.4). However, measurements of the average total cross-section for fast neutrons by BARSCHALL, MILLER, ADAIR, and others show departures from the monotonic decrease, with maxima occurring at energies of a few MeV. The positions of these maxima change slowly and smoothly with change of atomic weight. The "optical" model of nuclear interactions⁽⁹⁾ has had a striking success in explaining both the existence of the maxima and their behaviour, and has shown that the cause is the partial transparency of the nucleus to neutrons. The observed values can be explained when one takes into account the interference between the incident beams of neutrons, and neutrons which have penetrated into the nucleus, but have emerged again without forming a compound nucleus. The theory makes very few new assumptions, apart from assigning to the incident neutrons of energy ~ 1 MeV a mean free path in nuclear matter of $\sim 2 \times 10^{-12}$ cm (or about one nuclear diameter) before an interaction resulting in the formation of a compound nucleus will occur. There are theoretical grounds for believing that, as the neutron energy increases, the transparency of the nucleus falls, so that above about 5 MeV the strong-interaction model gives a reasonably adequate description.

In spite of the shortcomings of the simpler strong-interaction theory at lower energies, we shall continue to make use of it in the course of this chapter. Its accuracy may be gauged from Fig. 3.3.3: for instance, at 3 MeV the calculated and observed values of the total cross-section agree, for most elements, within a factor of less than two.

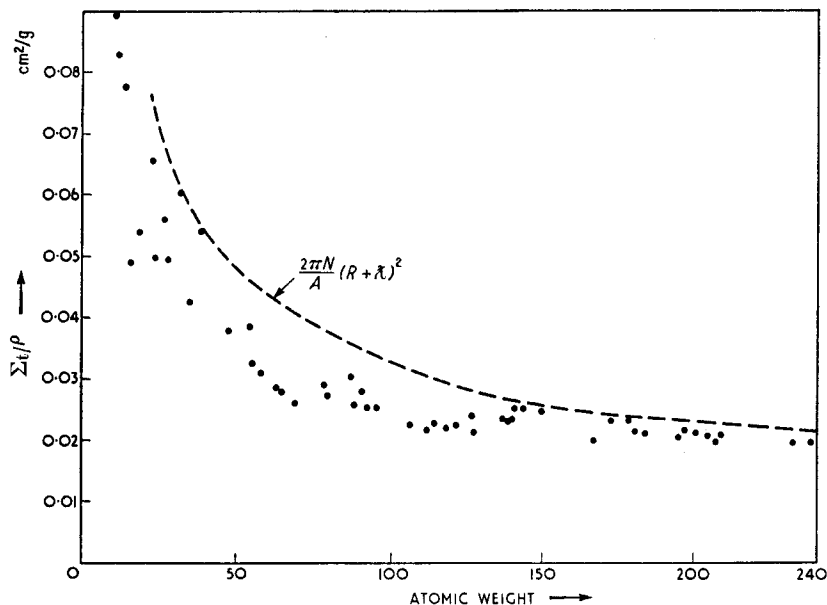


Fig. 3.3.3. Total cross-section (in cm^2 per gram) for 3 MeV neutrons as a function of atomic weight.

The broken curve was computed on the assumption that the atomic total cross-section is equal to $2\pi(R + \lambda)^2$ (i.e. the sum of the cross-sections for compound nucleus formation and for potential scattering).

TYPES OF REACTION BETWEEN NEUTRONS AND NUCLEI

3.4. SCATTERING

We shall deal first with scattering, because it is the most frequently occurring of all neutron reactions. There are two quite distinct mechanisms, of roughly equal importance:

- (i) The neutron enters the nucleus, forming a compound nucleus, and a neutron is subsequently emitted. This process is described as *capture scattering*. If the ingoing neutron has kinetic energy E_{n0} below the energy ϵ_1 of the first excited state of the target nucleus, the total kinetic energy of the neutron and nucleus after the collision must also necessarily equal E_{n0} (*capture elastic scattering*). If, on the other hand, E_{n0} is greater than the energies $\epsilon_1, \epsilon_2, \dots, \epsilon_r, \dots, \epsilon_q$ of the first q excited states, then in addition to capture elastic scattering there is the further possibility that the final kinetic energy of the system may have any of the values $E_{n0} - \epsilon_r$ ($1 \leq r \leq q$) (*capture inelastic scattering*). There is no essential difference between the mechanisms of the elastic and inelastic processes.
- (ii) The neutron is scattered by the nucleus, without having penetrated the nuclear surface. This scattering may be regarded as a reflection of the neutron wave by the potential jump at the nuclear surface, and is therefore known as *potential scattering*.

Slow neutrons ($E_n \leq 10$ eV).

When the wavelength of the neutron is large compared with the radius of the scattering nucleus, which is the case for slow neutrons, the cross-section

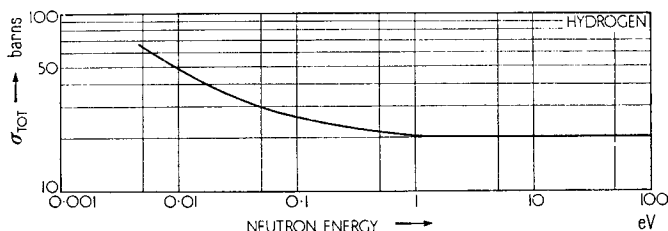


Fig. 3.4.1. Total cross-section of hydrogen in the low-energy region. The constant value above 1 eV is typical of the behaviour exhibited by many light elements when scattering is predominant. Below 1 eV the variation of the total cross-section is determined principally by the change from free-atom to bound-atom behaviour.

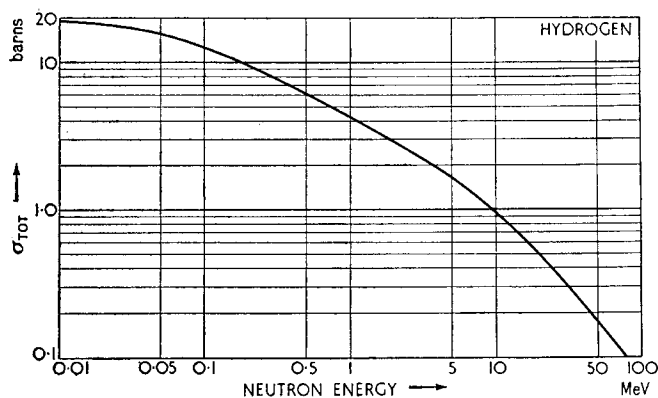


Fig. 3.4.2. Total cross-section of hydrogen in the intermediate and high-energy region.

Between 1.5 and 20 MeV the cross-section is represented to an accuracy of about 2 per cent by the formula

$$\sigma_H = 10.97 / (E_n + 1.66) \quad [\sigma_H \text{ in barns, } E_n \text{ in MeV}]$$

for potential scattering is approximately $4\pi R^2$; this is the cross-section which would be given by an impenetrable sphere of radius R . The actual observed cross-section differs from this value in a large number of cases owing to the effect of capture elastic scattering in nearby resonances; but far from a resonance the value is independent of neutron energy. This is in strong contrast to the behaviour of the capture cross-section, which tends to vary as the inverse of the neutron velocity (equation 3.5.1).

A cross-section approximately equal to $4\pi R^2$ is the value which applies to a nucleus free to recoil. However, neutrons of low energy are unable to transfer sufficient energy to a struck nucleus to free it from the chemical bonds holding it

to its neighbours. The struck nucleus then behaves as a bound atom, which results in the cross-section being modified by a factor which is small except for the lightest elements:

$$\sigma_{\text{bound}} = \sigma_{\text{free}}(A + 1)^2/A^2, \quad (3.4.1)$$

where A is the atomic weight of the nucleus. The effect of the transition from free to bound behaviour as the neutron energy is reduced is most marked in the case of hydrogen.

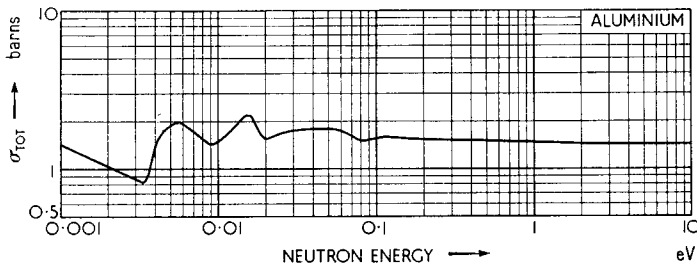


Fig. 3.4.3. Total cross-section of aluminium in the low-energy region.

Above 0.1 eV the nearly constant value indicates that scattering predominates. The oscillations in the curve between 0.003 eV and 0.1 eV arise from diffraction effects. At a low enough energy (i.e. for long wavelength neutrons) these effects are absent. The cross-section rises as the energy falls to 0.001 eV because capture begins to make an appreciable contribution for such very slow neutrons (see Section 3.5).

At energies below 1 eV the wavelength of the neutron becomes comparable with interatomic dimensions, and crystal diffraction effects then cause the observed scattering cross-section to vary rapidly with energy (Fig. 3.4.3). Although these effects are important in studies of the solid state⁽¹¹⁾ they are of small account in shielding work, and can safely be ignored.

In light elements, except at very low energies (thermal or sub-thermal, where, as we shall see later, the $E^{-1/2}$ dependence of capture makes itself felt) scattering is the predominating effect. There are four marked exceptions: the nuclei of He^3 , Li^6 , B^{10} , and N^{14} , which have large (n, p) or (n, α) cross-sections (Section 3.6).

In medium-heavy and heavy elements the relative importance of scattering is determined by competition between scattering and radiative capture. In very general terms, the gradual decrease of level spacing with increasing atomic weight tends to cause a gradual decrease in Γ_n (Section 3.3). At the same time Γ_γ remains much more nearly constant (Fig. 3.5.1). Consequently, in the middle of the periodic table the low-lying resonances are predominantly scattering resonances, but for the heaviest elements capture predominates. In the thermal region, the scattering and capture cross-sections are frequently of the same order.

Intermediate neutrons ($10 \text{ eV} \leq E_n \leq 0.5 \text{ MeV}$).

This is a transitional region, in which $\lambda/2\pi R$ decreases from a very large value at the low-energy end to about $4.45A^{-1/3}$ at 0.5 MeV. As a result, the potential scattering cross-section also decreases from its low-energy value of $4\pi R^2$ and

begins to approach more nearly the asymptotic high-energy value which, according to the strong interaction model, is $\pi(R + \lambda)^2$ —the same value as for compound nucleus formation, and therefore half the total cross-section. Another consequence is that for the heaviest elements potential scattering begins to become non-isotropic at the top end of the energy range, a behaviour which can be understood if the scattering is regarded as a diffraction of the neutron wave (equation 3.4.2).

Light elements. The cross-section remains constant until the first resonance is reached. On account of the large level spacing the chance of a resonance occurring at energies close to thermal is small, and resonances are not in fact observed at neutron energies of less than about 100 keV until atomic weights greater than about 20 are reached.

Medium-weight elements show gradually increasing numbers of resonances as one goes upwards through the periodic table. It is known that many of these are scattering resonances, i.e. that $\Gamma_n \gg \Gamma_\gamma$.

In heavy elements there are many resonances, those at the lower energies being mainly due to capture, and those at the upper energies mainly to scattering. For the heaviest nuclei inelastic scattering begins to make a small contribution to σ_{total} by the time the top of the intermediate energy range is reached.

Angular Momentum and the Angular Distribution of Scattered Neutrons⁽⁷³⁾

Neutrons passing at non-zero distances from the centre of a nucleus possess angular momentum with respect to the nucleus. In a scattering process the angular distribution of the scattered neutron is determined by its angular momentum and that of the incident neutron, together with the intrinsic angular momentum or spin of the target nucleus. Although the reasons why this should be so are beyond the scope of this book, we may notice that, for reactions involving low-energy neutrons, the scattered neutrons are isotropically distributed in the centre of mass system of co-ordinates (which, except for the lightest elements, is not very different from the laboratory system). Not until neutron energies of the order of $10A^{-\frac{1}{2}}$ MeV are reached does the contribution from reactions involving neutrons of higher angular momenta begin to make the distribution appreciably non-isotropic. Thus considerations of angular momentum enter into reactor shielding theory only when undegraded fission neutrons are being considered; elsewhere it is sufficient to assume that scattering is isotropic in the centre of mass system, just as in billiard-ball mechanics.

Scattering by Hydrogen

The lighter the scattering nucleus the higher is the neutron energy at which scattering becomes anisotropic in the centre of mass system. For the extreme case of hydrogen, scattering is isotropic in this system up to energies ~ 10 MeV. On transforming to the laboratory system of co-ordinates, however, it is found that scattering is mainly into the forward hemisphere; this result follows from the equality of the masses of the neutron and the hydrogen atom, and the application of the ordinary laws of mechanics (see equation 4.6.5).

On account of its small mass and the elementary nature of its nucleus (a proton) hydrogen possesses neutron scattering properties which give it a unique

position in neutron attenuation theory (equation 3.1.1). In the intermediate energy region its cross-section rises very rapidly as the neutron energy falls (Fig. 3.4.2), and this has the important practical consequence that in hydrogenous shields the build-up of low-energy neutrons in equilibrium with the faster neutrons is small. Collisions with hydrogen are the only important method by which neutrons of energy less than about 0.5 MeV can be degraded rapidly to thermal energies.

Fast Neutrons (0.5 MeV to 10 MeV)

(a) *Shadow or diffraction scattering.* At high energies the neutron wavelength becomes comparable with the radius of the nucleus, and potential scattering takes on a marked bias towards the forward direction. The explanation for this behaviour is found in the diffraction of the neutron wave by the nucleus. As with optical scattering by an opaque disc, the strong-interaction model gives the result that the majority of the neutrons are scattered into a cone of semi-angle $\sim \lambda/R$. The dependence of intensity on the angle of scattering is given, for an opaque nucleus, by the following formula of HAUSER and FESHBACH:

$$\frac{d\sigma(\theta)}{d\Omega} \simeq \frac{(R + \lambda)^2}{4} \cot^2 \left(\frac{\theta}{2} \right) \left\{ J_1 \left(\frac{R + \lambda}{\lambda} \sin \theta \right) \right\}^2, \quad (3.4.2)$$

where J_1 is the first-order Bessel function.

However, besides this diffraction scattering (sometimes called shape elastic scattering) there are two further ways in which neutrons may be elastically scattered. The first is due to the partial transparency of the nucleus to fast neutrons, to which we have already referred, and which results in what is effectively a refraction of the neutron wave. Secondly, compound elastic scattering (see below) is always energetically possible, though at high energies it is unimportant compared with inelastic scattering. The contributions from these three processes cannot be distinguished experimentally, and consequently the observed distributions differ appreciably, for neutron energies ~ 1 MeV, from those predicted by formula (3.4.2). For instance, WALT and BARSCHALL find that for 1 MeV neutrons the forward peak of the scattering distribution (shown in Fig. 3.4.5(a)) passes through a maximum in the region of $A = 140$, whereas according to equation (3.4.2) it should grow monotonically with increasing atomic weight. Other characteristic features of the same set of results are the flat distributions at $A \sim 60$, and the subsidiary peaks at $\sim 110^\circ$ for $A \sim 180$. Once again, the optical theory is able to reproduce most of the main features of the experimental distributions (Fig. 3.4.5(b)). At higher energies, around 10 MeV, the strong-interaction model which is the basis of equation (3.4.2) appears to be an adequate approximation. This is because the nucleus is much less transparent at these energies, so that there is a very much smaller contribution from the second and third processes mentioned above.

(b) *Capture scattering.* The highly excited compound nucleus formed by the interaction between a nucleus and a fast neutron can decay in one of several ways, of which the most important is by neutron emission; as mentioned on page 116, capture scattering may be either elastic or inelastic, depending

on the excitation energy ϵ of the residual nucleus. The quantity ϵ can have many possible values, each equal to the energy of excitation of a level of the nucleus. A most important factor, therefore, in determining the energy spectrum of the secondary neutrons is the level spacing of the residual nucleus. If the residual nucleus is left in a state of comparatively low excitation (most of the energy having been taken away by the neutron), then the level spacing is ~ 1 MeV for light elements, ~ 100 – 200 keV for nuclei of atomic weight ~ 100 , and ~ 50 – 100 keV for the heaviest nuclei. As ϵ increases, the level spacing becomes smaller, until the levels actually overlap. This occurs when ϵ is approximately 1 MeV above the neutron binding energy (for medium and heavy elements). The effect of this pattern of levels is to provide many more ways of leaving a nucleus in a state of high excitation than in one of lower excitation, thus favouring the emission of low-energy inelastically scattered neutrons. On the other hand, the more energetic the outgoing neutron the greater is the probability of its being able to leave the nucleus (equation 3.3.4). The result of these two opposing factors is that the spectrum of the outgoing neutrons is peaked at some intermediate energy.

These considerations can be made rather more quantitative with the aid of the statistical theory of the nucleus, which assumes that the level spacing is small, and that many levels are capable of being excited. The differential cross-section for the inelastic scattering of neutrons of initial energy E_{n0} into the energy range E_n to $E_n + dE_n$ (with $E_n = (E_{n0} - \epsilon) \ll E_{n0}$) is approximately

$$d\sigma(E_{n0}, E_n) \approx \sigma_{\text{cpd}}(n) \frac{E_n}{\theta^2} \exp(-E_n/\theta) dE_n \quad (3.4.3)$$

This expression confirms the peaked form which seemed plausible from elementary considerations. The most probable energy of the inelastically scattered neutrons is at $E_n = \theta$, where θ is the *nuclear temperature* corresponding to the excitation of the residual nucleus by an energy approximately equal to the kinetic energy of the original fast neutron:

$$\theta^2 = \epsilon/a \approx E_{n0}/a, \quad (3.4.3a)$$

where a is the parameter in the level-density formula (3.3.3). Equation (3.4.3) does not represent the high-energy end of the spectrum with any accuracy, since this corresponds to leaving the nucleus in a state of low excitation, where the levels are few and far apart, and where the statistical (or thermodynamic) approximation is invalid.

There are a number of practically important materials, such as lead, iron, and most lighter elements, in which the level spacing is so large that the statistical theory is inapplicable unless E_{n0} is greater than several MeV. For such elements the inelastic scattering cross-section varies with neutron energy in roughly the following manner (we make the simplifying assumption that scattering is the only important reaction). For neutrons of energy E_{n0} less than ϵ_1 , the energy of the first excited level of the target nucleus, the only reaction possible is capture elastic scattering, and the compound elastic cross-section σ_0 is therefore equal to the cross-section $\sigma_{\text{cpd}}(n)$ for compound nucleus formation. As E_{n0} increases

beyond ϵ_1 , excitation of the first level becomes a competing process. The inelastic cross-section σ_1 associated with excitation of this level grows at first rapidly with increasing energy once the threshold energy has been passed. At the same time the elastic cross-section shows a rapid decrease, since the total cross-section for compound nucleus formation does not change rapidly with neutron energy. If E_{n0} continues to increase in the energy band $\epsilon_1 < E_{n0} < \epsilon_2$, the value of σ_1 grows at the expense of σ_0 until, at ϵ_2 , a third process becomes possible, and σ_2 takes on a non-vanishing value.

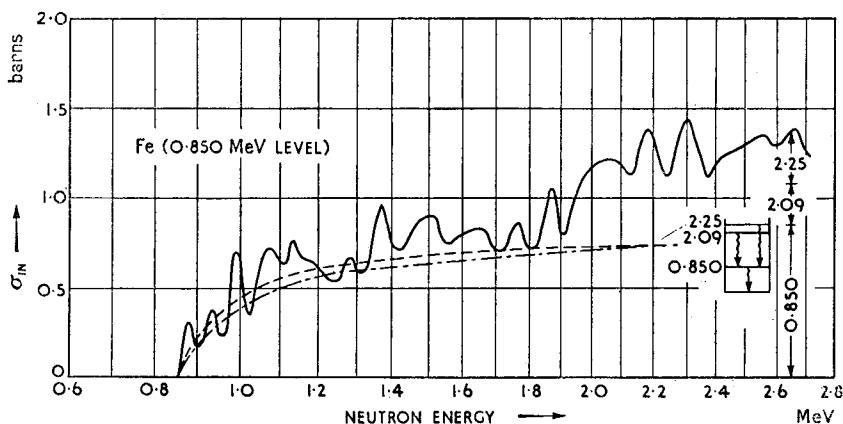


Fig. 3.4.4. Inelastic scattering by iron as a function of neutron energy. The dashed curves show the result of two calculations for scattering by the 0.850 MeV level. Above 2 MeV the other levels shown in the diagram also contribute. (KIEHN and GOODMAN⁽⁶⁷⁾.)

Experimental data regarding the excitation of individual levels accumulated very slowly before 1950, because the measurement of fast-neutron energies is difficult. However the steady improvement in techniques for gamma-ray observation has recently allowed a fresh approach to the problem.⁽¹³⁾ This depends on the fact that very soon (10^{-14} sec) after the neutron has been ejected, the residual excited nucleus returns to the ground state with the emission of one or more gamma-ray quanta. The total energy of the gamma rays is equal to the energy ϵ_r lost by the neutron in the inelastic scattering process (apart from the small amount of energy given to the recoiling nucleus). Thus by observing the numbers and energies of the gamma rays emitted in coincidence with the inelastically scattered neutron, direct measurement of the component cross-sections $\sigma_1, \sigma_2, \dots$, and of their dependence on E_{n0} is possible. Fig. 3.4.4 and many of the data given in the following compilation were obtained in this way. Although powerful, the method has its limitations: the interpretation of results may be confused if the higher levels decay with the emission of a cascade of soft gamma rays, rather than in one step; while in some rare cases the nucleus is left in a metastable state, and the gamma-ray emission is delayed; examples are the 4.5 hour indium activity and the gold-197 activity induced by inelastic scattering of neutrons of energy greater than 440 keV and 420 keV respectively.

COMPILATION OF INELASTIC SCATTERING DATA

Table 3.4.1. *Measured Values of Integral Inelastic Scattering Cross-section (Neutron of initial energy E_{n0} emerging with final energy $<E_t$)*

Note: Some of the measurements of σ_{in} for 14 MeV neutrons given below could be more correctly described as the cross-section for the emission of low-energy neutrons, without distinction between inelastically scattered neutrons and neutrons from the $(n, 2n)$ process. The results of reference 26 measure the inelastic portion only, and for this reason are appreciably lower than the other values quoted.

Element	Neutron energy		σ_{in} ($E_{n0} \rightarrow E_t$ or less)	Reference	Theoretical cross-section for compound nucleus formation
	Initially E_{n0} MeV	Finally less than E_t MeV			
Be	2.5	2.5	<0.014	2	0.79
	4.1	3.2	0.6 ± 0.1	23	
	14	~ 3	0.16 ± 0.07	2	
		~ 11	0.82 ± 0.03	2	
B	14	~ 3	0.24 ± 0.04	2	0.64
		~ 11	0.69 ± 0.10	2	
C	2.5	2.5	<0.006	2	0.74
	4.1	3.2	0.08 ± 0.1	23	
	14	~ 11	0.85 ± 0.02	2	
	14	11	0.52 ± 0.02	26	
		12	0.52 ± 0.2	24	
F	2.5	2.5	0.52 ± 0.18	2	1.4
Mg	2.5	2.5	0.75 ± 0.23	2	1.10
		~ 2.5	~ 0.6		
Al	4.1	3.2	0.7 ± 0.2	23	0.93
	14	~ 3	0.62 ± 0.07	2	
		~ 11	1.06 ± 0.05	2	
		11	0.79 ± 0.03	26	
		12	0.65 ± 0.17	24	
S	2.5	2.5	0.38 ± 0.1	2	0.90
Ti	4.1	3.2	1.2 ± 0.2	23	
Cr	2.5	2.5	1.2 ± 0.4	2	1.47
Fe	1.5	~ 0.5	0	2	1.58 1.52
		~ 0.9	0.6	2	
	2.5	2.5	1.9 ± 1.3	2	
	3.0	~ 0.75	0.3	2	
		~ 1.50	0.7	2	
		~ 2.25	1.1	2	
	4.1	3.2	1.5 ± 0.2	23	
	14	~ 2	0.78 ± 0.03	2	

(continued
on next
page)

Table 3.4.1.—continued

Element	Neutron energy		σ_{in} ($E_{n0} \rightarrow E_t$ or less)	Reference	Theoretical cross-section for compound nucleus formation
	Initially E_{n0} MeV	Finally less than E_t MeV			
Fe (continued)	14	~ 3	1.21 ± 0.03	2	
		~ 11	1.45 ± 0.02	2	
		12	1.3 ± 0.3	24	
		11	≥ 0.97	26	
Co	1.5	~ 0.5	(0)	2	1.62
		~ 0.9	(0.2)	2	
		~ 1.3	(0.8)	2	
Ni	1.5	~ 0.5	(0)	2	1.62
		~ 0.9	(0.1)	2	
		~ 1.3	(0.6)	2	
Cu	1.5	~ 0.5	(0.3)	2	1.65
		~ 0.9	(0.6)	2	
		~ 1.3	(0.9)	2	
		2.5	1.5 ± 0.7	2	
	2.5	~ 0.75	(0.6)	2	1.62
	3.0	~ 1.50	(1.3)	2	
		~ 2.25	(1.5)	2	
	14	11	0.36 ± 0.09	26	
Zn	4.1	3.2	1.7 ± 0.2	23	
	14	11	≥ 0.91	26	
	14	12	1.3 ± 0.3	24	
Zr	4.1	3.2	1.8 ± 0.2	23	
Ag	14	11	1.0 ± 0.2	26	
	14	12	1.6 ± 0.4	24	
Cd	4.1	3.2	2.1 ± 0.2	23	
	14	~ 2	1.14 ± 0.04	2	
		~ 3	1.66 ± 0.07	2	
		~ 11	1.89 ± 0.06	2	
	14	11	≥ 1.06	26	
	14	12	2.2 ± 0.6	24	
Sn	4.1	3.2	2.1 ± 0.2	23	
	14	11	≥ 0.92	26	
	14	12	1.8 ± 0.5	24	
Ta	1.5	~ 0.5	(1.4)	2	2.57
		~ 0.9	(2.0)	2	
		~ 1.3	(2.7)	2	
	4.1	3.2	2.7 ± 0.2	23	

Table 3.4.1.—continued

Element	Neutron energy		σ_{in} ($E_{n0} \rightarrow E_t$ or less)	Reference	Theoretical cross-section for compound nucleus formation
	Initially E_{n0} MeV	Finally less than E_t MeV			
W	1.5	~0.5	0.9	2	
		~0.9	2.1	2	
	3.0	~0.75	1.4	2	
		~1.50	2.4	2	
		~2.25	2.8	2	
	4.1	3.2	2.4 $\pm$ 0.3	23	
Au	3.0	~0.75	(2.1)	2	
		~1.50	(2.8)	2	
		~2.25	(3.0)	2	2.69
	4.1	3.2	2.7 $\pm$ 0.3	23	2.69
	14	11	$\geq$ 1.22	26	
	14	~2	1.47 $\pm$ 0.10	2	
		~3	2.06 $\pm$ 0.09	2	
		~11	2.51 $\pm$ 0.04	2	2.69
Pb	1.5	~0.5	0	2	
		~0.9	0.4	2	2.81
	2.5	~1	0.55	2	
		~2.5	1.3 $\pm$ 0.5	2	2.77
	3.0	~0.75	0.7	2	
		~1.50	1.2	2	2.77
		~2.25	1.6	2	
	4.1	3.2	1.9 $\pm$ 0.3	23	2.77
	14	~2	0.91 $\pm$ 0.06	2	
		~3	2.29 $\pm$ 0.04	2	
		~11	2.56 $\pm$ 0.05	2	
		11	$\geq$ 0.73	26	
		12	$\geq$ 2.6	2	2.77
	14	12	3.3 $\pm$ 0.8	24	
	14.5	~3	2.20 $\pm$ 0.17	2	
		~11	2.29 $\pm$ 0.12	2	2.77
Bi	2.5	~1	0.64	2	2.78
	4.1	3.2	2.2 $\pm$ 0.3	23	2.78
	14	~2	1.03 $\pm$ 0.11	2	
		~3	2.28 $\pm$ 0.08	2	
		~11	2.56 $\pm$ 0.05	2	
		11	$\geq$ 0.67	26	
		12	$\geq$ 3.3	2	2.78
		12	3.9 $\pm$ 1.0	24	

Table 3.4.2. Temperatures for Observed Distributions of
Inelastically Scattered Neutrons
(equation 3.4.3)

Element	E_{n0} (MeV)	Range of E_n (MeV)	θ (E_{n0}) (MeV)	Reference
B	11	>4	2.3 $\pm$ 0.3	2
		1-4	0.9 $\pm$ 0.1	2
C	14	0.5-4	1.04 $\pm$ 0.11	24
	14.8	0.5-4	0.93	25
Al	14	0.5-4	1.01 $\pm$ 0.10	24
	14.8	0.5-4	1.06	25
	15	>1	1.1 $\pm$ 0.1	2
Si	10.6	>2	1.3 $\pm$ 0.1	2
Fe	1.5	<0.9	None	2
	3.0	<2.25	None	2
	14	0.5-4	0.76 $\pm$ 0.08	24
	15	>1	0.6 $\pm$ 0.1	2
Co	10.5	>2	0.95 $\pm$ 0.1	2
Cu	14	0.5-4	0.77 $\pm$ 0.08	24
	14.8	0.5-4	0.82	25
Zn	14	0.5-4	0.73 $\pm$ 0.07	24
Pd	14	>2	0.85 $\pm$ 0.1	2
Ag	14	0.5-4	0.63 $\pm$ 0.06	24
Cd	14	0.5-4	0.66 $\pm$ 0.07	24
Sn	14	0.5-4	0.56 $\pm$ 0.06	24
	14.8	0.5-4	0.62	25
W	1.5	<0.9	0.35	2
	3.0	<2.25	0.50	2
Au	3.0	<2.25	(0.33)	2
	14	0.5-4	0.66 $\pm$ 0.07	24
Hg	14.6	>2	0.8 $\pm$ 0.1	2
Pb	4.3		None	2
	14	1-3	$\sim$ 0.8	2
	14	0.5-4	0.76 $\pm$ 0.08	24
	14.3	>2	0.78 $\pm$ 0.1	2
	14.8		0.75	25
	15	>1	0.7 $\pm$ 0.1	2
Bi	4.3	>1	None	2
	14	1-5	$\sim$ 0.9	2
	15	0.5-4	0.95 $\pm$ 0.10	24

Table 3.4.3 *Inelastic Scattering from Particular Energy Levels*

Most of the following results were obtained by observing the gamma rays emitted in the decay of the excited state produced by the inelastic-scattering collision. Experiments which measured the actual energy lost by the neutron are denoted by an asterisk. The cross-section is evaluated in most cases by assuming that the inelastically scattered neutrons are emitted isotropically.

Element	Neutron energy E_{n0} (MeV)	Observed gamma-ray energy or neutron energy-loss (MeV)	σ_{in}	Reference
C	14	4.43	0.3	14
	14	4.4	0.245 ± 0.035	21
O	14	6.13 ~7	0.2	14
	14	5.2 3.8 3.0		15
			0.52	
Al	2.5*	0.9	0.63 ± 0.25	16
	14	1.7 4.5 5.4	1.7	15 (See also 67)
Cr	2.5	1.42 ± 0.05	1.0 ± 0.2	17
	3.2	0.75 ± 0.03 0.97 ± 0.04 1.43 ± 0.06	0.027 0.10 0.73	18 (See also 67)
Mn	1.2	0.126	1	19
Fe	1.2	0.850	0.59 ± 0.06	19
	1.87*	0.85	0.82	20
	2.5	0.91 ± 0.15	1.0 ± 0.3	17
	2.5*	0.8	1.0 ± 0.25	16
	3.2	0.835 ± 0.015 1.17 ± 0.05 1.67 ± 0.07	1.18 0.39 0.33	18
	4.3*	0.85 2.1 2.7	0.9 ± 0.3 0.4 ± 0.2 0.2 ± 0.1	22 (See also 67)
Co	3.2	0.60 ± 0.02 1.15 ± 0.04 1.49 ± 0.07 1.7 ± 0.08 2.5 ± 0.1	0.20 0.82 0.53 0.15 0.19	18
Ni	2.5*	1.4	0.6 ± 0.25	16
	3.2	0.59 ± 0.02 1.33 ± 0.05 1.49 ± 0.06 2.66 ± 0.1	0.047 0.84 0.54 0.01	18 (See also 67)

Table 3.4.3.—continued

Element	Neutron energy E_{n0} (MeV)	Observed gamma-ray energy or neutron energy-loss (MeV)	σ_{in}	Reference
Cu	3.2	0.66 $\pm$ 0.03 0.96 $\pm$ 0.05 1.37 $\pm$ 0.06 1.9 $\pm$ 0.1	0.21 0.12 0.81 0.16	18
Zn	3.2	1.02 $\pm$ 0.05 1.3 $\pm$ 0.06 1.6 $\pm$ 0.07	1.3 0.03 0.02	18
Zr	3.2	0.69 $\pm$ 0.04 0.893 $\pm$ 0.04 1.14 $\pm$ 0.06 1.5 $\pm$ 0.07 2.17 $\pm$ 0.1	0.038 0.402 0.12 0.14 0.44	18
Mo	3.2	2.5 $\pm$ 0.2 1.4 $\pm$ 0.06 0.73 $\pm$ 0.03 Plus unresolved lines	0.059 0.33 0.66	18
Cd	3.2	0.57 $\pm$ 0.02 2.8 $\pm$ 0.02 Plus unresolved lines	0.69 0.016	18
In	2.5	0.92 $\pm$ 0.04 0.61 $\pm$ 0.06	0.4 $\pm$ 0.1 0.2 $\pm$ 0.1	17
	3.2	0.88 $\pm$ 0.04 1.15 $\pm$ 0.05 2.08 $\pm$ 0.1	0.27 0.10 0.16	18
Sn	3.2	0.69 $\pm$ 0.02 1.14 $\pm$ 0.05 2.0 $\pm$ 0.1	0.14 1.67 0.21	18
Ta	3.2	0.46 $\pm$ 0.03 1.44 $\pm$ 0.15	1.26 0.93	18
Pb	3.2	0.35 $\pm$ 0.02 0.52 $\pm$ 0.02 0.80 $\pm$ 0.03 1.1 $\pm$ 0.05 1.4 $\pm$ 0.10 2.2 $\pm$ 0.10	0.52 1.02 1.05 0.18 0.25 0.16	18 (See also 67)
Bi	2.5	0.85 1.58	1.2 $\pm$ 0.3 0.6 $\pm$ 0.2	17
	2.5*	0.9 $\pm$ 0.2 1.3 $\pm$ 0.2	0.75 $\pm$ 0.3 0.3 $\pm$ 0.15	16
	3.2	0.49 $\pm$ 0.02 0.94 $\pm$ 0.05 1.62 $\pm$ 0.07 2.6 $\pm$ 0.1	0.43 1.2 0.59 0.37	18 (See also 67)

The reader is also referred to the following papers presented at the Geneva Conference, 1955: papers P/581, P/588, and P/714.

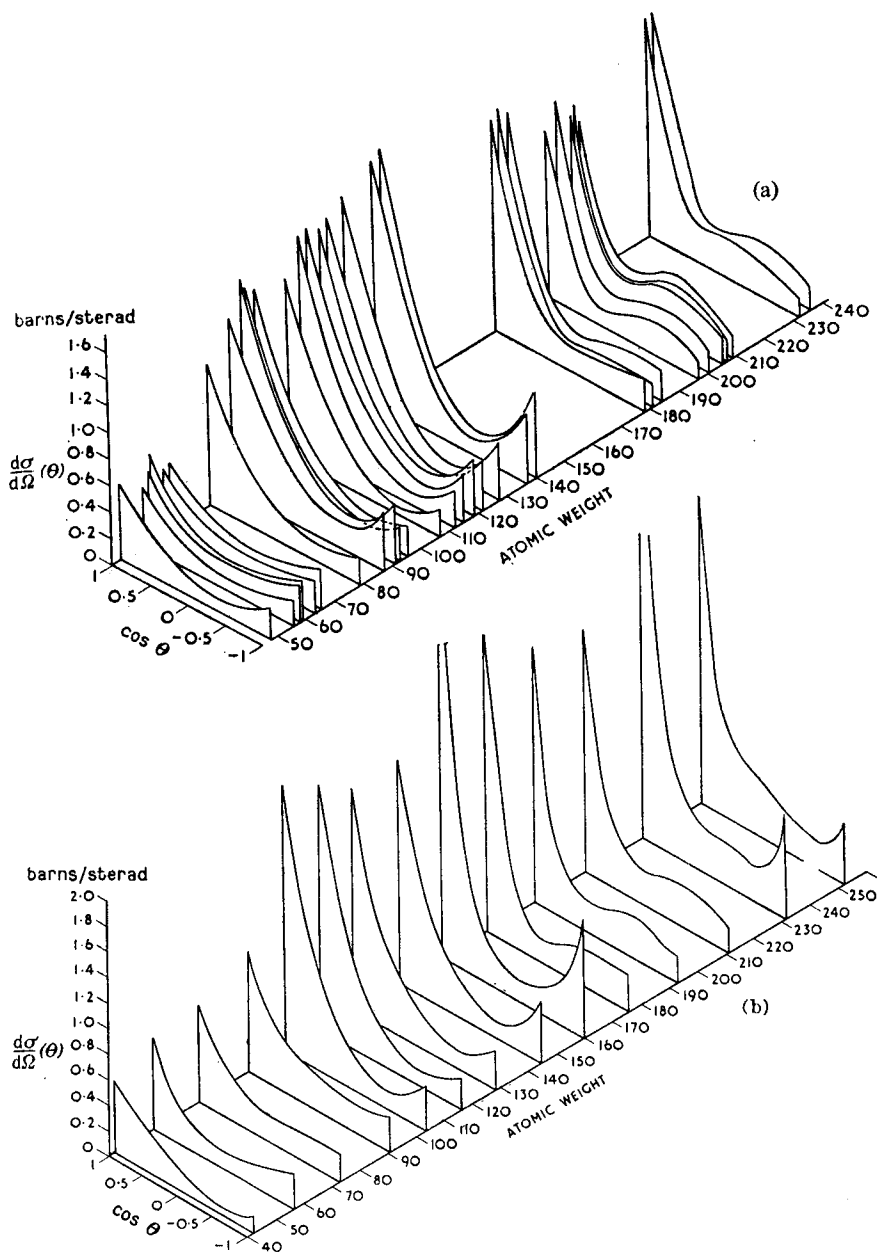


Fig. 3.4.5. Angular dependence of elastic scattering at 1 MeV as a function of $\cos \theta$ and atomic weight.

(a) Experimental (WALT and BARSCHALL⁽¹²⁾)

(b) Calculated from optical model (WEISSKOPF⁽⁹⁾)

Comprehensive data on angular distributions will be found in reference (73).

3.5. RADIATIVE CAPTURE

The capture of a neutron results in the formation of a compound nucleus with an excitation energy of the order of 7 MeV. The fate of this compound nucleus is determined by the relative probabilities for decay by gamma emission (radiative capture) and by neutron emission (scattering). The behaviour of the neutron width Γ_n has already been discussed in the previous sections. The gamma width Γ_γ behaves differently. Experimentally it is found that Γ_γ varies slowly with atomic weight (see Fig. 3.5.1), but shows remarkably little dependence on other nuclear properties such as spin. Since slow-neutron capture

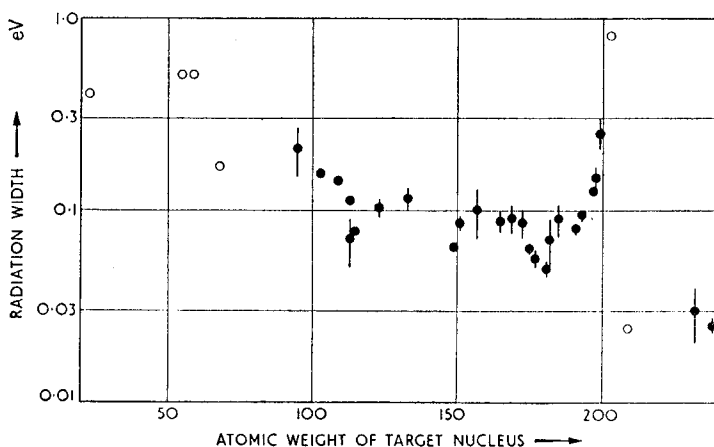


Fig. 3.5.1. Average radiation width as a function of atomic weight of the target nucleus. The open points are values computed from thermal cross-sections; the others are measurements of actual resonances. (LEVIN and HUGHES⁽²⁷⁾.)

results in a large excitation energy it seems unlikely that the radiation widths shown in Fig. 3.5.1 would change very much unless the kinetic energy of the neutron was of the order of 1 MeV or more.

The product nucleus resulting from the radiative de-excitation of the compound nucleus contains an excess of neutrons, and is therefore in many cases radioactive (see Section 3.8).

Dependence of Radiative Capture Cross-section on Energy

(a) *Slow neutrons.* In the neighbourhood of resonances the factor which determines the radiative capture cross-section $\sigma(n, \gamma)$ is the ratio Γ_γ/Γ_n . For heavy nuclei $\Gamma_\gamma \gg \Gamma_n$, for light nuclei $\Gamma_n \gg \Gamma_\gamma$. Away from a resonance $\sigma(n, \gamma)$ is still determined by the character of the nearest resonance, or, if there are several near enough to be effective, by their resultant effect. Far from a

resonance $\sigma(n, \gamma)$ varies as (neutron velocity) $^{-1}$ in the low-energy region. This follows from the single-level Breit-Wigner formula

$$\sigma(n, \gamma) = \pi \lambda_n^2 g \cdot \frac{\Gamma_\gamma \Gamma_n}{(E_n - E_r)^2 + \Gamma^2/4} \quad (3.5.1)$$

when $E_n \ll |E_r|$ and $|E_r| \gg \Gamma$, since $\lambda_n^2 \propto E_n^{-1}$ (equation 3.2.1), $\Gamma_n \propto E_n^{1/2}$ (equation 3.3.4), and Γ_γ is independent of neutron energy.

In a few cases of great practical importance a capture resonance occurs at neutron energies very close to thermal. The best-known example is cadmium, whose absorption cross-section is shown in Fig. 3.3.2. The effect of the 0.18 eV

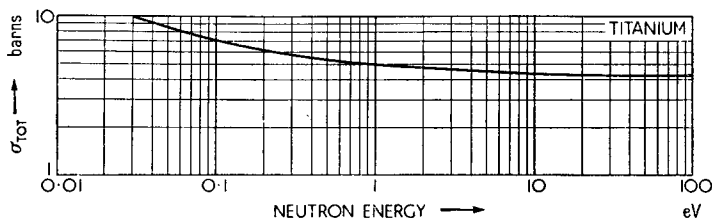


Fig. 3.5.2. Total cross-section of titanium in the low-energy region. The behaviour is typical of materials with moderate thermal capture when there is no resonance close to thermal energies. Above 10 eV the cross-section is constant, and is due to scattering. At lower energies capture (which is proportional to $E_n^{-1/2}$) becomes important.

capture resonance is so pronounced that 0.4 mm of cadmium will remove 99% of the neutrons from a thermal neutron beam. The cadmium cut-off coincides approximately with the cut-off of the thermal Maxwellian spectrum at room temperature (equation 3.2.3, Fig. 3.2.1), a fact which forms the basis for the very useful cadmium difference method of measuring thermal neutron fluxes in the presence of appreciable fluxes of faster neutrons. A neutron detector is first exposed to neutrons of all energies, and the observed intensity recorded; it is then covered with thin cadmium, which removes all the thermal neutrons, and the contribution due to epithermal neutrons is then measured. Owing to the very steep fall in the cross-section at just below 1 eV, the majority of epithermal neutrons will not have been appreciably attenuated in passing through the cadmium covering, and the thermal neutron contribution may therefore be obtained by a simple subtraction.

Tables of resonance parameters (i.e. values of g , Γ , Γ_γ , Γ_n , and E_r) for low-lying capture resonances are given in reference (2).

(b) *Intermediate neutrons.* In light elements radiative capture is an unimportant process. In the heavier elements, of atomic weight greater than about 100, a rough value for the cross-section averaged over many resonances is given by the following formula, which may be derived from equation (3.3.7),

$$\overline{\sigma(n, \gamma)} = \overline{\sigma_{\text{epd}}(n)} \cdot \frac{\Gamma_\gamma}{\Gamma} \sim \frac{500}{\sqrt{E_n(\text{eV})}} \frac{\Gamma_\gamma}{\Gamma} \quad (3.5.2)$$

For $E_n \geq 1$ keV

$E_n \leq 0.4$ MeV for $A = 100$

$E_n \leq 0.2$ MeV for $A = 240$

For all medium and heavy elements the gamma width is of the order of 0.1 eV, and is more or less independent of neutron energy; while the average neutron width $\overline{\Gamma}_n$ varies in the manner predicted by the very approximate equation (3.3.4). At some energy of the order of a few kilovolts the gamma and neutron widths will be equal. At higher energies the total width Γ is approximately equal to Γ_n , which has a $\sqrt{E_n}$ variation, so that the average

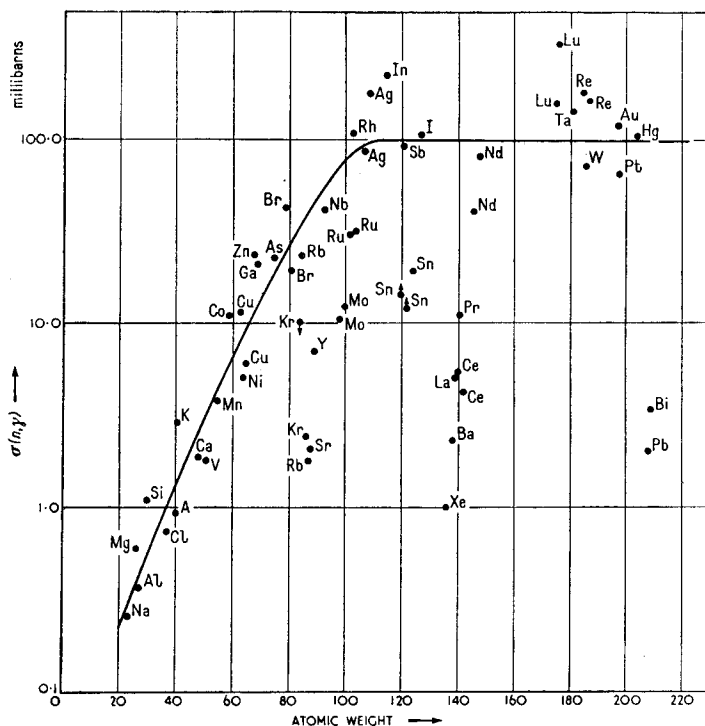


Fig. 3.5.3. Capture cross-sections at an effective energy of 1 MeV as a function of atomic weight. (HUGHES *et al.* ^(11,29).)

cross-section (equation 3.5.2) varies as $1/E_n$, in contrast to the $1/E_n^{1/2}$ law which prevails at very low energies. The formula reproduces at least the order of magnitude of the capture cross-sections observed for neutrons of energy about 1 MeV (Fig. 3.5.3).

(c) *Fast neutrons.* Formula (3.5.2) for the average radiative capture cross-section at energies of the order of 1 MeV may be written in the alternative form⁽²⁸⁾

$$\overline{\sigma(n, \gamma)} \sim 2\pi^2(\lambda + R)^2 \frac{\Gamma_\gamma}{D}, \quad (3.5.3)$$

where D is the average level spacing at the excitation energy of neutron capture. Remembering that D changes more than Γ_γ in going from one end of the periodic table to the other (equation 3.3.3 and Fig. 3.5.1), one would expect that for

neutrons of energy about 1 MeV the (n, γ) cross-section ought to vary with atomic weight in the following way: it should be small for light elements, and should increase at first rapidly with increasing atomic weight, and then more slowly. For the magic nuclei (which, as we have already noticed, have unusually large level spacings) the radiative capture cross-section would be expected to be abnormally low. These theoretical expectations have been confirmed in a very satisfactory manner by the systematic study of capture cross-sections which HUGHES, GARTH, EGGLER, and LEVIN^(11,29) carried out with fission neutrons (Fig. 3.5.3). The total cross-section of all elements at neutron energies ~ 1 MeV is a few barns, so that the cross-section for the (n, γ) process represents only a very small fraction of σ_{total} for fast neutrons; neutrons therefore cannot be absorbed efficiently while still fast. However, as we have just seen, there is no difficulty in absorbing them once they have been slowed down.

Gamma-ray Production resulting from Capture of Thermal Neutrons

The spectrum of the emitted radiation depends very markedly on the nuclear species involved, and in particular on the level spacing of the product nucleus. For light elements few levels lie between the ground state and the energy of excitation corresponding to neutron capture. In such cases the number of possible modes of decay is small, and the gamma-ray spectrum consists of a few well-defined gamma-ray lines. Somewhat heavier elements have levels that are closer together, and a greater variety of modes of de-excitation is possible. As an example, we may take the case of sodium, which is particularly straightforward because there is only one naturally occurring isotope, Na^{23} . The gamma rays observed in the de-excitation of the product nucleus Na^{24} can be fitted into the level scheme shown in Fig. 3.5.4; at least four different routes appear to be possible. (These gamma rays are, of course, quite distinct from those which are evolved much later during the radioactive decay of the product Na^{24} , following its beta transformation to Mg^{24} .) Still further up the periodic table, the medium-heavy elements have level spacings of no more than a few volts at excitations corresponding to the binding energy of a neutron (about 7 MeV). Consequently, there are very many levels lying between the ground state and the energy of excitation, and de-excitation can take place in a great variety of gamma-ray cascades; so that, in general, the gamma-ray spectrum appears to be continuous (Fig. 3.5.5). However, with some nuclides one particularly energetic mode of decay is found to predominate (iron is an example), so that the contribution from the other modes can then usually be ignored in shielding calculations. Among the heavy elements the magic number nuclei, such as lead, are unique in having unusually simple capture gamma-ray spectra, owing to their wide level spacing.

No comprehensive theory of the process of de-excitation by gamma emission has yet been developed. For medium-heavy elements BETHE⁽³¹⁾ has used a statistical treatment to show that the spectral distribution of the first-emitted quanta should be peaked at about 3 MeV. When a partially de-excited state of the compound nucleus has been reached by the emission of such an energetic primary photon, comparatively few levels remain between the new state and the ground state, and consequently the statistical theory cannot be applied to the

subsequent cascades of lower-energy radiation; however, one would in general expect the overall spectrum to be peaked at an energy lower than 3 MeV.

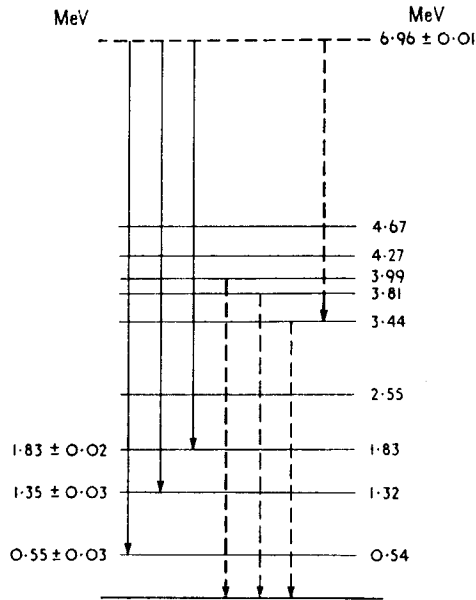


Fig. 3.5.4. Suggested level scheme for Na^{24} decaying to the ground state by gamma emission, following the absorption of a neutron by Na^{23} (KINSEY *et al.*⁽³⁰⁾).

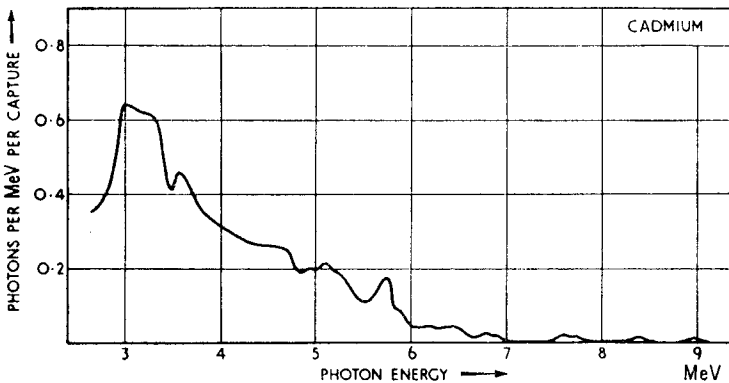


Fig. 3.5.5. Spectrum of gamma radiation emitted when neutrons are absorbed by cadmium (KINSEY *et al.*⁽³⁰⁾).

The experimental evidence is fairly comprehensive for gamma-ray energies above 3 MeV, a region in which the pair spectrometer provides a method of high resolution for observing gamma-ray spectra. At lower energies the situation is

not so satisfactory, principally because the resolution of the available instruments (magnetic lens spectrometer, crystal spectrometer, two-crystal spectrometer) is poorer, and the range over which they can be satisfactorily operated is limited. Consequently, the experimental results below 3 MeV are still incomplete, and are sometimes contradictory. The practical importance of this gap in our knowledge depends on the type of shield being used. As far as penetration through shields made of light and medium-heavy elements is concerned, the low-energy region is unimportant in comparison with the high energies. On the other hand, if a heavy-element gamma-ray shield—such as lead or barytes concrete—is being considered, the radiation which most concerns the designer is the fraction with photon energies around the value $E_{\min}$ at which the shield is most transparent; that is, 2–3 MeV (see Fig. 2.6.4). When this fraction is unknown, an overestimate of the shielding required for the capture radiation can be made by assuming that the whole of the binding energy (Table 3.3.1) is emitted in the form of quanta of energy equal to $E_{\min}$.

The following compilation summarizes the available data regarding the production of gamma radiation in the radiative capture of thermal neutrons. Additional information is given in Fig. 3.5.5 for cadmium, owing to its practical importance.

Table 3.5.1. *Summary of Experimental Data on Capture Gamma Rays*

A. Light Elements

Hydrogen ($Z = 1$)	One gamma ray per capture, 2.230 ± 0.007 MeV. No line with intensity $>20\%$ between 0.7 and 2.23 MeV.
Lithium (3)	Slow neutron (n, α) reaction with Li^6 unaccompanied by gamma radiation.
Beryllium (4)	6.81 MeV photon in $\sim 75\%$ of captures. Cascade of two photons, each about 3.41 MeV, in $\sim 25\%$ of captures.
Boron (5)	Slow neutron (n, α) reaction with B^{10} gives 0.478 MeV gamma ray in 94 % of captures, from decay of excited state of the Li^7 nucleus formed in the reaction.
Carbon (6)	4.951 MeV photon in $\sim 65\%$ of captures. 3.68 MeV photon in $\sim 26\%$ of captures.

B. Medium and Heavy Elements

Capturing element	Photons in designated (MeV) energy intervals per 100 captures					Highest energy gamma ray (MeV)
	0–1	1–3	3–5	5–7	>7	
Aluminium	?	>13	77	21	35	7.724
Antimony*	?	~ 80	36	12	0	6.80
Arsenic*	?	~ 80	47	22	1	7.30
Barium*	?	~ 80	75	14	1	9.23
Bismuth†	0	0	100	0	0	4.17
Cadmium*	>120	20	73	17	1	9.046
Calcium	?	50	60	101	2.4	7.83
Chlorine	?	20	13	18	21	8.56
Chromium*	>37	16	12	18	69	9.716

Table 3.5.1. B. (continued.)

Capturing element	Photons in designated (MeV) energy intervals per 100 captures					Highest energy gamma ray (MeV)
	0-1	1-3	3-5	5-7	>7	
Cobalt*	?	?	36	49	8	7.486
Copper*	?	?	>23	22	42	7.914
Fluorine	?	?	?	35	0	6.63
Gadolinium*	?	80	23	4	2	7.78
Gold*	?	?	66	38	0	6.494
Indium*	?	?	36	4	0	5.86
Iron*	?	<10	24	22	50 * *	10.16
Lead†	0	0	0	7	93	7.38
Magnesium	?	>59	110	25	11	9.216
Manganese*	?	?	>27	30	27	7.261
Mercury*	?	?	86	41	0	6.446
Molybdenum*	?	?	84	26	3	9.15
Nickel	?	?	>14	30	72	8.997
Niobium*	?	?	54	14	0	7.19
Nitrogen-14	?	<5	<35	90	39	10.8
Phosphorus	?	?	115	43	11	7.94
Platinum*	?	~120	45	15	1	7.920
Potassium	?	36	36	32	12	9.28
Praseodymium*	?	~80	34	8	0	5.83
Rhodium*	?	~70	38	10	0	6.792
Samarium*	?	~150	45	5	1	7.89
Scandium*	?	?	63	29	14	8.85
Selenium*	?	?	65	27	11	10.483
Silicon	?	>100	229	41	16	10.55
Silver*	?	~90	70	17	0.5	7.27
Sodium	~100	>50	61	29	0	6.41
Strontium*	?	~140	62	49	13	9.22
Sulphur	?	>19	80	91	8	8.64
Tantalum*	?	~50	26	2	0	6.07
Thallium*	?	~100	76	62	0	6.54
Tin*	?	?	139	33	4	9.35
Titanium*	>50	100	33	99	10	9.39
Tungsten*	?	?	53	14.5	0.5	7.42
Vanadium-51*	?	?	24	54	18	7.305
Zinc*	?	?	48	29	17	9.51
Zirconium*‡	?	?	113	35	4	8.66

(From MITTLEMAN and LIETKE⁽³²⁾, who give references. The reader is also referred to paper P/651, Geneva Conference 1955, by GROSHEV *et al.*)

* For elements marked with an asterisk, the table gives the number of unresolved photons; elsewhere only the intensity of resolved lines is given.

† For bismuth, the 4.17 MeV gamma ray is the only one present. For lead, the seven 5-7 MeV gamma rays are all 6.73 MeV.

‡ Capture-gamma-ray intensities for zirconium may be in error by 50%.

** For iron 36% of captures give a 7.64 MeV photon.

3.6. REACTIONS INVOLVING THE EMISSION OF CHARGED PARTICLES

Slow Neutrons

A proton or alpha particle may be emitted following the capture of a slow neutron, provided that energy is released in the reaction. In the language of nuclear physics, the Q of the reaction must be positive. For proton emission this quantity is given in MeV by

$$Q = 931(M(Z, A) + m_n - M(Z - 1, A) - m_{\text{proton}} - m_{\text{electron}}), \quad (3.6.1)$$

where the masses M are measured in atomic mass units, and include the masses of the orbital electrons. The energy Q appears as the kinetic energy of the reaction

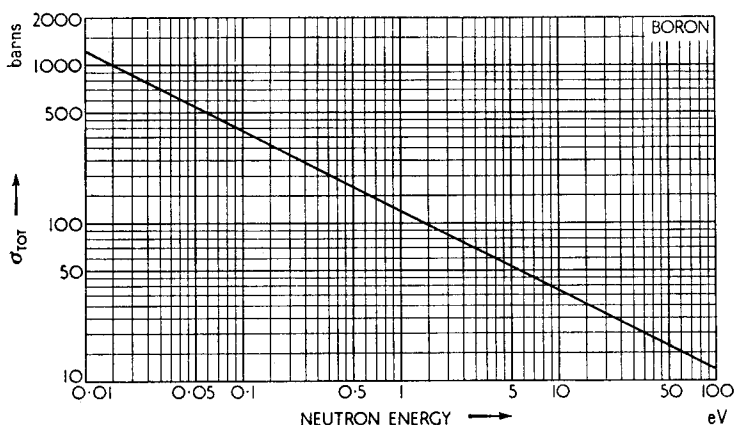


Fig. 3.6.1. Total cross-section of boron in the low-energy region. The overwhelming contribution is from the (n, α) reaction; scattering accounts for only 4 barns.

products. However, owing to the effect of the penetrability factor (equation 3.3.4) the reaction will have an insignificant cross-section unless Q is comparable with the height of the Coulomb barrier S_C (Fig. 3.3.1). For protons this is approximately equal to the potential energy of a charge $(Z - 1)$ units and a unit charge separated by a distance equal to the nuclear radius: $(Z - 1)$ is the charge on the product nucleus.

$$\begin{aligned} S_{C,p} &= (Z - 1) e^2 / R \\ &\approx 1.0 \times (Z - 1) A^{-1/3} \text{ MeV} \end{aligned} \quad (3.6.2)$$

For alpha particles

$$S_{C,\alpha} \approx 2(Z - 2) A^{-1/3} \text{ MeV} \quad (3.6.3)$$

Charged-particle reactions induced by slow neutrons tend to be confined to the lighter elements, for reasons already discussed. A notable exception is the

fission process which, in spite of the large nuclear charges carried by the fission fragments, produces enough energy to overcome the effect of the Coulomb barrier.

Owing to the large value of Q needed before a charged-particle reaction can proceed with a reasonably large cross-section, the partial width Γ_p or Γ_α , which can be very large, is also insensitive to small changes in the energy of the slow incoming neutron. Therefore, as will be seen from equation (3.3.5) and the subsequent discussion, the reaction cross-section can follow a $1/v$ law up to quite high neutron energies. In the case of the $B^{10}(n, \alpha)$ reaction, the cross-section is large and easily measurable, and the $1/v$ law is accurately obeyed up to energies of the order of 10^4 eV; consequently boron is widely used as a standard with which other cross-sections which are less easy to determine absolutely may be compared. In addition, the ease with which the emission of the alpha particle can be recorded makes the reaction particularly suitable for use in standard neutron detectors.

Table 3.6.1. Table of Charged Particle Reactions Induced by Slow Neutrons

Reaction	Q MeV	Isotopic cross-section at 2200 m/sec	Abundance of capturing isotope	Cross-section at 2200 m/sec per average atom
$He^3(n, p)H^3$	0.7637	5400 ± 300	$1-10 \times 10^{-5} \%$	—
$Li^8(n, \alpha)H^3$	4.784	945 ± 100	7.52 %	71.0 ± 1
$B^{10}(n, \alpha)Li^7$	2.791†	4030 ± 110	$\approx 18.8 \%$	760 ± 5
$N^{14}(n, p)C^{14}$	0.626	$1.75 \pm 0.05^*$	99.63 %	1.74 ± 0.05
$Cl^{35}(n, p)S^{35}$	0.62	0.30 ± 0.10	75.4 %	0.23 ± 0.08

* 6 % of neutron captures give $N^{14}(n, \gamma)$ reaction.

† In 94 % of neutron captures a 0.478 MeV photon is emitted, and the energy carried off by the short-range product nuclei is correspondingly reduced to 2.31 MeV.

The reactions with boron, and, to a lesser extent, with lithium, are widely used in shielding owing to their large cross-sections, to the fact that the reaction products are not radioactive, and, what is most important, to the absence of penetrating capture gamma radiation (Table 3.5.1).

Intermediate and Fast Neutrons

As the energy of the incident neutron increases, resonances for the formation of a compound nucleus occur, causing departures from the $1/v$ law. In addition, other reactions which were not energetically possible for slow neutrons ($Q < 0$) begin to be observed. The threshold energy at which a reaction of negative Q becomes possible is greater than $-Q$ by the factor $(A + 1)/A$, owing to part of the neutron energy being "wasted" as kinetic energy of the recoiling compound nucleus. As the neutron energy increases beyond the threshold value, the penetrability factor increases and the reaction cross-section rises accordingly. At still higher energies the cross-section drops once more, as competition from other fast neutron reactions, such as inelastic scattering and $(n, 2n)$ become

important. However, over a limited energy range the cross-sections are sufficiently flat for the reactions resulting in radioactive product nuclei to be important as threshold detectors of fast neutrons. For instance (Table 3.6.2, Fig. 3.6.2) by measuring the beta activity of P^{32} in a sample of sulphur that has been

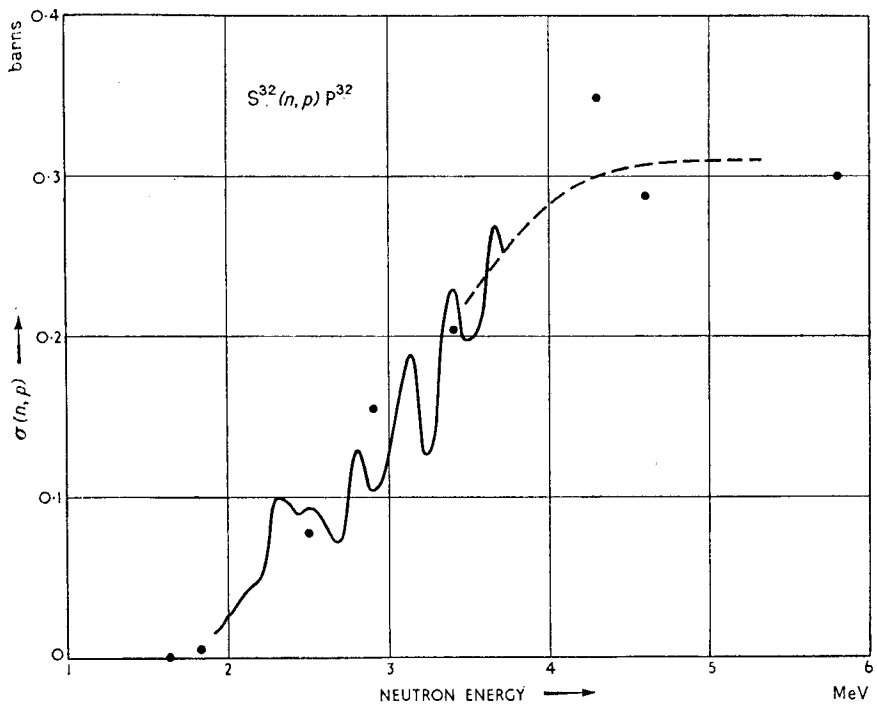


Fig. 3.6.2. Cross-section for the reaction $S^{32}(n, p)P^{32}$. The black circles are due to KLEMA and HANSON.⁽³³⁾ The line is drawn through the results of LÜSCHER *et al.*,⁽⁶⁹⁾ which have better energy resolution and which show some resonance structure.

Table 3.6.2 Table of Threshold Reactions of Use in Fast-neutron Detection

Reaction	Product nucleus half-life	Threshold energy (MeV)	Energy at which penetrability factor = 0.5	Threshold energy + Coulomb energy (MeV)
$P^{31}(n, p)S^{31}$	2.7 h	0.97	3.8	5.3
$S^{32}(n, p)P^{32}$	14.3 d	0.96	4.1	5.6
$Al^{27}(n, p)Mg^{27}$	10 m	1.96	4.5	5.9
$Si^{28}(n, p)Al^{28}$	2.3 m	2.7	5.4	6.9
$Fe^{56}(n, p)Mn^{56}$	2.6 h	3.0	7.6	9.4
$P^{31}(n, \alpha)Al^{28}$	2.3 m	0.91	8.3	9.8
$Al^{27}(n, \alpha)Na^{24}$	14.9 h	2.44	9.1	10.9

From FELD.⁽²⁾

irradiated in a fast neutron flux, the flux of neutrons of energies greater than about 2 MeV can be approximately determined. If several different detectors with widely different thresholds are used, it is possible to measure approximately the spectrum of fast neutrons existing, for instance, in a reactor shield. Although such methods give results that are necessarily imprecise, they are widely used on account of their simplicity.

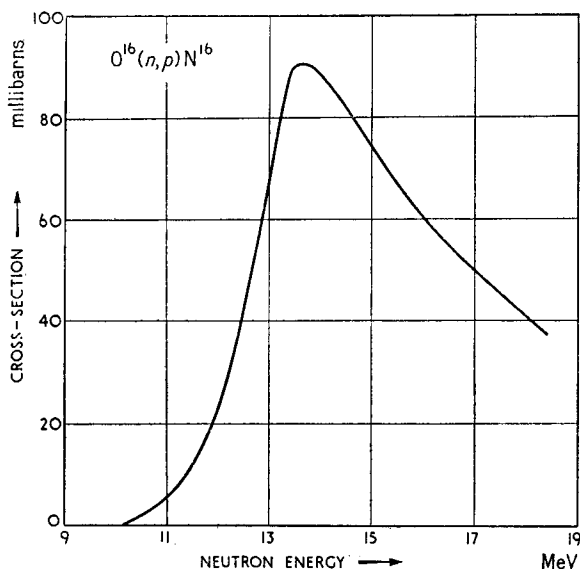
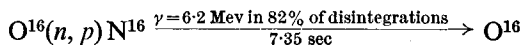


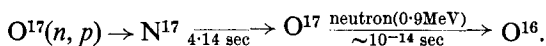
Fig. 3.6.3. Cross-section for the $O^{16}(n,p)N^{16}$ reaction as a function of neutron energy (MARTIN⁽⁸⁴⁾).

A reaction of considerable practical importance in water-cooled reactors is the formation of N^{16} by the reaction $O^{16}(n,p)N^{16}$. The product nucleus is short-lived, and emits very penetrating gamma radiation:



The threshold for this reaction is 10 MeV, which is below the maximum energy of neutrons formed in fission (Fig. 3.6.3). Consequently, the cross-section for this reaction in an undegraded fission spectrum is finite. The experimentally determined value averaged over the fission spectrum is $16 \mu\text{barns}$,⁽³⁵⁾ which corresponds to an effective cross-section of 46 millibarns for fission neutrons above 11.6 MeV. This value is large enough to give rise to a shielding problem in the out-of-pile portions of the primary coolant circuit.

A further (n,p) reaction which has to be taken into account in the design of water-cooled reactors is the formation of N^{17} from O^{17} , which is present in normal oxygen to the extent of 0.039%. The N^{17} decays within a few seconds to an excited state of O^{17} , which then emits a neutron:



The overall effect is therefore similar to the formation of a delayed neutron emitter. The value reported⁽³⁵⁾ for the $O^{17}(n, p)N^{17}$ reaction cross-section is 2.1 millibarns per O^{17} atom for fission neutrons above 10.0 MeV.

3.7. THE (*n*, 2*n*) REACTION

If the bombarding neutron has sufficient energy it is possible for the compound nucleus to emit two, or possibly more, neutrons. The threshold energy at which the (*n*, 2*n*) reaction becomes possible is just $(A + 1)/A$ times the binding energy of a neutron already in the target nucleus—the factor $(A + 1)/A$ being necessary to allow for the recoil energy given to the compound nucleus.

The (*n*, 2*n*) reaction competes with other possible modes of decay of the compound nucleus. For light elements these are principally elastic scattering and emission of charged particles, and for heavy elements inelastic scattering; at still higher energies there is also competition from more complex disintegrations, such as (*n*, 3*n*), (*n*, 2*n* + *p*), and so on.

Suppose that (*n*, 2*n*) and inelastic scattering are the only reactions which need be considered. After emission of the first neutron the daughter nucleus is left in an excited state, which may decay by emitting another neutron, or by gamma radiation. From our previous discussion of neutron and radiation widths we see that, if the excitation is at least a few tens of kilovolts above the neutron binding energy, the second neutron will almost always be emitted, giving an (*n*, 2*n*) reaction. But if the first neutron carries off too much of the available energy the residual excitation is insufficient for emission of a second neutron, so we have the case of inelastic scattering instead. Thus the (*n*, 2*n*) cross-section

$$\sigma(n, 2n) \approx \sigma_{\text{cpd}}(n) \times P, \quad (3.7.1)$$

where $\sigma_{\text{cpd}}(n)$ is the cross-section for the formation of the initial compound nucleus, given roughly by equation (3.3.7), and *P* is the probability that the first neutron leaves the daughter nucleus excited above the neutron binding energy.

By integrating (3.4.3) with respect to E_n , from 0 to δ , one finds that

$$P \approx \left\{ 1 - \left(1 + \frac{\delta}{\theta} \right) \exp \left(-\frac{\delta}{\theta} \right) \right\}, \quad (\lambda \ll R) \quad (3.7.2)$$

where $\delta = E_{n0} - E_{\text{threshold}}$ is the surplus energy of the incoming neutron over the threshold of the (*n*, 2*n*) reaction, and θ is the nuclear temperature of the intermediate nucleus after the first neutron has been emitted. The temperature is related to the approximately Maxwellian energy distribution of the neutrons emitted in the first step, and to the level spacing in the excited intermediate nucleus:

$$\theta \approx (\epsilon^*/a)^{1/2}$$

where ϵ^* is the mean excitation energy, and *a* is given in Table 3.3.2.

3.8. ARTIFICIAL RADIOACTIVITY RESULTING FROM NEUTRON REACTIONS

The majority of materials, when exposed to the intense fluxes of neutrons in a nuclear reactor, become extremely radioactive, to such an extent that the withdrawal of most materials from a reactor can be performed safely only when they

are shielded. Some of the elaborate techniques for remote handling which have had to be developed will be mentioned later in the book (Chapter 6); but here we are concerned with the rather more physical aspects of the matter, and particularly with the different types of artificial radioactivity that are commonly encountered.

Stable and Unstable Nuclei

As we have already mentioned, the stability of a given nucleus is intimately connected with the ratio R of the number of protons Z and the number of neutrons $A-Z$ which it contains. For all naturally occurring isotopes R varies within a comparatively small range. For most light elements the mean value $\bar{R}$, averaged for all the stable isotopes in a limited region of the periodic table, is close to unity. For heavier elements $\bar{R}$ falls steadily as Z increases, finally reaching a value of ≈ 0.64 for the heaviest elements. Each element has only a few isotopes, in some cases only one, which are stable. If the value of R is changed, by (for instance) subjecting the nucleus to an (n, γ) or an $(n, 2n)$ reaction, the product nucleus may be stable or radioactive. In the latter case it reaches stability by decaying in a way which is characteristic of the particular isotope. In general, neither the decay scheme for a given radioactive isotope nor its exact half-life can be predicted theoretically. However, a very large number of decay schemes have been investigated experimentally, and standard compilations of nuclear data are available⁽⁷⁴⁾ showing which isotopes are unstable, how they decay, and what are their half-lives and formation cross-sections; this information is given in abbreviated form, in Appendix III. The use of this information in reactor engineering is described in Chapter 6.

Decay by Negative Electron Emission (beta decay)

Radioactive nuclei formed by the addition of neutrons usually reach stability by beta decay, which may be regarded as the transformation of the excess neutron within the nucleus to a proton and a beta particle, or electron:

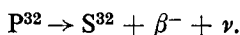


The symbol ν represents the neutrino, which, as we have already explained (Section 2.11) must also be emitted if energy and angular momentum are to be conserved in the process. In comparison with some other nuclear reactions (which may be complete within times as short as 10^{-14} sec), beta decay is a slow process, beta lifetimes being measured in minutes or even years.

In the simplest type of beta decay the electron and neutrino carry away all the energy of the reaction Q . This energy, which in this case is also equal to the maximum energy of the electrons, is determined in the usual way by the mass differences of the parent and product nuclei. If atomic mass units (a.m.u.) are used, Q is given in MeV by

$$Q = 931(M_{\text{parent}} - M_{\text{product}}), \quad (3.8.1)$$

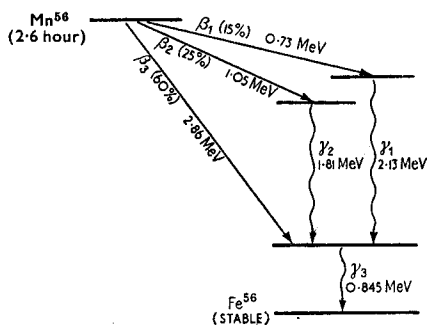
the masses of the orbital electrons cancelling out. Consider the pure beta decay



The P^{32} atom is heavier than the S^{32} atom by 0.00185 a.m.u., and the electron therefore has a maximum energy of

$$931 \times 0.00185 = 1.72 \text{ MeV.}$$

Pure beta decay is, however, not the only possibility. A beta decay process can also leave the product nucleus in an excited state, which within an immeasurably short time then decays to the ground state with the emission of one or more photons. These gamma rays may carry off a substantial fraction of the decay energy Q , and the beta particle is then correspondingly less energetic. It may also be possible for the reaction to proceed in any one of several ways, each of which leaves the product nucleus in a particular excited state. Probabilities can then be assigned for each mode of decay. Thus manganese-56 has the following rather complex decay scheme.*



From the point of view of shielding, the gamma rays emitted by the excited Fe^{56} are far more important than the beta particles which they follow.

Internal Conversion

The transition of a nucleus from an excited state to a lower state of the same nucleus is not necessarily associated with gamma-ray emission. In some cases the transition energy E_γ is absorbed in the ejection of orbital electrons from the atom, a process known as *internal conversion*. The electrons, which are known as *internal conversion electrons*, are emitted with energies $E_\gamma - E_K$, $E_\gamma - E_L \dots$, where E_K , E_L are the binding energies of the electrons in the K , $L \dots$ shells. The process is superficially analogous to the photoelectric effect. The degree of conversion is described by the *conversion coefficient*, which is defined in one of two alternative ways. According to the more usual convention, it means the ratio of the number of conversion electrons leaving the atom to the number of photons leaving the atom; defined in this way the coefficient can vary from zero to infinity. In the older literature, it is defined as the number of conversion electrons leaving the atom to the number of

* This is the conventional method of writing decay schemes. Excitation energy increases upwards, and atomic number from left to right. Transitions involving the emission of beta particles are therefore denoted by arrows sloping downwards and to the right; transitions involving the emission of positrons by arrows sloping downwards and to the left; and gamma transitions by vertical arrows.

disintegrating nuclei, and can then vary from zero to one. There is a general tendency for large conversion coefficients to be associated with the lowest energy and least penetrating gamma radiation, and vice versa; so that from the point of view of shielding the effect is of small importance.

Positron Emission

Some nuclear reactions, e.g. $(n, 2n)$, produce daughter nuclei with an excess of protons (or a deficit of neutrons). Such nuclei, if unstable, may reach stability by the emission of a positron, which is the consequence of the transformation of one of the protons of the nucleus into a neutron:



This reaction is endothermic, and cannot proceed unless energy can be absorbed from some other particle. When the proton is not isolated, but is contained in a nucleus, this energy deficit can be supplied from the difference in masses of the parent and product nuclei. The total energy available for the transition is determined by an equation which is slightly different from (3.8.1),

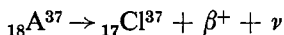
$$\begin{aligned} Q &= 931(M_{\text{parent}} - Zm_{e-}) - (M_{\text{product}} - (Z - 1)m_{e-} - m_{e+}) \\ &= 931(M_{\text{parent}} - M_{\text{product}} - 2m_{e-}), \end{aligned} \quad (3.8.2)$$

the masses of both electron and positron being equal. This equation shows that, unless the mass of the parent atom exceeds the mass of the daughter atom by more than $1862m_e = 1.02 \text{ MeV}$, positron emission cannot take place.

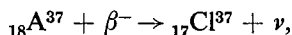
Orbital Electron Capture

There is, however, an alternative process by which a nucleus can reduce its proton/neutron ratio, even when positron emission is forbidden on energy grounds. This is the process of *orbital electron capture*, in which (neglecting the orbital energy of the captured electron) an extra 1.02 MeV are available.

For instance, the reaction



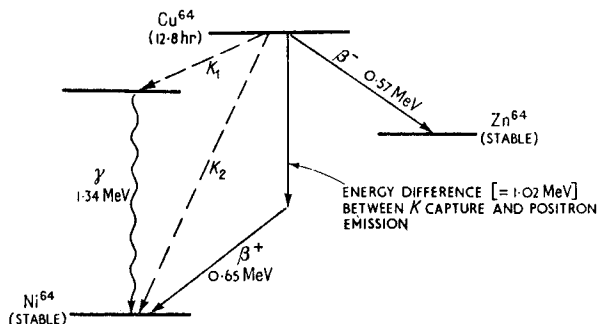
does not occur, because the masses of the products on the right-hand side of the equation exceed the mass of the A^{37} nucleus by 0.0003 a.m.u. , equivalent to an energy deficit of 0.3 MeV . The alternative is for the argon to capture an orbital electron,



a reaction which liberates 0.7 MeV , and which is therefore energetically possible. The process of orbital electron capture usually involves a K electron, and for this reason the process is sometimes loosely described as K capture. It leaves a gap in the orbital electron structure, which is immediately filled by another electron. This, it will be remembered, is the mechanism by which X-rays are produced. Thus orbital electron capture is always accompanied by the emission of X-rays; but since their energy is comparatively low, even for heavy elements (see Fig. 2.6.4) it is unnecessary to take them into account in shielding calculations.

When both positron emission and electron capture are energetically possible, it is common for both processes to occur in competition with each other. It is

also possible for both to exist side by side with the negative electron decay of the same nuclide. Thus Cu^{64} decays in all three ways:



It must be remembered, when calculating the shielding required for any positron emitter, that the eventual annihilation of the positron liberates two 0.51 MeV photons (Section 2.4); a reminder to this effect is not always included in compilations of the properties of radioactive isotopes.

Isomeric Transitions

In a relatively small number of nuclear reactions the daughter product is left in a metastable excited state, which instead of decaying instantaneously to the ground state, persists for a period which may be measured in hours; we have already mentioned the examples of metastable indium-115 and gold-197 formed by inelastic neutron scattering. When a metastable state decays by gamma emission to the ground state the transition involves no change of mass number or atomic number. A transition of this kind is described as an isomeric transition (usually abbreviated I.T. in tables of nuclear data).

Decay by Neutron Emission

A few examples are known in which beta decay is followed by neutron emission. The most important are the decay of N^{17} (Section 3.6), and the delayed neutron emitters formed in fission (Section 3.9).

Activation Cross-sections.

In general both the type of activity induced by neutron absorption, and the cross-section for inducing the activity (usually referred to as the *activation cross-section*) are different for the different isotopes of a given element. Since it is very unusual for separated isotopes to be used, tables of activation cross-sections (e.g. Appendix III) normally quote the activation cross-section per average atom, a quantity which is the product of the appropriate isotopic cross-section and the relative abundance of the isotope which gives rise to the particular activity. For elements with only one isotope (e.g. Na), and when the neutron energy is low, the reaction following neutron absorption can usually take only one course, and the activation cross-section is then equal to the absorption cross-section.

SOURCES OF NEUTRONS

3.9. FISSION NEUTRONS

The neutrons resulting from the fission process may be classified as either prompt or delayed. More than 99% of the neutrons are prompt; that is, they are emitted within a very short time of the initiation of the fission process. This time has been shown experimentally^(36, 37) to be less than 10^{-8} sec and is believed on theoretical grounds to be of the order of 10^{-15} sec. The energy-range covered by the prompt neutrons is very wide, extending from a few tens of kilovolts to about 18 MeV.⁽³⁸⁻⁴¹⁾ The differential energy-spectrum is well

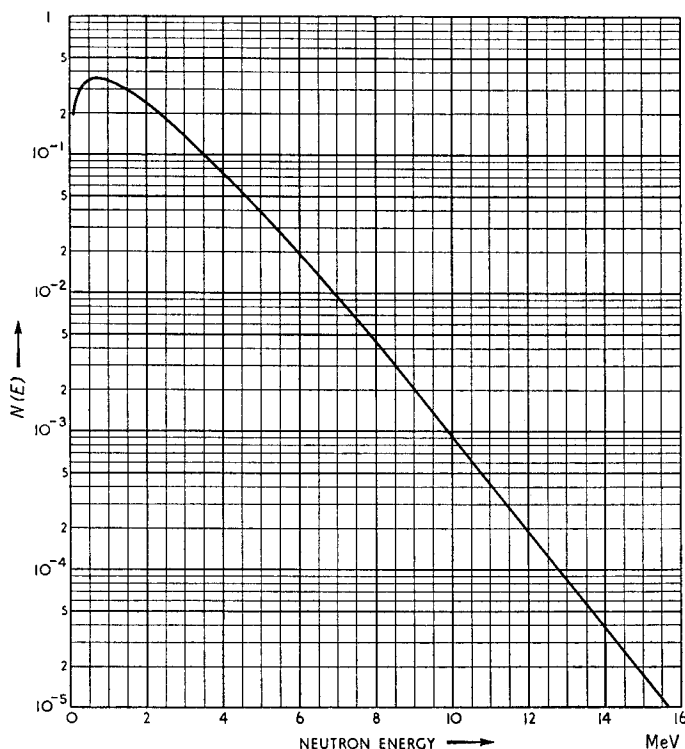


Fig. 3.9.1. Energy distribution of neutrons emitted during the fission of U^{235}

represented, for the case of slow-neutron-induced fission of U^{235} , by a semi-empirical formula originally derived from a suggestion by FEATHER,⁽⁴²⁾ and later improved by WATT,⁽⁴³⁾

$$N(E_n) dE_n = \sqrt{2/\pi e} (\sinh(2E_n)^{1/2}) \cdot e^{-E_n} dE_n, \quad (3.9.1)$$

where E_n is the neutron energy in MeV (see Fig. 3.9.1). This formula is normalized to unit neutron emission, the constant $\sqrt{2/\pi e}$ being numerically equal

to 0.484. Over the energy range 4 to 12 MeV the spectrum is well fitted by the approximate expression

$$N(E_n) dE_n \approx 1.4 e^{-0.72E_n} \quad (3.9.1a)$$

The energy spectrum of the neutrons from thermal fission of Pu^{239} is similar.^(38, 44)

In order to be able to calculate the total number of prompt neutrons of a given energy emitted by a nuclear reactor per unit time, two further factors are needed. The first, the number of fissions per unit time, is obtained directly from the known heat output of the reactor, and the energy released per fission (~ 200 MeV), using the conversion factors given in Appendix I. The second is the average number of prompt neutrons emitted per fission. This is very nearly equal to the total number of neutrons per fission, a quantity which is conventionally denoted by the symbol ν , and which is given for the four most important thermally fissile nuclides in Table 3.9.1.

Table 3.9.1. Number of Prompt and Delayed Neutrons per Fission induced by Thermal Neutrons⁽⁴⁵⁾

Nuclide	ν	β
U^{233}	2.52 ± 0.06	0.0024
U^{235}	2.48 ± 0.05	0.0073
Pu^{239}	2.92 ± 0.06	0.0027
Pu^{241}	$3.10 \pm 0.1^*$	—

* Prompt neutrons only were measured for Pu^{241} .

The *delayed* neutrons, which form a small fraction ($\beta = \nu_{\text{delayed}} \div \nu_{\text{total}}$) of the total neutron emission per fission, are associated with the beta decay of certain fission products in which the daughter nucleus may be left with an excitation greater than the binding energy of its last neutron. At least six delayed neutron periods have been identified.⁽⁴⁶⁻⁴⁸⁾ Within the experimental errors, the same half-lives have been reported for three of these in thermal fission of U^{233} , U^{235} , and Pu^{239} and fast fission of Th^{232} , U^{233} , U^{235} , U^{238} , and Pu^{239} . Three other periods show apparently different half-lives when observed in thermal and fast fission. In addition to these six periods, three very-low-intensity periods of long half-life have been reported⁽⁴⁹⁾ in U^{235} . Some experimental results on delayed neutrons from thermal fission are given in Table 3.9.2 together with estimates of the mean energies of the different groups and the chemical identification of the beta-emitting fission products, where this is known.

Although delayed neutrons are of fundamental importance to reactor control, their low yield and relatively small kinetic energy mean that they can be safely ignored in very nearly all shielding problems. The only important exceptions are reactors with circulating fuel, such as aqueous homogeneous reactors.

Table 3.9.2. Delayed Neutrons from Thermal Fission

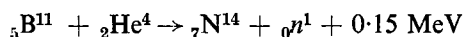
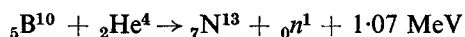
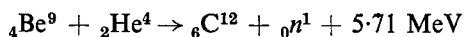
Group index	Chemical identification	Approximate mean energy keV	U^{233}		U^{235}		Pu^{239}	
			Half-life sec	Relative yield $= a_i/a$	Half-life sec	Relative yield $= a_i/a$	Half-life sec	Relative yield $= a_i/a$
1	Bf^{87}	250 ± 60	55.6 ± 0.2	0.075 ± 0.002	55.6 ± 0.2	0.034 ± 0.009	55.0 ± 0.27	0.030 ± 0.001
2	I^{137}	560 ± 60	22.0 ± 0.2	0.238 ± 0.019	22.0 ± 0.2	0.220 ± 0.023	22.4 ± 0.33	0.294 ± 0.007
3	$Bf^{89}?$	430 ± 60	4.51 ± 0.1	0.350 ± 0.019	4.51 ± 0.1	0.282 ± 0.017	4.5 ± 0.13	0.322 ± 0.009
4	—	620 ± 60	1.52 ± 0.05	0.254 ± 0.019	1.52 ± 0.05	0.319 ± 0.017	1.05 ± 0.04	0.354 ± 0.021
5	—	420 ± 60	0.43 ± 0.05	0.083 ± 0.024	0.43 ± 0.05	0.112 ± 0.011	—	—
Total delayed neutrons				0.0024		0.0075		0.0027
Total fission neutrons								
References	50	51	52		46, 47		47, 53	

 $\Sigma a_i \equiv a$ = total delayed neutrons per fission.

In the absence of any delayed neutron sources in the fuel the shielding for the out-of-pile fuel circuit would be determined purely by the gamma activity of the fission products. The presence of delayed fission neutrons (and also of other neutron sources such as the O^{17*} activity mentioned in Section 3.6), makes it necessary to include some neutron absorbing material in the shielding. A hydrogenous material is necessary, since the energy of the delayed neutrons is too low for inelastic scattering by heavy elements to be effective.

3.10. (α, n) SOURCES

The neutron sources commonly employed for calibrating laboratory equipment make use of the exoergic reactions between beryllium or boron and alpha particles:⁽⁷⁰⁾



[Main source of neutrons from boron.]

The existence of these reactions enables neutron sources to be made simply by mixing a naturally occurring alpha emitter (such as radium or polonium) with boron or beryllium powder. The intensity that is obtainable depends on a number of factors, the most important of which is the number of alpha particles emitted per second by the radioactive substance; consequently, one uses a material of short half-life, such as polonium ($t_{1/2} = 138.2$ days), when maximum intensity is required. However, the source is then less suitable as a standard for calibration of laboratory apparatus than when a material of longer life is used. According to ANDERSON⁽⁵⁴⁾ the intensity of a radium-beryllium source (using radium bromide) is given approximately by the expression

$$\text{Yield} = \left\{ 1.7 \times 10^7 \frac{\text{Mass Be}}{\text{Mass Be} + \text{Mass RaBr}_2} \right\} n \text{ sec}^{-1} \text{ g}^{-1} \text{ of radium.}$$

The intensity of radium-boron sources is somewhat lower:

$$\text{Yield} = \left\{ 6.8 \times 10^6 \frac{\text{Mass boron}}{\text{Mass boron} + \text{Mass RaBr}_2} \right\} n \text{ sec}^{-1} \text{ g}^{-1}.$$

The fraction in these expressions allows for the effect of competition between the wanted (α, n) process and the unwanted ionization process, by which the alpha particle also loses energy. The dependence of the source intensity on the degree of mixing which these expressions imply is a disadvantage when the source is intended for use as an absolute standard, since one can never be sure that shaking has not changed the source strength. It is preferable to use a source of definite chemical composition, such as the intermetallic compound PuBe_{13} formed between plutonium and beryllium, which incidentally has the further merit of being much safer to handle than most other plutonium compounds.

The energy spectrum of the neutrons emitted from an alpha-beryllium source is a continuous distribution extending from very low energies up to a maximum energy which is slightly less than the sum of the α -particle energy and the Q of the reaction (Fig. 3.10.2). The distribution shows several peaks, which are associated with events in which the C^{12} product nuclei are left either in the ground state or in one of the excited states at 4.5 MeV and 7.5 MeV. Although the energy retained by each C^{12} nucleus is well defined, the neutrons emitted in the peaks are not monoenergetic. The spread in energy arises from several causes, the two most important being the dynamics of the collision between the alpha particle and the Be^9 nucleus (i.e. the amount of kinetic energy taken up by the recoiling product nucleus), and the fact that the alpha particles have a variable

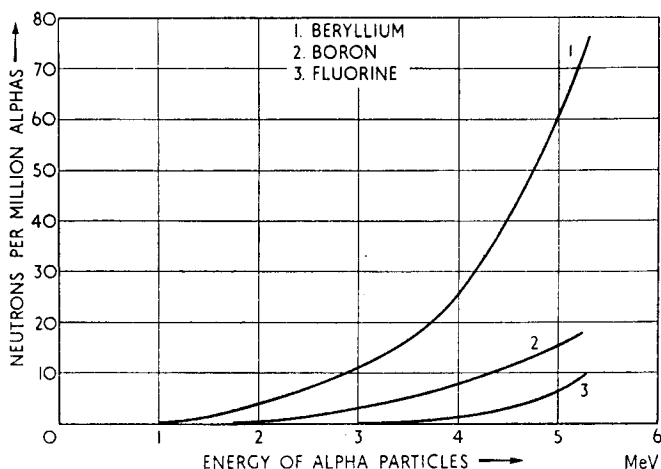


Fig. 3.10.1. Neutron yields for alpha particles falling on thick targets of beryllium, boron, and fluorine. (SEGRÉ and WIEGAND⁽⁵⁴⁾.)

energy, depending on how much they have been slowed down by ionization before the collision with the beryllium nucleus takes place. Measured energy distributions from sources of Po-Be, Ra-Be, and Pu-Be are shown in Fig. 3.10.2. In each case the area under the curve has been normalized to unity. It will be seen that the distribution is peaked at a considerably higher energy than the fission spectrum, though the upper energy limit is lower. In the case of radium and beryllium, some of the neutrons are attributable to (γ, n) reactions (see Section 3.11); sources using polonium and plutonium emit very much smaller amounts of gamma radiation, which for many experimental applications is a distinct advantage.^(10, 71, 72)

A note on the shielding of small laboratory sources of neutrons will be found in Section 6.13.

3.11. (γ, n) SOURCES

A neutron can be liberated by a (γ, n) interaction between a nucleus and a photon whose energy is greater than the binding energy of a neutron in the

nucleus. It will be seen from Table 3.3.1 that for practically all elements the gamma-ray energy required is ~ 6 MeV or more. The only exceptions are deuterium and beryllium, both of which have abnormally low (γ, n) thresholds—2.227 MeV for deuterium and 1.665 MeV for beryllium. These two values are below the photon energies emitted by several easily-produced radioactive nuclei (e.g. Na^{24}), and consequently a second type of laboratory neutron source can be made either by mixing these materials with an emitter of energetic gamma rays

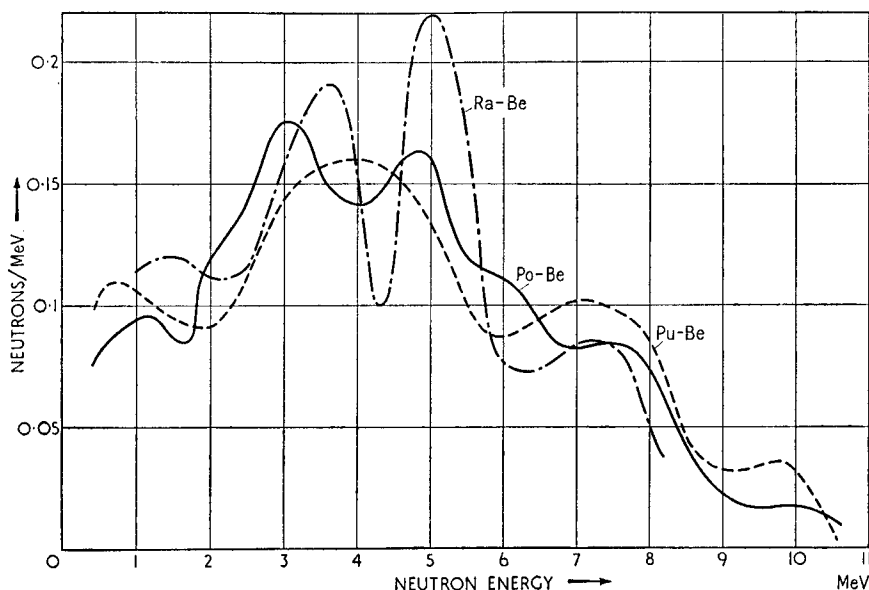


Fig. 3.10.2. Energy distribution of neutrons emitted from sources of radium-beryllium,⁽⁵⁵⁾ polonium-beryllium,⁽⁵⁶⁾ and plutonium-beryllium.⁽⁵⁷⁾

as in the commonly-used antimony-beryllium source, or by placing the gamma-ray source inside a vessel containing heavy water or beryllium. The latter method is more convenient when gamma-emitters of short half-life are to be used (see Table 3.11.1).

The energy of the neutrons produced in such a *photoneutron* source depends on the binding energy S_n MeV of the neutron, on the gamma-ray energy E_γ MeV, and on two other factors which affect the amount of energy taken away by the recoiling product nucleus—the mass A of the target atom, and the angle θ between the direction of emission of the neutron and the direction of the original gamma ray:⁽²⁾

$$E_n = \left[\frac{A-1}{A} \right] \left[E_\gamma - S_n - \frac{E_\gamma^2}{1862(A-1)} \right] + \Delta \cos \theta,$$

where

$$\Delta \cong E_\gamma \left[\frac{2(A-1)(E_\gamma - S_n)}{931A^3} \right]^{\frac{1}{2}}.$$

The inherent spread in energy ($= 2\Delta$) is a small fraction of E_n (except for very light elements and close to the threshold energy); consequently the neutrons produced by the (γ, n) process can for many practical purposes be regarded as monoenergetic, though it is necessary to remember that in thick sources a further energy spread can be caused by the normal moderating action of deuterium and beryllium.

Table 3.11.1. Photoneutron Yields from Sources in which the Gamma Emitter is separated from the D_2O or $Be^{(2)}$

Source	E_γ (MeV)	Energy of emitted neutrons (MeV)	Half-life	Yield from 1 gram of Be or D_2O at 1 cm from 1 curie source (neutrons per sec)
$Na^{24} + Be$	2.76	0.83	14.8 h	130,000
$Na^{24} + D_2O$	2.76	0.22	14.8 h	270,000
Ra + Be	Many	Wide spread	1620 yr	30,000
Ra + D_2O	2.42	0.1	1620 yr	1,000
Sb + Be	1.7	0.024	60 days	190,000

Antimony-beryllium sources are frequently prepared from intimate mixtures of equal volumes of the two constituents. The neutron output of a typical source containing 28 g of antimony and 8 g of beryllium, compressed into a cylinder 0.8 inches in diameter and $1\frac{1}{4}$ inches long, is about 1×10^6 neutrons per second, when the mixture contains one curie of radioantimony. For a given activity per unit mass of mixture, the neutron output of such a source is roughly proportional to the square of the mass employed.

(γ, n) Cross-Section at High Energy

Energetic neutrons can be conveniently produced by utilizing the energetic bremsstrahlung produced at the target of a high-energy electron accelerator. It is found that the cross-section for neutron production by high-energy gamma rays exhibits a well-marked systematic dependence on gamma energy and atomic weight for all except the lightest nuclei. As the gamma energy increases, the cross-section rises from zero at the threshold to a maximum value at a gamma energy of ~ 20 MeV, and then falls again. The peak cross-section increases as the atomic weight of the target increases, roughly as $A^{3/2}$ (variations proportional to $A^{3/2}$ and $A^{5/3}$ have both been reported). The peak energy decreases as A increases, and is roughly given by $38.4A^{-0.186}$ MeV.⁽⁶¹⁾ The physical explanation of this resonance-like behaviour of the cross-section (Fig. 3.11.3) in this energy range is that the electric field of the photon causes the protons in the nucleus to oscillate as a group with respect to the neutrons, and the oscillations build up when the frequency of the gamma radiation corresponds to the period of oscillation of the protons.

In principle the yield of neutrons from a target on which energetic electrons are incident can be computed by first working out the bremsstrahlung spectrum (Section 2.12), and then using the cross-section curves given below. This,

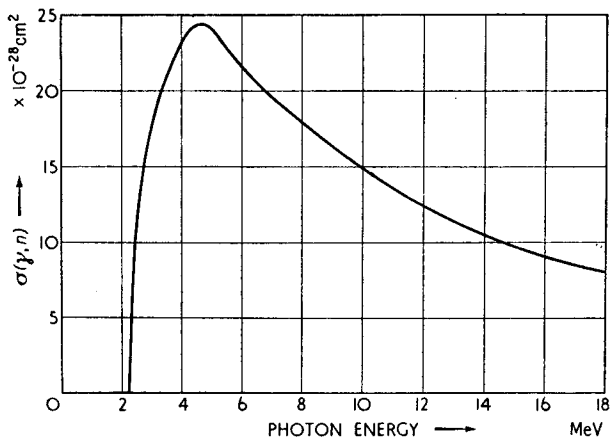


Fig. 3.11.1. Cross-section for the reaction $\text{D}(\gamma, n)$ as a function of gamma-ray energy.^(58,59,68)

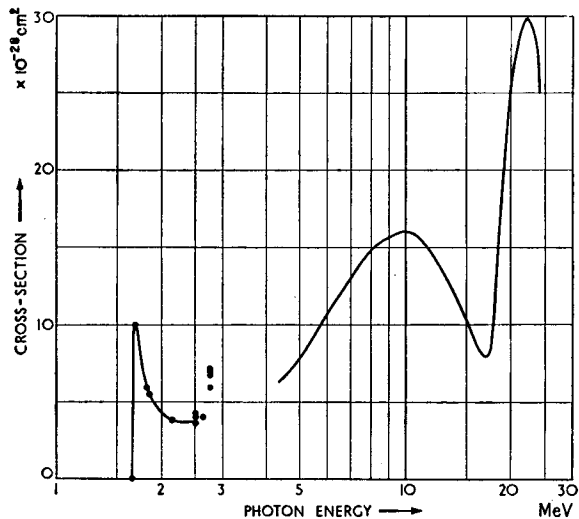


Fig. 3.11.2. Cross-section for the reaction $\text{Be}(\gamma, n)$ as a function of gamma-ray energy.^(60,61)

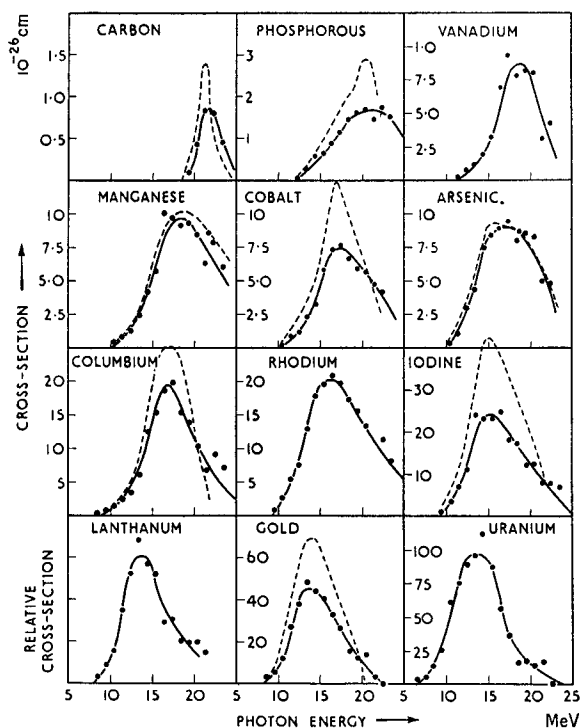


Fig. 3.11.3. Cross-sections of a number of elements for photoneutron production by high-energy gamma radiation (from NATHANS and HALPERN⁽⁶¹⁾ and MONTALBETTI *et al.*⁽⁶²⁾ (dashed lines)).

however, is an extremely tedious process, and it is more convenient to make use of direct measurements such as those of PRICE and KERST⁽⁶³⁾ shown in Fig. 3.11.4. The curves at high energies⁽⁶⁶⁾ are not very different in shape from those shown in this figure: the majority of the neutron production is still due to photons with energies in the region of 20 MeV, owing to the rapid reduction in the cross-section above the resonance energy.

(γ , n) Processes in Reactor Shields

It will be seen from the curves already given that the cross-section for photoneutron production is two or three orders of magnitude less than the cross-sections for the other processes by which gamma rays are attenuated (Fig. 2.5.3). Consequently, when mixed sources of photons and neutrons are being shielded, as in a reactor, the pattern of neutron attenuation will be modified by photoneutron production only if the gamma intensity is very much greater than the neutron intensity. In concrete shields the effect of photoneutron production is usually trivial, because the energetic photons and fast neutrons start with very roughly similar intensities at the inner face of the shield, and both kinds of radiation are (as we shall see later) attenuated at approximately the same

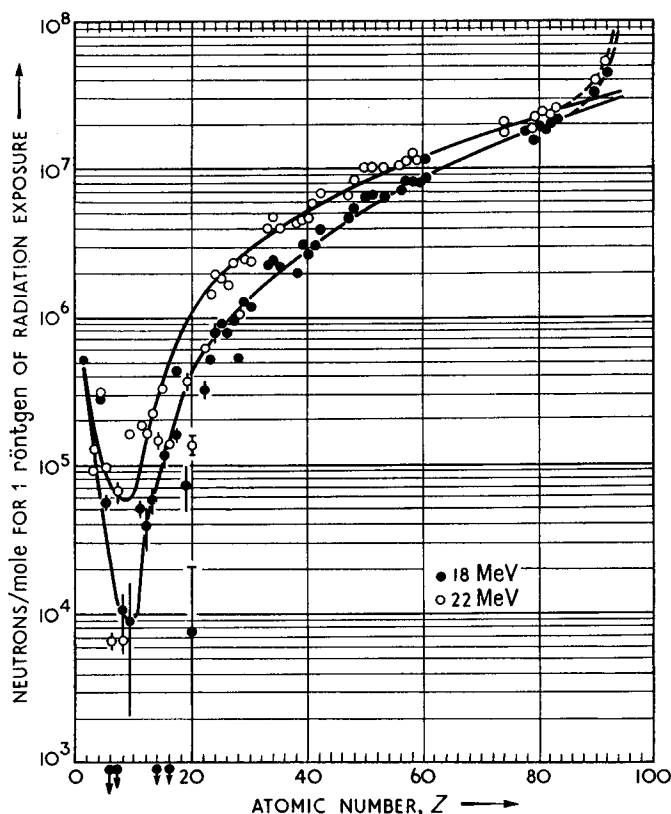


Fig. 3.11.4. Neutron yields in neutrons per mole for 1 röntgen of primary gamma-ray beam exposure, measured in a thick-walled Victoreen ionization chamber.

The neutron yields are plotted as a function of the atomic number of the target material (PRICE and KERST⁽⁶³⁾). Similar results have been obtained by MONTALBETTI *et al.*⁽⁶³⁾ Values for 50-MeV gamma rays have been reported by BALDWIN and ELDER⁽⁶⁴⁾ (reproduced from ref. 65).

rate. However, when the entire shield consists of water, which attenuates neutrons far more rapidly than gamma radiation, photoneutron production in the small proportion of deuterium (0.016%) would cause the neutron intensity at great depths to be governed principally by the gamma intensity (Fig. 6.12.2).

Besides the neutron sources described above, others which require the production of beams of accelerated charged particles (protons, deuterons, etc.) are also commonly used in nuclear physics laboratories. Their characteristics are described in a review article by FELD.⁽²⁾

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CHAPTER FOUR

NEUTRON ATTENUATION IN THICK SHIELDS

4.1. INTRODUCTION

THE outline of neutron physics given in Chapter 3 was presented from the point of view of the life-history of an individual neutron. In practice, however, we are not concerned with individual histories, but with the statistical behaviour of a very large population. The analytical methods which describe the statistical features of the passage of neutrons through matter are conveniently described by the general term *transport theory*. If we examine the methods which have been developed for calculating the penetration of neutrons through matter we find that, with the possible exception of the "moment" method,^(1, 2, 3) which we shall later mention in Section 4.10, almost all are severely handicapped in practice because of the oversimplifying assumptions which are made. Thus, in spite of much elegant and instructive work by various authors⁽⁴⁻⁶⁾ their methods have not been generally applied to shielding calculations. Nevertheless, as a result of this theoretical work, the physical background of the mechanics of neutron penetration is now well understood, and forms the basis of the less exact but more useful semi-empirical theories which will later be described. The reader should consult the references given above for further information on these analytical theories; but some discussion will be given here of two of the simpler methods which do have direct application, namely, diffusion theory and "age" theory. In the sections which follow we shall not attempt to describe these methods in any detail, since excellent accounts are readily available;^(7, 8) but we shall place particular emphasis on their conditions of applicability, and on their use as an aid to the understanding of more practical methods. We shall confine our attention to neutron energies below about 15 MeV, this being roughly the upper energy limit for fission neutrons.

Definitions and Notation

A physical definition of the term flux has already been given (pp. 20 and 104); but before proceeding further it is helpful to express this quantity in more formal mathematical terms, and also to introduce the related quantities current and neutron density. The definitions and notation which follow appear to be the most commonly accepted versions. Vector quantities are indicated by heavy type. Initially we shall assume that all neutrons have the same speed v and will reserve until later the case of variable v .

The probable number of neutrons in a volume element dV at the point $\mathbf{r}$ which are moving inside the elementary cone $d\Omega$ about a direction defined by the unit vector $\mathbf{\Omega}$ is given by $n(\mathbf{r}, \mathbf{\Omega}) dV d\Omega$; the quantity $n(\mathbf{r}, \mathbf{\Omega})$ is known as

the neutron angular density. The neutron density, $n(\mathbf{r})$, is then given by

$$n(\mathbf{r}) = \int n(\mathbf{r}, \mathbf{\Omega}) d\mathbf{\Omega} \quad (4.1.1)$$

where the integration is taken over all directions. It represents the total number of neutrons per unit volume at the point $\mathbf{r}$, and has the dimensions of $(\text{length})^{-3}$.

The neutron angular flux, $\phi(\mathbf{r}, \mathbf{\Omega})$ is defined by $\phi(\mathbf{r}, \mathbf{\Omega}) = n(\mathbf{r}, \mathbf{\Omega})v$; it represents the number of neutrons crossing unit area normal to $\mathbf{\Omega}$ per unit time and per unit solid angle.

The neutron flux is then

$$\phi(\mathbf{r}) = \int \phi(\mathbf{r}, \mathbf{\Omega}) d\mathbf{\Omega} \quad (4.1.2)$$

This may be interpreted physically as the number of neutrons which cross in unit time a sphere of unit cross-sectional area centred at the point $\mathbf{r}$; it has the dimensions $(\text{length})^{-2} (\text{time})^{-1}$. The product of the neutron flux, and Σ_i , the macroscopic cross-section for some neutron interaction (page 104) gives the number of such events which occur per unit volume per unit time at the point $\mathbf{r}$.

Finally, we define the angular current, $\mathbf{j}(\mathbf{r}, \mathbf{\Omega})$. This is a vector quantity, and is given by

$$\mathbf{j}(\mathbf{r}, \mathbf{\Omega}) = \phi(\mathbf{r}, \mathbf{\Omega})\mathbf{\Omega} = n(\mathbf{r}, \mathbf{\Omega})v\mathbf{\Omega} \quad (4.1.3)$$

It represents the angular distribution of velocity vectors, and has the same scalar magnitude as $\phi(\mathbf{r}, \mathbf{\Omega})$. The integral of $\mathbf{j}(\mathbf{r}, \mathbf{\Omega})$ over all directions gives the net current $\mathbf{j}(\mathbf{r})$:

$$\mathbf{j}(\mathbf{r}) = \int \phi(\mathbf{r}, \mathbf{\Omega})\mathbf{\Omega} d\mathbf{\Omega} \quad (4.1.4)$$

Thus $\mathbf{j}(\mathbf{r})$ is such that $\mathbf{j} \cdot \mathbf{n}$ is the net number of neutrons crossing unit area normal to $\mathbf{n}$ per unit time ($\mathbf{n}$ being a unit vector). If $\mathbf{n}$ is parallel to $\mathbf{j}$, then this number is $|\mathbf{j}|$. In general it is not possible to determine the direction of $\mathbf{j}(\mathbf{r})$ without a knowledge of $\phi(\mathbf{r}, \mathbf{\Omega})$, but in systems in which there is symmetry about an axis through $\mathbf{r}$, $\mathbf{j}(\mathbf{r})$ will always be directed along this axis.

4.2. DIFFUSION THEORY

The first question we shall consider is the behaviour of neutrons which are already in thermal equilibrium with their surroundings. Although this is chronologically the last stage before the neutrons are finally absorbed, it is a convenient point of departure for our consideration of neutron shielding, since the neutrons may be regarded (at least to a first approximation) as diffusing without energy-loss. Thus the definitions given in the previous section apply, provided a suitable average value of v is chosen (see Section 3.2).

A complete description of these neutrons would require a knowledge of $\phi(\mathbf{r}, \mathbf{\Omega})$ for all directions $\mathbf{\Omega}$ at all points throughout the system; such a calculation would in general require the refined methods of transport theory. For shielding applications, however, it is usually enough to know $\phi(\mathbf{r})$ and $\mathbf{j}(\mathbf{r})$; these quantities can be obtained, provided certain conditions are fulfilled, by means of the simplified theory known as *diffusion theory*.

The basic assumption of diffusion theory is that, in a uniform medium,

$$\mathbf{j} = -D \text{grad } \phi, \quad (4.2.1)$$

where D —the *diffusion constant*—is a constant having the dimensions of a length. The relation (4.2.1) is known in other branches of physics as Fick's law. In the case of neutron diffusion it holds exactly only in regions which are not closer to boundaries than two or three mean free paths. In this sense a boundary is any surface where the neutron scattering and absorbing properties undergo a sudden change. Regions containing, or near to, neutron sources also cannot be treated by diffusion theory, unless the sources are uniform and isotropic throughout the whole volume under consideration; however, for our purposes the law holds sufficiently well for extended sources whose strength is a function of position, provided the source strength per unit volume does not change greatly over distances of the order of a mean free path.

In the regions where Fick's law applies the neutron flux satisfies the simple differential equation

$$\nabla^2 \phi(\mathbf{r}) - \frac{\phi(\mathbf{r})}{L^2} = -\frac{S(\mathbf{r})}{D} \quad (4.2.2)$$

where ∇^2 is the Laplacian operator. In Cartesian coordinates

$$\nabla^2 \equiv \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right)$$

and in the spherical coordinates r, θ, ψ

$$\nabla^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left[\sin \theta \frac{\partial}{\partial \theta} \right] + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \psi^2}$$

The quantity L appearing in equation (4.2.2) is another characteristic constant with dimensions of a length, and is known as the *diffusion length*, while the equation itself is the *diffusion equation*. The quantity $S(\mathbf{r})$ is the neutron source density—the number of neutrons produced per unit volume per unit time at the point $\mathbf{r}$ —and is assumed to be a slowly varying function of $\mathbf{r}$. If, for instance, we are interested in the diffusion of thermal neutrons in a shield, then $S(\mathbf{r})$ describes the rate at which thermal neutrons are produced as a result of the slowing down of fast neutrons. (See Section 4.7.)

The quantities D and L are determined by the following properties of the material within which the neutrons diffuse: Σ_s and Σ_c , the macroscopic scattering and capture cross-sections per unit volume (measured in units such as cm^{-1}) and $\bar{\mu}_0$, the average cosine of the scattering angle per collision measured in the *laboratory* system of coordinates. As we have seen in Section 3.4, neutrons of energy less than a few hundred keV are scattered isotropically in the *centre-of-mass* system of coordinates (that is, the system in which the centre-of-mass of the colliding particles is at rest). When this is so, and if the medium consists

of a single element of atomic weight A , $\bar{\mu}_0$ is given by

$$\bar{\mu}_0 = \frac{2}{3A} \quad (4.2.3)$$

For a mixture of elements in which the i th element has an atomic scattering cross-section equal to $\sigma_{i,s}$

$$\bar{\mu}_0 = \frac{\sum_i \left[N_i \sigma_{i,s} \left(\frac{2}{3A_i} \right) \right]}{\sum_i [N_i \sigma_{i,s}]} \quad (4.2.4)$$

where the abundance of the i th type of atom is denoted by N_i and the sum is taken over all the constituents of the mixture. Simple expressions for D and L can be obtained when Σ_c is small compared with the total macroscopic cross-section Σ_t (which equals $(\Sigma_s + \Sigma_c)$). To the first order in Σ_c/Σ_t :

$$\frac{1}{D} = 3\Sigma_t(1 - \bar{\mu}_0) \left(1 - \frac{4\Sigma_c}{5\Sigma_t} + \frac{\Sigma_c}{\Sigma_t} \frac{\bar{\mu}_0}{1 - \bar{\mu}_0} + \dots \right). \quad (4.2.5)$$

To the same approximation

$$\frac{1}{L^2} = 3\Sigma_t \Sigma_c (1 - \bar{\mu}_0) \left[1 - \frac{4\Sigma_c}{5\Sigma_t} + \frac{\Sigma_c}{\Sigma_t} \frac{\bar{\mu}_0}{1 - \bar{\mu}_0} + \dots \right] = \frac{\Sigma_c}{D} \quad (4.2.6)$$

In the crudest approximation, when Σ_c/Σ_t is neglected in comparison with unity,

$$D = l_{tr}/3 \quad (4.2.7)$$

and thus

$$L^2 = (l_{tr} l_c/3) \quad (4.2.8)$$

where l_{tr} is the *transport mean free path* given by $l_{tr} = \frac{1}{\Sigma_s(1 - \bar{\mu}_0)}$ and $l_c = \frac{1}{\Sigma_c}$ is the *capture mean free path*.

Formulae (4.2.7) and (4.2.8) are usually adequate for most applications; but if Σ_c/Σ_t is not small, the asymptotic value of the diffusion length far from any sources is given by the positive real root of the transcendental equation:

$$\frac{L\Sigma_s}{2} \ln \left(\frac{L\Sigma_t + 1}{L\Sigma_t - 1} \right) = \frac{1 + 3L^2\Sigma_s\Sigma_c\bar{\mu}_0}{1 + 3L^2\Sigma_t\Sigma_c\bar{\mu}_0} \quad (4.2.9)$$

which may also be written

$$L\Sigma_s \tanh^{-1} \left(\frac{1}{L\Sigma_t} \right) = \frac{1 + 3L^2\Sigma_s\Sigma_c\bar{\mu}_0}{1 + 3L^2\Sigma_t\Sigma_c\bar{\mu}_0} \quad (4.2.10)$$

When L has been found, D is then obtained from the relationship

$$D = L^2\Sigma_c \quad (4.2.11)$$

Formulae (4.2.9) and (4.2.11) reduce to (4.2.6) and (4.2.5) for small Σ_c/Σ_t . In practice one is rarely concerned with monoenergetic neutrons, and more

usually one needs the average diffusion length for neutrons in a fairly narrow band of energies, such as (for instance) thermal neutrons. As explained on page 106, it is then important to choose the correct method of averaging the cross-sections needed in the above equations.

For isotropic scattering in the laboratory system, which holds for low-energy neutrons in all except the lightest elements, we may write $\bar{\mu}_0 = 0$.

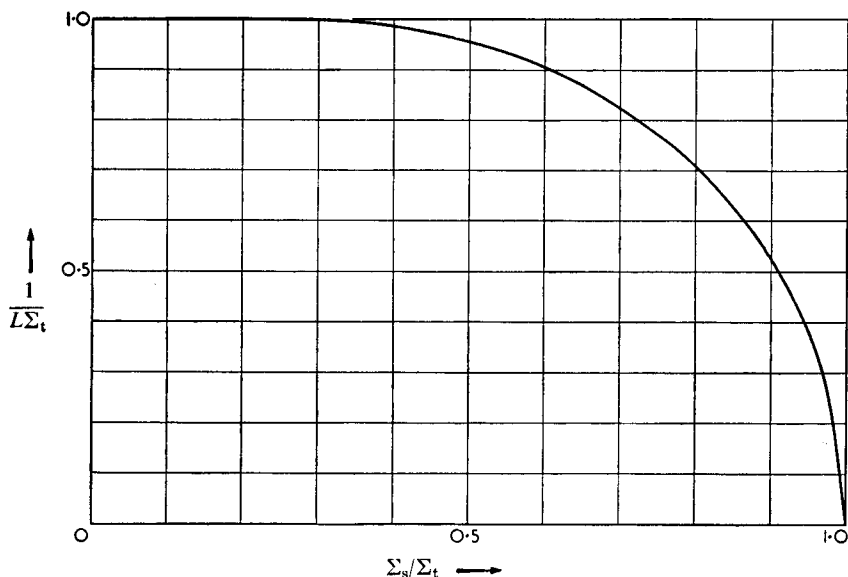


Fig. 4.2.1. The dependence of the diffusion length L on the ratio Σ_s/Σ_t .

By plotting the results of equation (4.2.9) in the form shown in Fig. 4.2.1 we see that the diffusion length far from a source then approximates to $(\Sigma_t)^{-1}$ when the medium is heavily capturing.

4.3. THE SOLUTION OF THE DIFFUSION EQUATION IN PLANE GEOMETRY

In order to discuss the properties of the diffusion equation at greater length it is convenient to consider particular geometrical arrangements. We shall consider first the special case of plane geometry, which is of great significance in shielding work, and shall deal with spherical geometry in Section 4.4. In plane geometry there is only one spatial variable (which we shall denote by z), which is the distance from an arbitrarily placed infinite plane. In this case equation (4.2.2) becomes

$$\frac{d^2\phi(z)}{dz^2} - \frac{\phi(z)}{L^2} = -\frac{S(z)}{D}, \quad (4.3.1)$$

while equation (4.2.1) becomes

$$j(z) = -D \frac{d\phi(z)}{dz}, \quad (4.3.2)$$

where j is directed along the direction of positive z .

The general solution of equation (4.3.1) contains two arbitrary constants, which are fixed by the boundary conditions. These conditions must, in a sense, be approximate, since they involve values of the flux on the boundaries and (in

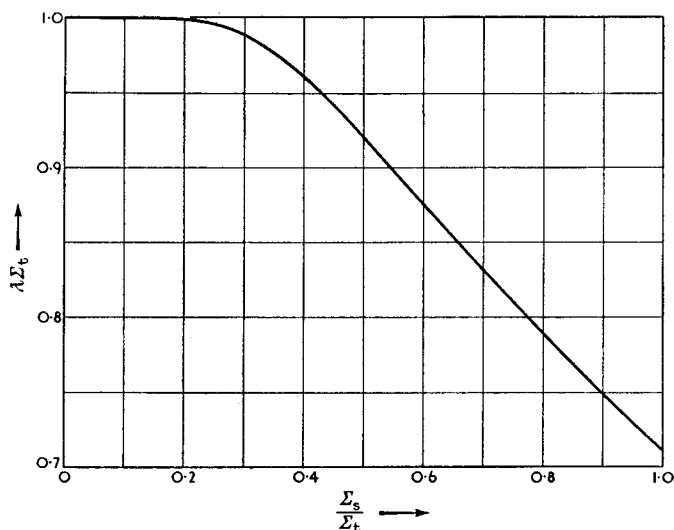


Fig. 4.3.1. The dependence of the linear extrapolation length on (Σ_s/Σ_t) , the ratio of the scattering and total cross-sections.⁽⁹⁾

some cases) at sources, where the diffusion equation itself is not valid. However, those given below have been found to be satisfactory in practice, and are also physically reasonable.

Condition (i). At a boundary between two media with different scattering or capture properties the conditions to be satisfied are:

$$(a) \quad \phi(z) \text{ is continuous}, \quad (4.3.3)$$

$$(b) \quad j(z) \text{ is continuous}. \quad (4.3.4)$$

Condition (ii). At a boundary between a medium which both scatters and captures neutrons and a vacuum (or a second medium which captures but does not scatter) the flux must fall off in such a way that linear extrapolation from the boundary would result in the flux vanishing at a distance λ outside the scattering-capturing medium. The distance λ is known as the *linear extrapolation length* and its value depends on the properties of the scattering-capturing medium. Fig. 4.3.1 shows $\lambda \Sigma_t$ as a function of Σ_s/Σ_t for the case of isotropic scattering.⁽⁹⁾ It will be seen that λ tends to $0.71/\Sigma_t = 0.71/l_t$ as Σ_s/Σ_t tends to one; that is, as the probability of capture tends to zero. For almost all cases capture cross-sections are small enough for $\lambda = 0.71/\Sigma_t$ to be a good approximation.

Condition (iii). In an infinite medium the flux must be finite wherever the diffusion equation applies. This condition also holds, of course, in finite media, but it is only in infinite media that it needs to be applied.

We are now in a position to solve equation (4.3.1) and we give below the solution in a number of cases of interest. More extensive tabulations are given in references 7, 8, and 10.

(a) *Isotropic Plane Source in an Infinite Medium*

Since there are no distributed sources we can put $S(z) \equiv 0$ in (4.3.1) and incorporate the plane source in the boundary condition. Fig. 4.3.2 represents

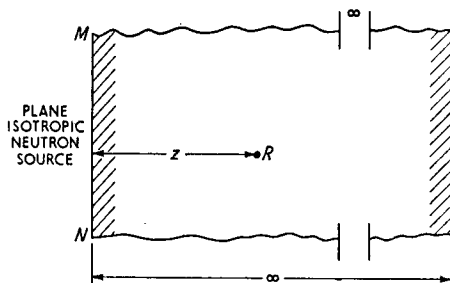


Fig. 4.3.2.

the problem; the plane source MN is taken as the reference plane. The diffusion equation is now

$$\frac{d^2\phi}{dz^2} - \frac{\phi}{L^2} = 0, \quad (4.3.5)$$

and the general solution is

$$\phi = Be^{-\frac{z}{L}} + Ce^{+\frac{z}{L}}, \quad (4.3.6)$$

where B and C are arbitrary constants. Since the medium is infinite C must be zero by condition (iii) above. At the boundary MN , where the source strength is S neutrons per unit area per unit time, we have, by symmetry, a net current of $S/2$ in the positive z direction.

Thus

$$\begin{aligned} \frac{S}{2} &= -D \left. \frac{d\phi}{dz} \right|_{z=0} \\ &= BD/L \end{aligned} \quad (4.3.7)$$

whence $B = SL/2D$.

The complete solution for the flux at the point R , distant z from the source plane, is therefore given by

$$\phi(z) = \frac{SL}{2D} e^{-\frac{z}{L}} \quad (4.3.8)$$

and hence the magnitude of the current in the z direction is

$$j(z) = \frac{S}{2} e^{-\frac{z}{L}} \quad (4.3.9)$$

(b) *Infinite Medium with Given Net Current across a Plane*

This case arises when the current entering a thick shield is known and it is required to find the flux inside. If the given net current is $I \text{ cm}^{-2} \text{ sec}^{-1}$ across MN in Fig. 4.3.2 the flux is found to be

$$\phi(z) = \frac{IL}{D} e^{-\frac{z}{L}} \quad (4.3.10)$$

while the magnitude of the current is

$$j(z) = I e^{-\frac{z}{L}}.$$

(c) *Isotropic Source in a Finite Medium*

Fig. 4.3.3 represents a finite slab of thickness $2h$ with a plane source MN

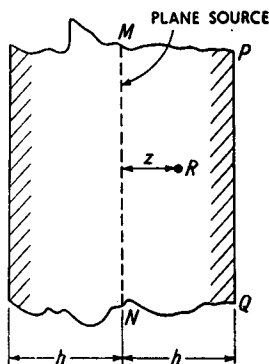


Fig. 4.3.3.

situated at the centre. We wish to find the flux at a point R in the region $MNPQ$. The distance variable z is measured from MN as before, and the source strength is again taken to be S neutrons per unit area per unit time. Since we are dealing with a finite medium the constant C in the general solution (4.3.6) cannot in this case be put equal to zero, and the two boundary conditions are

$$\frac{S}{2} = -D \left. \frac{d\phi}{dz} \right|_{z=0} \quad (4.3.11)$$

as before, and

$$\phi(h) = -\lambda \left. \frac{d\phi}{dz} \right|_{z=h} \quad (4.3.12)$$

from condition (ii) above. Alternatively, when $\lambda/L \ll 1$, this last condition may be replaced by the very nearly equivalent condition

$$\begin{aligned} \phi(a) &= 0 \\ a &= (h + \lambda) \end{aligned} \quad (4.3.13)$$

where

Condition (4.3.13) makes the flux vanish at a distance given by the true slab half-thickness plus the extrapolation distance. Applying (4.3.11) and (4.3.13) to the general solution (4.3.6) we obtain as the final expression for the flux:

$$\phi(z) = \frac{SL \left(e^{-\frac{z}{L}} - e^{-\frac{1}{L}(2a-z)} \right)}{2D(1 + e^{-2a/L})} \quad (4.3.13)$$

The magnitude of the current is found to be

$$j(z) = \frac{S}{2} \left(\frac{e^{-\frac{z}{L}} + e^{-\frac{1}{L}(2a-z)}}{1 + e^{-2a/L}} \right) \quad (4.3.14)$$

(d) Problems with Distributed Sources

The general solution of equation (4.3.1) is given below for a number of simple source functions $S(z)$. The constants B and C are found from the appropriate boundary conditions as before.

(i) $S(z) = \text{constant} = \kappa$ (say) neutrons per unit volume per unit time.

$$\text{Solution: } \phi(z) = Be^{-z/L} + Ce^{+z/L} + \frac{\kappa L^2}{D}; \quad (4.3.15)$$

(ii) $S(z) = a + bz$

$$\text{Solution: } \phi(z) = Be^{-z/L} + Ce^{+z/L} + \frac{L^2}{D} (a + bz); \quad (4.3.16)$$

(iii) $S(z) = \kappa e^{\nu z}$, where $|\nu| \ll \frac{1}{L}$

$$\text{Solution: } \phi(z) = Be^{-z/L} + Ce^{+z/L} + \frac{\kappa e^{\nu z}}{D \left(\frac{1}{L^2} - \nu^2 \right)}. \quad (4.3.17)$$

4.4 THE SOLUTION OF THE DIFFUSION EQUATION IN SPHERICAL GEOMETRY

Although the majority of neutron shielding problems are concerned with media in the form of thick slabs, for which we can use the plane geometry solutions given above, there are also a number of problems (such as the shielding of reactors with very small cores) for which spherical geometry is more appropriate. In this case the diffusion equation (4.2.2) becomes

$$\frac{d^2\phi}{dr^2} + \frac{2}{r} \frac{d\phi}{dr} - \frac{\phi(r)}{L^2} = -\frac{S(r)}{D} \quad (4.4.1)$$

with $\mathbf{j}(r) = -D \frac{d\phi}{dr}$ directed along the radius vector through the point r .

The general solution of equation (4.4.1) when the medium contains no sources ($S(r) = 0$) is

$$\phi(r) = \frac{B}{r} e^{-r/L} + \frac{C}{r} e^{+r/L} \quad (4.4.2)$$

Solutions for the cases corresponding to those given in the previous section are:

(a) *Isotropic Point Source in an Infinite Medium*

If the source strength is S neutrons per unit time,

$$\phi(r) = \frac{Se^{-r/L}}{4\pi Dr}; \quad (4.4.3)$$

$$j(r) = \frac{S}{4\pi r} \left[\frac{1}{r} + \frac{1}{L} \right] e^{-r/L}. \quad (4.4.4)$$

(b) *Infinite Medium with Given Net Current across a Boundary*

If the current across the boundary at $r = R$ is given as I per unit area per unit time, then the flux inside the medium for $r > R$ is

$$\phi(r) = \frac{IR^2L}{D(R+L)r} e^{-(\frac{r-R}{L})}, \quad (4.4.5)$$

while

$$j(r) = \frac{IR^2L}{(R+L)r} \left(\frac{1}{r} + \frac{1}{L} \right) e^{-(\frac{r-R}{L})}. \quad (4.4.6)$$

(c) *Point Source in a Finite Medium*

For a point source of strength S per unit time, in a finite medium of radius R , the flux is given by

$$\phi(r) = \frac{S}{4\pi Dr} \left[\frac{e^{(\frac{a-r}{L})} - e^{-(\frac{a-r}{L})}}{e^{a/L} - e^{-a/L}} \right] \quad (4.4.7)$$

and the current by

$$j(r) = \frac{S}{4\pi r(e^{a/L} - e^{-a/L})} \left[\left(\frac{1}{r} + \frac{1}{L} \right) e^{(\frac{a-r}{L})} - \left(\frac{1}{r} - \frac{1}{L} \right) e^{-(\frac{a-r}{L})} \right], \quad (4.4.8)$$

where $a = R + \lambda$.

(d) *Problems with Distributed Sources*

(i) $S(r) = \text{constant} = \kappa$, say.

$$\text{Solution: } \phi(r) = \frac{B}{r} e^{-r/L} + \frac{C}{r} e^{+r/L} + \frac{\kappa L^2}{D}, \quad (4.4.9)$$

(ii) $S(r) = \frac{a}{r} + b$

$$\text{Solution: } \phi(r) = \frac{B}{r} e^{-r/L} + \frac{C}{r} e^{+r/L} + \frac{L^2}{D} \left(\frac{a}{r} + b \right), \quad (4.4.10)$$

(iii) $S(r) = \frac{\kappa e^{\nu r}}{r}$, with $|\nu| \ll 1/L$

$$\text{Solution: } \phi(r) = \frac{B}{r} e^{-r/L} + \frac{C}{r} e^{+r/L} + \frac{\kappa e^{\nu r}}{rD \left(\frac{1}{L^2} - \nu^2 \right)}. \quad (4.4.11)$$

4.5. THE PHYSICAL MEANING OF DIFFUSION LENGTH

We have seen (equation 4.3.8) that neutrons leaving a plane source in an infinite capturing medium and diffusing without energy loss are attenuated exponentially in such a way that the flux is reduced by a factor $1/e$ in traversing a thickness L of material, where L is the diffusion length. Additional physical interpretations of L are provided by the first and second moments of the spatial distribution of the flux. Thus the first moment, or arithmetic mean $\bar{z}$, which is defined as

$$\bar{z} = \frac{\int_0^\infty z e^{-z/L} dz}{\int_0^\infty e^{-z/L} dz}, \quad (4.5.1)$$

is the average crowflight distance normal to the source plane that a neutron from a plane source penetrates before it is captured. Evaluation of expression (4.5.1) gives $\bar{z} = L$. The second moment, or mean square, is defined as

$$\bar{z}^2 = \frac{\int_0^\infty z^2 e^{-z/L} dz}{\int_0^\infty e^{-z/L} dz}, \quad (4.5.2)$$

and may be shown to be equal to $2L^2$ for plane geometry. Similar calculations for a point source in an infinite medium give

$$\bar{r} = 2L \quad \text{and} \quad \bar{r}^2 = 6L^2 \quad (4.5.3)$$

In Table 4.5.1 below are listed values of the thermal diffusion length in some commonly used reactor materials, together with other quantities of interest. A value for pure D_2O is quoted; the normal value for L , when slightly contaminated with H_2O , is about 100 cm. It should be noted that small variations

Table 4.5.1. Thermal Diffusion Properties of Various Materials

Material	Density (g cm ⁻³)	L (cm)	Σ_c (cm ⁻¹)	D (cm)
H ₂ O	1.0	2.85	0.0197	0.16
D ₂ O	1.1	176	2.6×10^{-5}	0.80
Be	1.85	20.8	1.1×10^{-3}	0.48
C	1.6	52	3.2×10^{-4}	0.87
Fe	7.87	1.1	0.19	—
Pb	11.3	13.6	0.0050	0.92
Portland concrete	2.30	7.04*	—	—

* Measured value. See also Section 6.6.

in the composition of a material such as concrete can have a very marked effect on Σ_e and hence on the diffusion length; for this reason the value given should be taken as a rough guide only.

4.6. THE SLOWING DOWN OF NEUTRONS

We must now consider how the fission neutrons produced in the reactor can be slowed down. As explained in the previous chapter, there are two principal mechanisms by which neutrons can lose energy. The first, inelastic scattering, has already been described in detail. Although the data given in Chapter 3 cannot at present be used in an exact fashion in slowing-down theory, they serve to indicate the relative effectiveness of the various elements in causing fast neutrons to suffer large energy losses.

The other important mechanism is elastic scattering. In an elastic collision (page 116) between a neutron and a nucleus, kinetic energy and momentum are conserved; in addition, at the energies with which we are concerned (<15 MeV), the neutrons are non-relativistic. We can therefore make use of ordinary Newtonian mechanics to describe the result of a collision. The mathematical consequences of these physical laws have already been treated very clearly in a number of books,^(4, 7) and we shall content ourselves with summarizing the results obtained.

It is convenient to define two frames of reference: (i) the *laboratory system*, which is the system in which the target nucleus is at rest before collision, and (ii) the *centre of mass (C. of M., or C.G.) system*, which is the frame of reference in which the centre of mass of the two colliding bodies is at rest. The usefulness of the C. of M. system arises from the fact that for elastic scattering the probability distribution of the angle of scattering in this frame is essentially isotropic over a wide range of neutron energies (Section 3.4). Deviations from the isotropic law are small at energies of less than a few MeV for light elements, or several hundred keV for heavy elements. Since collisions with light elements are a very important factor in slowing down fast neutrons, the theory can be justifiably based on the assumption of isotropic scattering, except when there is appreciable diffraction scattering by heavy elements (page 120); the modifying effect of this latter phenomenon will be dealt with later in Section 4.8.

If θ is the angle of scattering in the C. of M. system, and A is the atomic weight of the target nucleus, the ratio of the neutron energy after collision to that before, E_2/E_1 , is given by

$$\frac{E_2}{E_1} = \frac{A^2 + 2A \cos \theta + 1}{(A + 1)^2} \quad (4.6.1)$$

$$(0 \leq \theta \leq \pi)$$

If ψ is the angle of scattering in the laboratory system,

$$\cos \psi = \frac{A \cos \theta + 1}{(A^2 + 2A \cos \theta + 1)^{\frac{1}{2}}} \quad (4.6.2)$$

Note that no assumption has been made at this stage about the angular distribution of scattering.

Equation (4.6.1) shows that the maximum possible energy loss in a collision with a nucleus of given mass occurs for $\theta = \pi$, when $\cos \theta = -1$, and

$$\frac{E_2}{E_1} = \left[\frac{A-1}{A+1} \right]^2. \quad (4.6.3)$$

Furthermore, the ratio E_2/E_1 decreases as A is reduced. In the extreme case of scattering by hydrogen, $A = 1$, and (4.6.3) gives

$$\frac{E_2}{E_1} = 0, \quad (4.6.4)$$

so that it is possible for a neutron to lose all its energy in one collision. It is this feature which causes hydrogen to be so valuable as a slowing-down material. For heavy elements, with $A \gg 1$, E_2/E_1 is very close to 1 for all θ , and many elastic collisions are needed to reduce the neutron energy appreciably.

It is easily seen from equation (4.6.2) that ψ is always less than θ . The difference becomes more marked as A is reduced, and in the case of hydrogen

$$\cos \psi = \left(\frac{1 + \cos \theta}{2} \right)^{\frac{1}{2}} = \cos \frac{\theta}{2},$$

so that

$$\psi = \theta/2 \quad (4.6.5)$$

Thus scattering by hydrogen is always into the forward hemisphere. This forward scattering lessens the effectiveness of the very light elements as shielding media, because it increases the average distance which neutrons migrate from their point of origin before being completely slowed down; however, it is not sufficient to outweigh the advantages which have already been mentioned.

If we now assume that the scattering is isotropic in the C. of M. system, and define $p(\theta)$ as the probability that a neutron will be scattered into an elementary solid angle $d\Omega$ between the angles θ and $\theta + d\theta$, we have

$$p(\theta) d\theta = \frac{d\Omega}{4\pi} = \frac{2\pi \sin \theta d\theta}{4\pi} = \frac{\sin \theta d\theta}{2}. \quad (4.6.6)$$

Alternatively, if we use the symbol μ to denote $\cos \theta$,

$$p(\mu) d\mu = -p(\theta) \cdot \frac{d\theta}{d\mu} \cdot d\mu = \frac{d\mu}{2}. \quad (4.6.7)$$

(The negative sign is introduced since μ decreases as θ increases.)

Considering now the probability that a neutron of initial energy E_1 will be scattered into an energy range between E_2 and $E_2 + dE_2$, we have

$$p(E_2) dE_2 = -p(\theta) \frac{d\theta}{dE_2} dE_2,$$

and from (4.6.1) we find

$$\frac{d\theta}{dE_2} = \frac{-(1+A)^2}{2AE_1 \sin \theta};$$

thus

$$p(E_2) dE_2 = \frac{(1+A)^2}{4AE_1} \cdot dE_2. \quad (4.6.8)$$

Since $p(E_2)$ is independent of E_2 we see that the energy of the scattered neutron has an equal probability of lying anywhere within the range permitted by equation (4.6.1).

Logarithmic decrement and lethargy. It is convenient to give a special symbol ξ to the average change in the logarithm of the neutron energy, or the *average logarithmic decrement*, per scattering collision. It is the average of the quantity $(\ln E_1 - \ln E_2) = \ln(E_1/E_2)$ over all energies E_2 that are possible given an initial energy E_1 ; i.e.

$$\xi = \frac{\int \ln \frac{E_1}{E_2} p(E_2) dE_2}{\int p(E_2) dE_2}, \quad (4.6.9)$$

where the integration runs from $E_2 = E_1 \left(\frac{A-1}{A+1} \right)^2$ to $E_2 = E_1$, in accordance with (4.6.1). Substitution from (4.6.8) gives

$$\xi = 1 + \frac{(A-1)^2}{2A} \ln \left(\frac{A-1}{A+1} \right), \quad (4.6.10)$$

which, for $A \gg 1$, may be written approximately as

$$\xi = \frac{2}{A}. \quad (4.6.11)$$

It should be noted that ξ is dependent only on A , the atomic weight of the scattering nucleus, and not on the neutron energy. This suggests the use of a new energy variable $u = \ln E_0/E$, where E_0 is some constant reference value. Then we see that the average change in u per collision is constant, and equal to ξ , so that energy is lost in approximately equal steps in u . The quantity u is known as the *lethargy* of the neutron, and is in direct one-to-one relationship with the energy E .

Average cosine of scattering angle. Just as for thermal neutrons, the average cosine of the angle of scattering in the laboratory system is of importance in the theory. It is defined as

$$\bar{\mu}_0 = \frac{\int_0^\pi \cos \psi p(\theta) d\theta}{\int_0^\pi p(\theta) d\theta}, \quad (4.6.12)$$

which, after substitution from (4.6.2) and (4.6.6), may be shown to be equal to $\frac{2}{3A}$.

4.7. AGE THEORY

In a later section, in which we shall consider methods of calculating the attenuation of neutrons in thick shields, we shall need to make use of certain results of the well-known theory of slowing down known as "age theory."

Comprehensive descriptions of this method are readily available in the literature, (refs. 4, 7, and 10), and we shall therefore give only a very brief review.

Age theory treats the succession of discrete energy losses in individual collisions by which neutrons are actually slowed down as equivalent to a continuous slowing-down process which results in the same average rate of energy loss. It provides an approximate value for the flux of neutrons, as a function of space and energy, due to a given source of fast neutrons in an elastically scattering and weakly capturing medium. Its usefulness for shielding calculations is limited by the fact that its accuracy decreases with distance from the source, and that it breaks down completely at distances which are much smaller than those usually encountered in reactor shields. For this reason it plays only a subsidiary role in shielding calculations, as will later become apparent.

In Section 4.1 we defined the neutron flux $\phi(r)$ for the case of monoenergetic neutrons; we now introduce the idea of energy dependence, and define $\phi(u, r)$ as the flux of neutrons per unit interval of u , the lethargy. Considering first a simplified system consisting of a non-capturing medium with plane symmetry and isotropic scattering in the centre of mass system, age theory shows that $\phi(u, r)$ satisfies the approximate equation

$$\frac{1}{3\Sigma_s(u)[1 - \bar{\mu}_0(u)]} \frac{\partial^2 \phi(u, z)}{\partial z^2} = \frac{\partial}{\partial u} [\Sigma_s(u) \bar{\xi}(u) \phi(u, z)], \quad (4.7.1)$$

where $\Sigma_s(u)$ is the macroscopic scattering cross-section (also equal to the total cross-section in this case) for neutrons of lethargy u ; and $\bar{\mu}_0(u)$ is the average cosine of the angle of scattering, as defined in equation (4.6.12), and is a function of u only when the medium contains a mixture of elements, as is assumed here. The quantity $\bar{\xi}(u)$ is the average change in lethargy per collision, and is also a function of energy. It is given by the sum

$$\frac{\sum_i [\Sigma_{is}(u) \xi_i]}{\sum_i [\Sigma_{is}(u)]} = \frac{\sum_i [\Sigma_{is}(u) \xi_i]}{\Sigma_s(u)} \quad (4.7.2)$$

where $\Sigma_{is}(u)$ is the macroscopic scattering cross-section for the i th element in the mixture, and ξ_i is defined for each element by formula (4.6.10).

Equation (4.7.1) may be simplified considerably by the introduction of two further quantities: (i) the *slowing down density* $q(u, z)$, defined by

$$q(u, z) \equiv \Sigma_s(u) \bar{\xi}(u) \phi(u, z) \quad (4.7.3)$$

and (ii) the *age* $\tau(u)$, defined by

$$\tau(u) = \int_{u_0}^u \frac{du'}{3\Sigma_s^2(u' \bar{\xi})(u')[1 - \bar{\mu}_0(u')]}, \quad (4.7.4)$$

where the integration is taken from u_0 , the lethargy of the source neutrons. If

these two quantities are substituted into equation (4.7.1) the following very simple form is obtained:

$$\frac{\partial^2}{\partial z^2} q(\tau, z) = \frac{\partial q}{\partial \tau}(\tau, z). \quad (4.7.5)$$

There is a clear analogy between this equation and that of the classical time-dependent heat conduction equation with q in place of the temperature and τ instead of time. This correspondence has given rise to the term "age" for τ , although it should be noted that it has the dimensions of (length)².

The physical interpretation of $q(\tau, z) \equiv q(u, z)$ may be seen from equation (4.7.3), since $\Sigma_s(u)\phi(u, z)$ represents the number of collisions occurring per unit time per unit volume and per unit lethargy interval, and $\bar{\xi}(u)$ is the average increment in u which occurs per collision. Thus $q(u, z)$ is the number of neutrons per unit volume per unit time which are degraded in energy past the lethargy point u (or, alternatively, past the energy point E corresponding to u). We see therefore that if τ has the value appropriate to thermal energy, then $q(\tau, z)$ is exactly the source function for neutrons becoming thermal in the medium. The physical meaning of τ is discussed on the next page.

Equation (4.7.5) has been extensively studied for a variety of source functions and boundary conditions, mainly in connection with heat transfer problems; a comprehensive tabulation of the solutions for the neutron case is given in reference (10). As an illustration we give here the solution for a case which is of importance in shielding. We assume a plane isotropic source of mono-energetic fast neutrons in an infinite medium, with S neutrons per unit time leaving the source plane at $z = 0$. The appropriate solution of equation (4.7.5) is then

$$q(\tau, z) = \frac{Se^{-\frac{z^2}{4\tau}}}{(4\pi\tau)^{\frac{1}{2}}}, \quad (4.7.6)$$

where τ is a function of the energy of the neutrons defined by equation (4.7.4). (The corresponding solution for a point source is given in equation (4.7.12)). The flux $\phi(u, z)$ may now be found from equation (4.7.3), which gives the value per unit interval of u . Thus the total flux of neutrons at the point z between the source lethargy u_0 and any other given lethargy u is

$$\phi(z) = \int_{u_0}^u \frac{S \exp[-z^2/4\tau(u')]}{\Sigma_s(u')\bar{\xi}(u')[4\pi\tau(u')]^{\frac{1}{2}}} du'. \quad (4.7.7)$$

These results can easily be extended to the general case of a source which emits neutrons of many energies. If the source strength is given by $F(u)$ neutrons per unit interval of u , over the range from u_1 to u_2 , then equation (4.7.6) becomes

$$q(\tau, z) = \int_{u_1}^{u_2} \frac{F(u_0)e^{-\frac{z^2}{4\tau}}}{[4\pi\tau]^{\frac{1}{2}}} du_0, \quad (4.7.8)$$

where τ is defined as in equation (4.7.4). This may also be written as

$$q(\tau, z) = \int_{u_1}^{u_2} \frac{F(u_0) \exp\{-z^2/4[\tau(u) - \tau(u_0)]\}}{\{4\pi[\tau(u) - \tau(u_0)]\}^{\frac{1}{2}}} du_0, \quad (4.7.9)$$

where $\tau(u)$ is now defined as

$$\int_{u_0}^u \frac{du'}{3\Sigma_s^2(u')\bar{\xi}(u')[1 - \bar{\mu}_0(u')]} \quad (4.7.10)$$

Capturing media. So far we have confined the discussion to a non-capturing medium. Provided capture is small, or more specifically if $\Sigma_c(u) \ll \bar{\xi}(u)\Sigma_s(u)$ over the range of u considered, we may incorporate the effects of capture by means of a simple multiplicative function. Thus, if capture is included, we have, instead of equation (4.7.6):

$$q(\tau, z) = \frac{Se^{-z^2/4\tau}}{[4\pi\tau]^{\frac{1}{2}}} \exp \left[- \int_{u_0}^u \frac{\Sigma_c(u')}{\bar{\xi}(u')\Sigma_s(u')} du' \right]. \quad (4.7.11)$$

Conditions of validity. Turning now to the conditions which must be satisfied for age theory to apply, we find that these at first sight appear formidable. Apart from the limitation on capture which has already been mentioned, three other basic assumptions are necessary:

- (i) The fractional change of mean free path must be small over a lethargy region comparable with ξ ;
- (ii) the average number of collisions occurring between the source energy and the energy of interest must be large compared with unity;
- (iii) the theory may only be used up to distances from the source of the order of $\tau\Sigma_s$.

Condition (ii) implies that $(u - u_0) \gg \xi$ and this, as well as the first condition, means that age theory may be a poor approximation in media containing a high proportion of hydrogen, for which $\xi = 1$. These conditions are often held against the use of age theory in hydrogenous materials such as water or concrete; but although the objection is valid if high accuracy is important, it is not so restrictive in shielding applications. For example, HOLTE⁽¹¹⁾ shows that even in water, which is the worst case likely to be met with, age theory agrees with the exact theory to within 30 per cent for all distances within the range permitted by condition (iii). This third condition is, in fact, the major obstacle in the use of the theory for shielding. For this reason the theory is mainly of use in estimating the neutron flux over the first one or two decades of attenuation. But it also has applications at greater distances for finding the "build-up" of neutrons of energy less than the source energy, as will be seen in Section 4.9.

Physical meaning of age. For the case of a point source of strength S neutrons per unit time in an infinite medium, the solution analogous to (4.7.6) is

$$q(\tau, r) = \frac{Se^{-r^2/4\tau}}{[4\pi\tau]^{3/2}}, \quad (4.7.12)$$

where r is measured from the source. We may look for a physical meaning for τ by evaluating the second moment of the slowing down density. Thus

$$\overline{z^2(\tau)} = \frac{\int_0^\infty q(\tau, z)z^2 dz}{\int_0^\infty q(\tau, z) dz} = 2\tau, \quad (4.7.13)$$

and similarly

$$\overline{r^2(\tau)} = 6\tau. \quad (4.7.14)$$

These relationships are similar to those already found in the case of diffusion theory in equations (4.5.2) and (4.5.3), with L , the diffusion length, replaced by $\sqrt{\tau}$. This quantity, $\sqrt{\tau}$, is often called the *slowing down length*, and is given the alternative notation L_s .

Evaluation of the first moment $\bar{z}$ or $\bar{r}$ gives

$$\bar{z} = 2\sqrt{\frac{\tau}{\pi}} = \frac{2L_s}{\sqrt{\pi}}$$

and

$$\bar{r} = 4\sqrt{\frac{\tau}{\pi}}.$$

Thus when τ corresponds to the energy of thermal neutrons, the slowing-down length, in the case of neutrons leaving a plane source, represents $\sqrt{\pi}/2$ times the average distance (normal to the plane), to which a neutron penetrates before becoming thermal. The slowing-down length of a material is therefore an important quantity, determining its effectiveness in moderating neutrons. Some values of L_s are given in Table 4.7.1.

Table 4.7.1. *Slowing Down Lengths*

Material	Density (g cm ⁻³)	Age from fission to indium resonance (cm ²)	Age to thermal (cm ²)	L_s to thermal (cm)
H ₂ O	1.0	30.4	31.4	5.6
D ₂ O (0.16% H ₂ O)	1.1	105	120	11.0
Be	1.85	80	97	9.8
C	1.6	311	364	19

Calculated age (2 MeV to thermal) for Portland concrete = 137 cm².

4.8. THE ATTENUATION OF THE FAST COMPONENT

It will be seen from the summary of neutron physics given in the previous chapter that neutron total cross-sections, averaged over many adjacent resonances, show a general tendency to decrease with increasing energy. This phenomenon has a decisive effect on the penetration of neutrons, and enables the calculation of the flux or dose inside a shield, due to an inflowing current of

neutrons from a reactor, to be divided naturally into three stages. First, and most fundamental, is the determination of how the flux of the important "fast" neutrons (i.e. $E_n > 0.5$ MeV) varies with the distance into the shield. Next comes the calculation of the flux of neutrons of lower energy arising from fast neutron collisions; this flux is naturally determined to a very considerable extent by the fast neutron distribution, but it also depends on the slowing-down and capturing properties of the medium. By analogy with the gamma-ray case (Section 2.7), this stage may be regarded as the calculation of a kind of neutron "build-up-factor." Lastly, the contribution to the total flux from neutrons which entered the shield with energies below 0.5 MeV must be determined. Although in most practical cases encountered in nuclear engineering these slower neutrons are more numerous than the fast neutrons, they are much more rapidly attenuated, and their effect is confined to the region close to the source. Thus they have little effect on the thickness of a reactor shield, though they must be taken into account when the heating of the part of the shield nearest the reactor is being considered (Chapter 6). When these three steps have been completed there still remains the calculation of the distribution of thermal neutrons, which arises both from the thermal flux that is incident on the shield, and from those neutrons which have slowed down inside it.

As already explained, the processes by which fast neutrons are attenuated in thick shields are exceedingly complex; consequently the exact calculation of the first two stages of this scheme is difficult mathematically, and those methods which attempt it are not easy to apply. However, there are cruder but simpler ways which have been found to be useful in practice; we shall describe them here, not because they have any claim to be perfected methods, but for want of anything better.

Physical Basis of the Removal Cross-section Method

In order to understand the physical basis of these semi-empirical methods, it is useful to consider the qualitative behaviour of the fast neutrons. Consider a parallel beam of neutrons, not necessarily all of the same energy, but confined to an energy region which is above 0.5 MeV and is a few MeV in width. This beam is assumed to enter a thick shield in a direction normal to its surface, and we consider how the flux of neutrons with energies above 0.5 MeV varies with depth.

Neutrons can be removed from the fast group either by absorption, or by scattering into a lower energy region, the latter being by far the most important process. If the shielding medium contains hydrogen or other very light nuclei, then only one or two collisions with these will suffice to remove neutrons from the fast group. Inelastic scattering by heavier elements has a similar effect, and the inelastic cross-section is of the order of half the total cross-section at these energies (Table 3.4.1). The neutrons can therefore be regarded as diffusing in a strongly "capturing" medium which scatters without energy loss, and whose fictitious "capture" cross-section is given approximately by the sum of the cross-sections for collisions with hydrogen, and for capture plus inelastic scattering in heavier elements. We have seen earlier (equation 4.2.10) that in absorbing media in which scattering is isotropic in the laboratory system ($\bar{\mu}_0 = 0$)

the neutron flux far from the source falls off as e^{-Kz} , where K is given by the formula

$$\tanh^{-1} \left(\frac{K}{\Sigma_t} \right) = \frac{K}{\Sigma_s}.$$

For very strong absorption, as in the present case where Σ_s/Σ_t may be less than 0.5, we see from Fig. 4.2.1 that K is almost equal to Σ_t . The physical interpretation of this result is that even elastic scattering acts like absorption, since it increases the distance that a neutron must travel in order to penetrate the shield, and thereby increases its chance of absorption. This admittedly non-rigorous argument suggests that, in the particular case of a medium in which the scattering is isotropic, the total cross-section acts as though it were nearly all pure absorption, and that it can therefore be regarded as a "removal cross-section."⁽³⁷⁾

In practice, however, elastic scattering of fast neutrons may not be isotropic in the laboratory system, owing to the effect of potential or shadow scattering (Section 3.4). This behaviour is very marked for the scattering of neutrons in the 5 to 10 MeV region by medium- and heavy-weight nuclei. These are, of course, just the neutrons which are of crucial importance in thick shields. As a result the true removal cross-section is somewhat less than the total cross-section, and can be regarded as being roughly equal to the total cross-section minus the cross-section for scattering into the shadow cone. The concept of a removal cross-section is a useful one, and has formed the basis of a series of measurements at the Oak Ridge laboratory, the results of which are summarized in Table 4.8.1.

In the discussion so far it has been implicitly assumed that the removal cross-section is energy independent over the source energy range. If this is not so, as is usually the case, then the flux of neutrons in the fast group will vary with distance as $N(z)$, where

$$N(z) = \int F(E) \exp(-\Sigma_{\text{rem}}(E) \cdot z) dE; \quad (4.8.1)$$

$F(E)$ is proportional to the source spectrum, and the integration is taken over the whole range of energies covered by the "fast" group of source neutrons.

Since, in most shielding materials, the total cross-section continues to decrease with increasing energy over the fission spectrum range, and the forward scattering also becomes more pronounced at higher energies, it is to be expected that

$\left| \frac{d}{dz} (\ln N(z)) \right|$ will slowly decrease as z increases, due to the "hardening" of the neutrons remaining in the beam. In fact, although this hardening behaviour is observed, the phenomenon is not very marked because of the sharp fall-off in the fission spectrum at high energies (Fig. 3.9.1), and in practice it is found that $N(z)$ can be fitted experimentally to a simple exponential with a constant average value of Σ_{rem} without serious error. This rule holds rather less well for media in which hydrogen is the principal fast neutron attenuator, owing to its

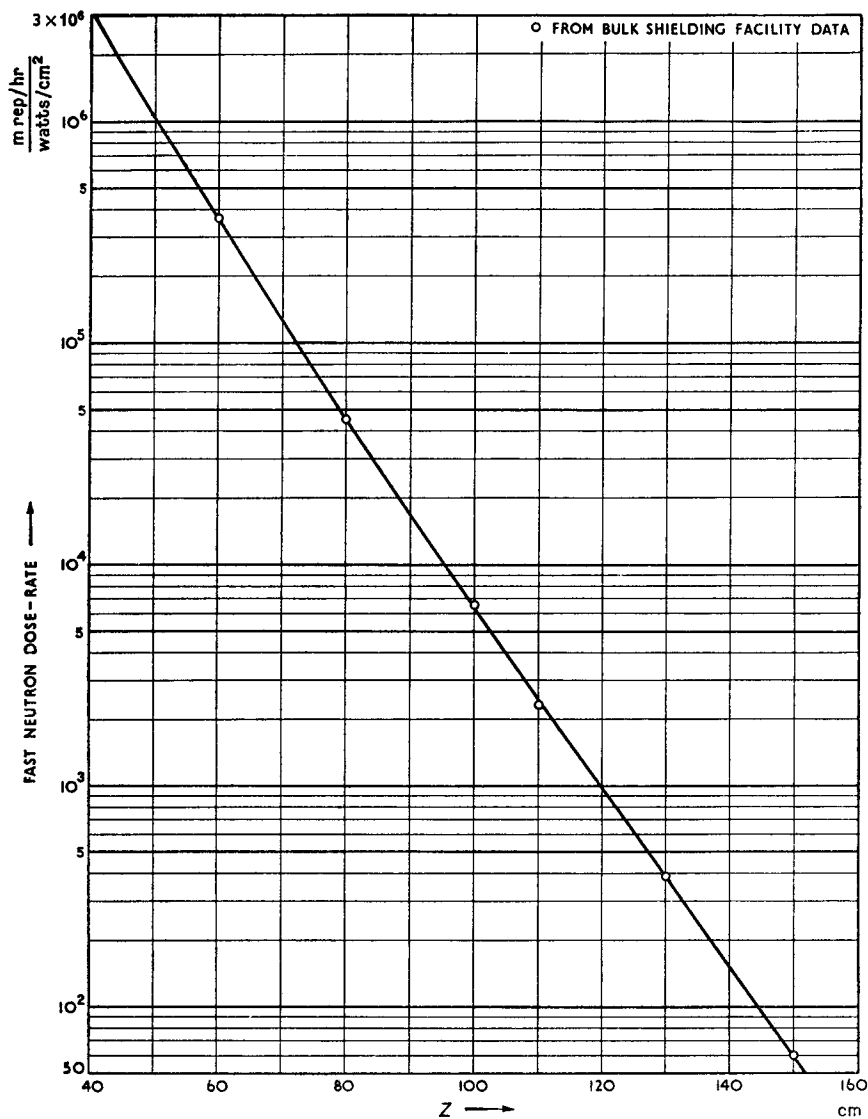


Fig. 4.8.1. Attenuation of fast neutron dose due to a plane collimated fission source in a water shield⁽⁸⁾ (see text). A fast neutron dose-rate in water of 1 mrep hr^{-1} is associated with a thermal flux of about $400 \text{ n cm}^{-2} \text{ sec}^{-1}$ at 50 cm from a fission source, or about $300 \text{ n cm}^{-2} \text{ sec}^{-1}$ at considerably greater distances.⁽³⁵⁾

rapid change of total cross-section with neutron energy in the MeV region (Fig. 3.4.2.). Thus in water the relaxation length of fission neutrons increases steadily with distance from the source, although over the range of penetrations shown in Fig. 4.8.1 the effect is not very pronounced.

Observed Attenuation of Fast Neutrons in Water

Fig. 4.8.1 is derived from observations on the American Bulk Shielding Facility. The ordinate is equal to $4\pi z^2 \cdot G(z)$, where $G(z)$ is the biological dose-rate (in mrep hr⁻¹) at a distance z cm from a point fission source generating 1 watt of heat, and emitting neutrons isotropically. Assuming 200 MeV/fission, and 2.48 neutrons per fission of U²³⁵, this corresponds to a source emitting 7.7×10^{10} fission neutrons sec⁻¹. The dose-rate $G(z)$ is entirely due to recoil protons, as measured by a Hurst-Ritchie type of fast neutron dosimeter.⁽²⁷⁾

We can alternatively regard the data in Fig. 4.8.1 as giving the dose-rate which would be observed in water at a distance z from an infinite plane source generating 1 watt cm⁻², and emitting 7.7×10^{10} fission neutrons cm⁻² sec⁻¹, *all collimated in the +z direction*.* This alternative interpretation of the data is perhaps the more useful one for quick calculations, since a substantial measure of collimation of fast neutrons certainly exists at considerable depths in most practical shields; and in any case, the assumption that all source neutrons are collimated in the outward direction has the advantage that it leads to an estimate of the required shield thickness which is on the safe side.

A curve showing the attenuation in water of 1 MeV neutrons (from the decay of N¹⁷) is given on page 313.

Experimentally Determined Removal Cross-sections for Fission Neutrons

In the form in which we have defined a removal cross-section, it refers only to removal from the fast energy group. Those neutrons which are scattered to lower energies continue to diffuse whilst slowing down, and may add a considerable "build-up" to the contribution of the fast flux. In order to make a direct experimental measurement of a removal cross-section the detector should ideally have a high energy threshold, in order to measure only the fast neutron flux. In practice, this may be difficult because of the low efficiency of such counters and the low counting-rate which would therefore be involved. The procedure adopted at Oak Ridge in measuring removal cross-sections is to use a composite shield made up of slabs of the element in question immersed in water. The detector is separated by about 140 cm of water from the nearest slab of the element being studied, and is on the side away from the source. Under these circumstances, build-up is small, and an equilibrium spectrum exists; thus an effective removal cross-section for fast neutrons can be determined using thermal neutron detectors of high sensitivity. This method removes the experimental difficulty which would otherwise be caused by the low sensitivity of fast neutron counters. However, such a measurement also includes the effect of water in removing the fast neutrons. As was stated above, this cannot

* The attention of the reader is drawn to this unusual interpretation of the meaning of a plane source of strength 1 watt cm⁻². In most other applications the phrase refers to a source which emits neutrons isotropically into both hemispheres.

be fitted by a simple exponential with an average removal cross-section; so that it must, in some way, be separated out. It is found that this may be done by fitting the measured biological dose, $I(z)$ (dose rather than flux being the quantity that is measured at Oak Ridge) to the expression

$$I(z) = I_0 D(t_w) \exp(-\Sigma_{\text{rem}} t_h), \quad (4.8.2)$$

where I_0 is the fast neutron dose incident on the experimental shield,

t_w is the total thickness of water between source and detector,

t_h is the total thickness of the material being studied,

and $D(t_w)$ is the observed attenuation of the dose in a thickness t_w of water alone, which is obtained from a separate experiment (see Fig. 4.8.1).

Published values of experimental removal cross-sections obtained in this way are given in Table 4.8.1. In some cases the values were derived from a study of compounds of elements, by assuming that removal cross-sections are additive.

Table 4.8.1. Experimentally Determined Values of Removal Cross-sections for Use in Equation (4.8.2)

Material	Cross-section in barns per atom or molecule	
	1 MeV neutrons (from N ¹⁷)	Fission neutrons
Aluminium	1.1	1.31 ± 0.05
Boron		(0.97) ± 0.10
Beryllium		1.07 ± 0.06
Bismuth		3.49 ± 0.35
Carbon		0.81 ± 0.05
Chlorine		(1.2) ± 0.08
Copper		2.04 ± 0.11
Fluorine		(1.29) ± 0.06
Iron		1.98 ± 0.08
Lithium		1.01 ± 0.04
Nickel		1.89 ± 0.10
Oxygen		(0.99) ± 0.10
Lead		3.53 ± 0.30
Tungsten		2.51 ± 0.55
Uranium		3.6 ± 0.4
C ₇ F ₁₆		26.3 ± 0.8
C ₂ F ₃ Cl		6.66 ± 0.8
CH ₂		2.84 ± 0.11
B ₂ C		4.7 ± 0.3
B ₂ O ₃		4.30 ± 0.41
D ₂ O		2.76 ± 0.11

Effective removal cross-sections given in parentheses were derived from analysis of compounds of the elements. From CHAPMAN and STORRS.⁽²⁶⁾

The experimentally determined removal cross-section is approximately equal to 0.6–0.7 times the total cross-section at 8 MeV. The removal cross-section per unit mass shows a smooth variation with atomic weight (Fig. 4.8.2). This regularity gives very strong grounds for believing that, apart from the general

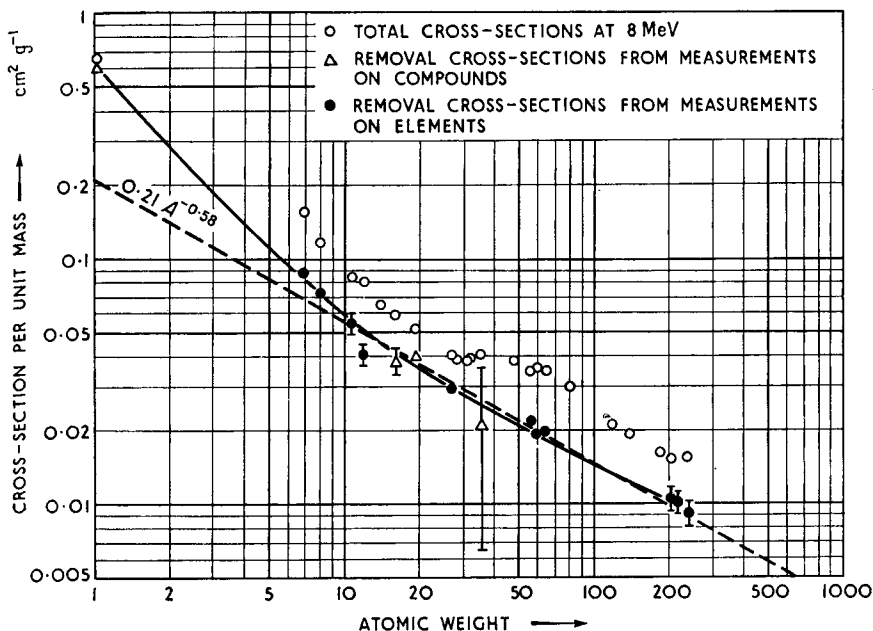


Fig. 4.8.2. Removal cross-sections per unit mass for fission neutrons, as a function of atomic weight.

Over the range $8 \leq A \leq 240$ the values are well fitted by the function $0.21A^{-0.58}$ (CHAPMAN and STORRS⁽²⁵⁾).

trend in favour of hydrogen and the other light elements, there are no special nuclear reasons for preferring any one element to its neighbour in the table of elements. However, in practice the choice is also affected by the purely geometrical influence of density on shield weight (page 279). This is particularly true of small propulsion reactors, towards which neutron shielding studies tend to be directed. By taking advantage of the relation between removal and total cross-sections already mentioned, we can use the much more numerous total cross-section data to illustrate the effect of density on the choice of materials. Because density shows a periodic variation with atomic weight, the total cross-section per unit volume also has maximum values in the neighbourhood of $A = 60, 100, 150$, and 190 . The choice for shielding materials thus falls naturally on to the less expensive elements situated near the peaks shown in Fig. 4.8.3. In particular, the most generally useful constructional material, iron, is also an unusually good attenuator of neutrons. Because of the large density of iron, mixtures of iron and water have a smaller relaxation length for fission neutrons than water itself.

Application of Removal Cross-sections to Shielding Calculations

The method of applying these experimental removal cross-sections depends on the composition of the shield. The simplest case is when the shield contains a fairly large proportion of water (hydrogen content greater than about 20% by

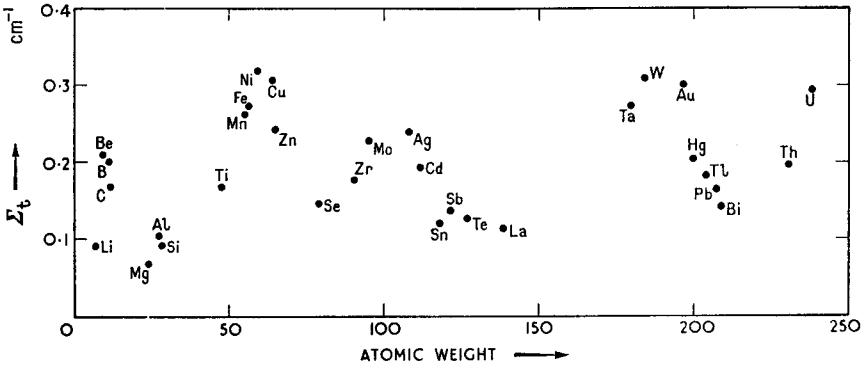


Fig. 4.8.3. Macroscopic total cross-sections at 8 MeV as a function of atomic weight.

number of atoms), when the attenuation of neutron dose due to neutrons of all energies may be found from a formula similar to (4.8.2).

Thus for a shield made up of slabs of different materials (including water) the neutron dose at the distance z from a plane collimated source of strength I_0 watts cm^{-2} (as defined on page 179) is given by

$$I(z) = I_0 D(t_w) \exp \left\{ - \sum_i (\Sigma_{i,\text{rem}} t_{hi}) \right\} \quad (4.8.3)$$

where

$\Sigma_{i,\text{rem}}$ is the macroscopic removal cross-section for the i th material,
 t_{hi} is the total thickness of the i th material between the source and the detector, and

t_w and $D(t_w)$ are as defined previously.

$D(t_w)$ may be obtained from Fig. 4.8.1, in which the values given are normalized to 1 watt cm^{-2} of fission neutrons from a plane collimated source.

As written, equation (4.8.2) is directly applicable to the case of a thick shield made up of alternate layers of heavy material and water, but it can be generalized for homogeneous mixtures by modifying t_{hi} and t_w to $A_i \rho t / \rho_i$ and $A_w \rho t$ respectively, where

A_i is the fraction by weight of the i th heavy material in the shield,

A_w is the fraction by weight of water,

ρ_i is the intrinsic density of the i th heavy material in g cm^{-3} ,

ρ is the actual density of the shield in g cm^{-3} , and

t is the total thickness of shield between source and detector.

The same formula can also be used for estimating the attenuation of other quantities such as neutron flux, provided suitable experimental information on the attenuation in water is available for substitution in place of $D(t_w)$. Shields

of quite intricate construction can be treated as homogeneous mixtures provided the relaxation length for fast neutrons is appreciably larger than the size of the units forming the shield.

It is necessary to make two comments on the use of equation (4.8.3). It will be recalled that the removal cross-sections given in Table 4.8.1 were determined for slab absorbers separated from the point of interest by a large thickness of water. This is therefore the condition in which the values can be used with least error. When substantially different arrangements are to be used, the meaning and the limitations of the removal cross-section technique must be kept clearly in mind. Close to an absorber the neutron spectrum is disturbed, and does not have an equilibrium form. The detector used for the experimental measurement has a response which depends on the neutron energy spectrum as well as on the total flux of neutrons, so that if it had been positioned in the non-equilibrium region—which extends a distance of about 40 cm into the water from the nearest absorbing slab—different results would have been obtained. Thus in this non-equilibrium region an analysis based on removal cross-sections does not give reliable results.

Secondly, although the measurements were made with the absorbers close to a fission source, in practice one needs to know the effect of absorbers that are distributed through water or some other hydrogenous shield. It is found empirically⁽²⁸⁾ that one must then reduce the removal cross-section by 10%. The necessity for this correction can be understood by considering how the cross-sections are measured. In the usual arrangement, the absorber and detector are separated by 140 cm of water, and neutrons which have lost a substantial amount of energy by inelastic scattering are consequently not recorded. When the absorber is uniformly dispersed through the water, however, some of these neutrons can reach the detector, which is equivalent to a reduction in the effective removal cross-section.

Estimation of Removal Cross-sections from Fundamental Nuclear Data

The method of removal cross-sections described above is at present limited by the lack of information on fast neutron spectra other than a fission spectrum. It is possible, however, to make a fairly crude calculation of the attenuation of fast neutrons, based on the qualitative description given in this section, and using readily available data on total cross-sections and angular distributions. We have seen that Σ_{rem} can be considered to be equivalent to Σ_t minus the cross-section for scattering into the forward peak of the angular distribution. This suggests that a rough estimate of Σ_{rem} could be obtained from the analogy of a transport cross-section, by writing

$$\Sigma_{\text{rem}}(E) = \Sigma_t(E) - 2\pi \int_{-1}^{+1} \Sigma_s(E, \mu) \mu \, d\mu, \quad (4.8.4)$$

where $\Sigma_s(E, \mu)$ is the differential cross-section for elastic scattering of neutrons of energy E through an angle $\cos^{-1} \mu$;

$$\text{i.e.} \quad \Sigma_{\text{rem}}(E) = \Sigma_t(E) - \Sigma_s(E) \overline{\cos \theta(E)}, \quad (4.8.5)$$

where $\overline{\cos \theta(E)}$ is the average cosine of the angle of scattering as defined in

equation (4.6.12); it will be a fairly rapidly varying function of E over the fission energy range.

This definition applies to all media except hydrogen which, as stated earlier, should be treated as though a collision causes total removal from the beam. Thus if Σ_{rem} is to be calculated for a composite homogeneous shield which contains hydrogen, expression (4.8.5) becomes:

$$\Sigma_{\text{rem}}(E) = \sum_i [\Sigma_{i,t}(E)] - \sum_{i \neq \text{H}} [\Sigma_{i,s}(E) \overline{\cos \theta_i(E)}], \quad (4.8.6)$$

where the second summation is taken over all elements, i , in the shield except hydrogen. In spite of the approximate nature of this expression for Σ_{rem} it has been found to give good agreement with experiment in the cases where it has been applied.

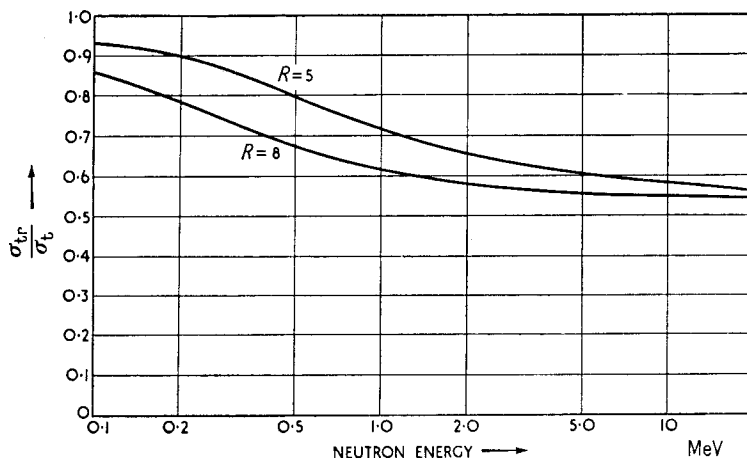


Fig. 4.8.4. Calculated variation of $\frac{\sigma_{\text{tr}}}{\sigma_{\text{t}}} = \frac{\sigma_{\text{rem}}}{\sigma_{\text{t}}}$ with neutron energy for two values of the nuclear radius R . The radius is given in units of 10^{-13} cm.

The calculation of removal cross-sections as described above requires a knowledge of the angular distribution of elastic scattering for neutron energies in the range from 1 to about 15 MeV. A summary of experimental data on angular distributions is given in ref. (13), which covers most elements of interest in shielding; but the neutron energies for which cross-sections are given are very limited, being confined to one or two spot values in the energy range of interest. The doubtful regions can be covered with the aid of theoretical calculations. FESHBACH and WEISSKOPF⁽¹⁴⁾ have published calculations of the transport cross-section per atom, which, in their definition, is identical with σ_{rem} , as a function of energy and nuclear radius R . Figure 4.8.4, which was obtained from their results, shows calculated values of $\frac{\sigma_{\text{rem}}(E)}{\sigma_{\text{t}}(E)}$ for $R = 5 \cdot 10^{-13}$ cm and $R = 8 \cdot 10^{-13}$ cm; these correspond to nuclear masses of about 40 and 150

respectively. These curves will be found helpful in interpolating for values of $\sigma_{\text{rem}}(E)$ not obtainable from experimental measurements, although the method of calculation used (hard sphere model) is now regarded as superseded by the more accurate optical theory (page 111), for which results of calculated angular distributions are also available.⁽²³⁾ The curves in Fig. 4.8.4 are reasonably accurate above about 4 MeV, which is the region of interest for reactor shielding.

Attenuation of the Fast Component

We are now in a position to deal with the first of the three stages mentioned in the introduction to this section, that is, the attenuation of the fast component.

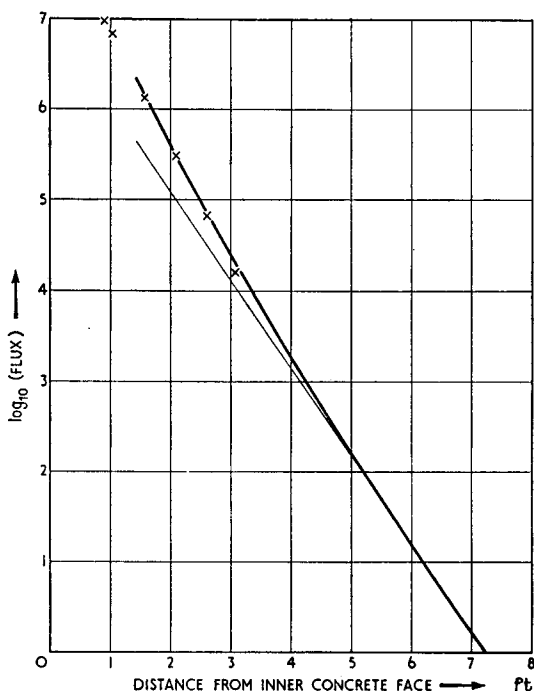


Fig. 4.8.5. The attenuation of the fast neutron component from a reactor in a Portland concrete shield of density 2.4 g cm^{-3} .

The crosses show experimental points; the solid line is a theoretical curve, calculated as described in the text; the thin line shows the asymptotic gradient.

If there is an isotropic point source of fast neutrons of strength S neutrons per unit time and with an energy spectrum proportional to $F(E)$ per unit interval of E , the flux of fast neutrons ($>1 \text{ MeV}$) at a point distant r from the source is, for large r ,

$$\phi(r) \sim \frac{S}{4\pi Dr} \frac{\int_1^\infty F(E) \exp(-\Sigma_{\text{rem}}(E) \cdot r) dE}{\int_1^\infty F(E) dE}, \quad (4.8.7)$$

where the coefficient D is a length of magnitude roughly equal to the total mean free path averaged over the source energy spectrum (cf. equation 4.4.3). Any attempt at a more accurate definition is hardly justified in view of the approximations involved in the definition of Σ_{rem} . This expression may easily be generalized to a distributed source of neutrons by a further integration over the volume of the source (e.g. a reactor core or fission plate). The main usefulness of this treatment is in finding the relaxation length (or e -folding length) of the fast flux rather than the absolute magnitude of ϕ . An example of the results obtained by this method is given in Fig. 4.8.5, which shows the attenuation of fast neutrons from a reactor in a Portland concrete shield. The flux measurements were made using the $\text{S}^{32}(n,p)\text{P}^{32}$ reaction as a threshold detector (page 139). The theoretical curve was obtained with $F(E)$ (equation 4.8.1) proportional to the fission spectrum; it is normalized to fit the experimental point at 1.6 ft.

4.9. BUILD-UP OF NEUTRONS OF LOWER ENERGIES

The total collision cross-section for neutrons which have been removed from the fast neutron group and are in the process of slowing down to thermal energies will almost certainly be greater than the effective removal cross-section for fast neutrons (as given by Table 4.8.1 or equation 4.8.5), with the result that these lower energy neutrons are more rapidly attenuated than the fast neutrons. Under these circumstances the flux of these lower energy neutrons per unit energy at a given energy E can be approximately represented by the expression

$$\Phi(E, z) = B(E, z) \cdot \phi(z), \quad (4.9.1)$$

where $\phi(z)$ is the flux of fast neutrons, and $B(E, z)$ is the "build-up factor" appropriate to the quantity Φ (i.e. dose b.u.f., energy b.u.f., etc.). Except in the non-equilibrium region near the source, $B(E, z)$ will vary fairly slowly with z in hydrogenous shields, so that its contribution to the rate of variation of Φ with z can be regarded as a second-order effect compared with the variation of $\phi(z)$. This implies that fairly simple methods of calculating $B(E, z)$, such as age-diffusion theory, should prove adequate for shielding needs. It has already been pointed out that $\phi(z)$ can usually be fitted to a simple exponential $\lambda e^{-\Sigma_{\text{rem}} z}$, and we shall assume that this has been done. The distribution of generating sources for the lower energy "build-up" neutrons is therefore given by $\lambda \Sigma_{\text{rem}} \exp(-\Sigma_{\text{rem}} z)$ (if we neglect true absorption, as is usually justified). This source term may then be substituted into the age equation, so that, in plane geometry, and in the non-capturing case, we have

$$\frac{\partial^2 q(z, \tau)}{\partial z^2} = \frac{\partial q}{\partial \tau} - \lambda \Sigma_{\text{rem}} \exp(-\Sigma_{\text{rem}} z) \cdot \delta(\tau), \quad (4.9.2)$$

where $q(z, \tau)$ is the slowing-down density of neutrons of age $\tau(u)$, $\delta(\tau)$ is the Dirac delta-function, and $\tau(u)$ is given as a function of the lethargy u by the usual definition

$$\tau(u) = \int_{u_0}^u \frac{du}{3\xi\Sigma_s\Sigma_{\text{tr}}}. \quad (4.9.3)$$

We have assumed in formulating equation (4.9.2) that all neutrons leave the fast group with the same lethargy u_0 (see below). The solution of (4.9.2) at large distances is given by

$$q(z, \tau) = \lambda \Sigma_{\text{rem}} \exp \left(-\Sigma_{\text{rem}} z + (\Sigma_{\text{rem}})^2 \tau(u) \right) \quad (4.9.4)$$

This result can be modified to take account of capture, provided this is small,

i.e. if $\frac{\Sigma_c}{\xi \Sigma_s} \ll 1$. The flux of neutrons per unit lethargy interval is then

$$\phi(z, u) = \frac{q(z, \tau)}{\xi \Sigma_s} = \frac{\lambda \Sigma_{\text{rem}}}{\xi \Sigma_s} \exp \left(-\Sigma_{\text{rem}} z + (\Sigma_{\text{rem}})^2 \tau(u) - h(u) \right), \quad (4.9.5)$$

where $h(u) = \int_{u_0}^u \frac{\Sigma_c}{\xi \Sigma_s} du$ is the term representing the absorption. The build-up of neutrons $B(u)$ per unit lethargy interval is

$$B(u) = \frac{\Sigma_{\text{rem}}}{\xi \Sigma_s} \exp \left((\Sigma_{\text{rem}})^2 \tau(u) - h(u) \right). \quad (4.9.6)$$

Note that in this simplified theory $B(u, z)$ is found to be independent of z . The build-up of neutrons of all energies between u_0 and u_1 , say, is $\int_{u_0}^{u_1} B(u) du$.

It is difficult to give a precise value to the most suitable u_0 to take in this theory, but for reactor shielding one should use a value corresponding to about the nuclear temperature (Table 3.4.2) when the removal from the fast beam is predominantly due to inelastic scattering, and about 4 or 5 MeV when it is predominantly due to hydrogen.

If the mean scattering cross-section for neutrons in the process of slowing down is not large compared with Σ_{rem} , as may be the case when the shield contains only small amounts of hydrogen, the age theory approach described above will underestimate the build-up. This eventuality is discussed in reference (15) by HURWITZ (to whom the authors are indebted for many of the ideas introduced in this section).

We turn now to the last stage in the calculation of neutron penetration, namely the flux due to those neutrons which enter the shield with energies less than about 1 MeV. These neutrons will only be of importance in the inner regions of the shield, and we have already indicated how they may be calculated by means of age theory using equations (4.7.8) and (4.7.9). These require a knowledge of the source energy spectrum $F(u)$ per unit interval of u , which may not always be known. A reasonable approximation for neutrons emerging from a reflector is to assume a $1/E$ spectrum. This is the energy spectrum which is attained by neutrons slowing down in an infinite non-capturing medium—conditions which are satisfied approximately by a thick reflector. An example of such a spectrum is shown in Fig. 3.2.1. If the source per unit area per unit energy interval of E is S/E over the range $E_2 \leq E \leq E_1$, then $F(u)$ is constant and equal to S over the range $u_1 \leq u \leq u_2$, where u_1 corresponds with E_1 and u_2 with E_2 .

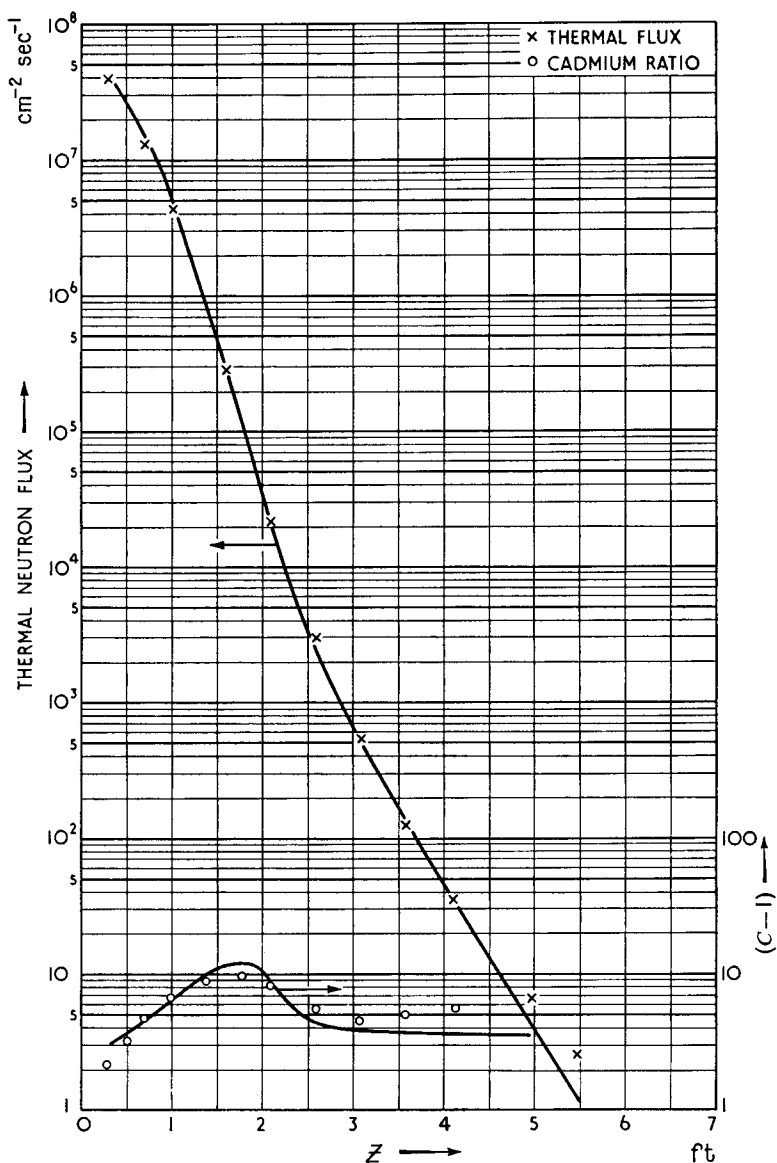


Fig. 4.9.1. Thermal flux in the Portland concrete shield of a natural uranium graphite moderated reactor, as a function of z , the distance from the inner face of the concrete. The full line is the result of the theoretical calculations referred to in the text. Also shown are the calculated and measured values of $(C - 1)$ for indium foils, where C is the cadmium ratio (defined on page 332).

Concrete density = 2.4 g cm^{-3} .

Hydrogen content = 0.63% by weight.

Calculated L^2 (2 MeV to thermal) = 137 cm^2 .

Thickness of graphite reflector = 3 feet.

Thickness of steel thermal shield = 6 inches.

When all the above stages have been carried out, the thermal neutron source strength can be evaluated by using the appropriate value of the age, and the thermal flux can, if necessary, be calculated from diffusion theory. If all that is required is the rate at which thermal neutrons are captured, it is usually adequate in a highly capturing medium to neglect the diffusion of neutrons after thermalization, and to equate the capture rate to the thermal neutron source strength at the same point.

Fig. 4.9.1 illustrates some further results of the semi-empirical method applied to the problem of neutron attenuation in Portland concrete. The experimental points shown are obtained from measurements of the thermal neutron flux (using the cadmium difference method with indium foils) in the concrete shield of a graphite moderated thermal reactor.

The theoretical curve for the thermal neutron flux was obtained from the fitting together of the results of two separate calculations. The first part of the curve (over the range from zero to two feet) was found by means of age theory, as described above, assuming a $1/E$ incident spectrum, and using the resulting distribution of resonance neutrons as a source function for a diffusion theory calculation of the thermal neutron flux.

The second part of the curve (for distances greater than three feet) is the theoretical curve already shown in Fig. 4.8.5, obtained from a theoretically determined $\Sigma_{\text{rem}}(E)$. To obtain the overall theoretical curve, the two contributions $A(z)$ and $B(z)$, say, were added together with a coupling constant λ so that the thermal flux $\phi(z) = A(z) + \lambda B(z)$, and λ was adjusted to give the best fit. This process of addition is physically reasonable since, as already mentioned, the observed thermal neutrons arise from two distinct sources; namely the intermediate energy neutrons below 1 MeV and the fast flux above 1 MeV. One could alternatively avoid the need for an arbitrary coupling constant by estimating the magnitude of the incident intermediate flux from reactor data, by using the removal cross-section technique to determine the absolute value of the flux of fast neutrons existing in the shield, and by subsequently proceeding as described above. This method, which would be unavoidable in a shielding calculation on a new type of reactor, is unlikely to give results of very high accuracy. Consequently, in preparing Fig. 4.9.1 we have retained the fitted coupling constant λ , in order to show more clearly how well this simple theory explains the observed thermal fluxes and cadmium ratios.

Although the preceding discussion shows that the mechanism of attenuation in hydrogenous media is now well understood, less is known about the behaviour of intermediate neutrons in non-hydrogenous media, especially those of medium and large atomic weight. However, this gap in our knowledge is of minor importance since there are very few applications in which it is absolutely essential to eliminate hydrogen from any large part of a fast neutron shield.

4.10. THE MOMENT METHOD

To conclude our discussion of the attenuation of neutrons by thick shields we shall refer briefly to the moment method. This type of calculation has already been mentioned in connection with gamma-ray penetration (page 43).

The same technique is also applicable to neutron attenuation, and the results of a number of calculations have been published.^(2, 3, 16, 17) The computational problem is more severe than in the gamma case, because of the rapid changes in neutron cross-sections with energy, and because neutrons undergo many

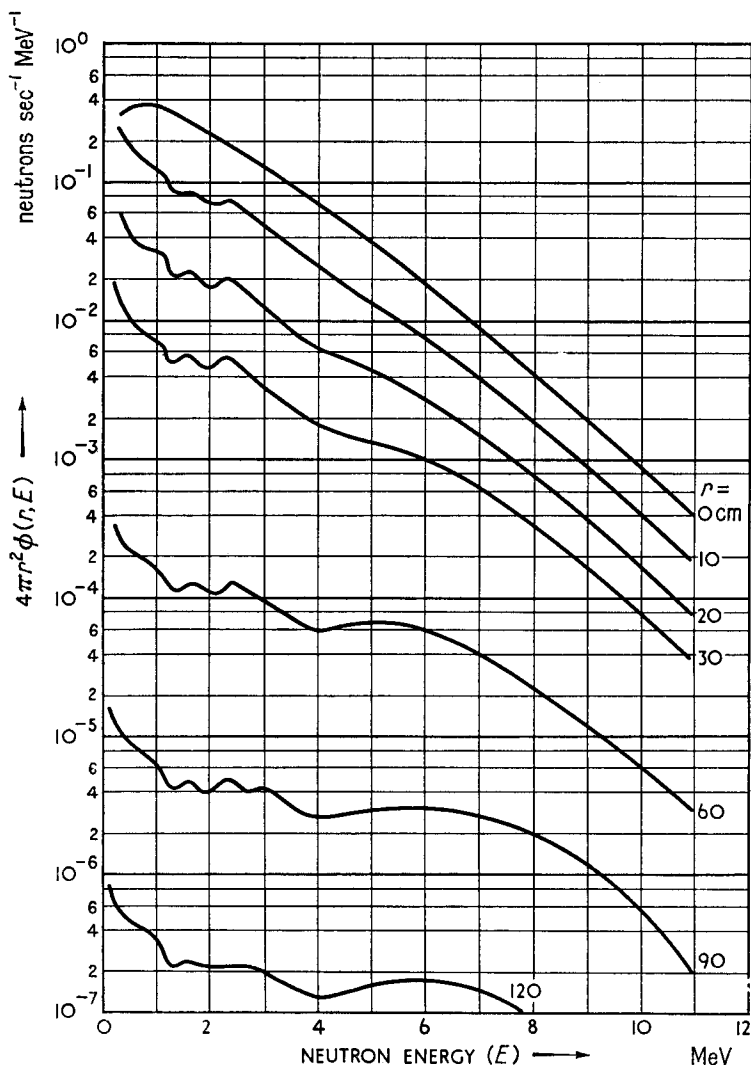


FIG. 4.10.1. The calculated energy spectrum of the neutron flux ϕ due to a point fission source in water, as calculated by the moment method.

The ordinate gives the absolute value of the differential spectrum at a distance r from a source emitting one neutron per second. Note the nearly constant (equilibrium) form of the spectrum beyond $r = 60$ cm. Compare with Fig. 6.8.1. (From ARONSON *et al.*⁽³⁾) Measured values are given in reference (36).

collisions before absorption; consequently the use of a high-speed computer with ample memory capacity is essential.

The main achievement of the moment method has been the determination of the fast neutron flux (in both energy and space) by a method which correctly

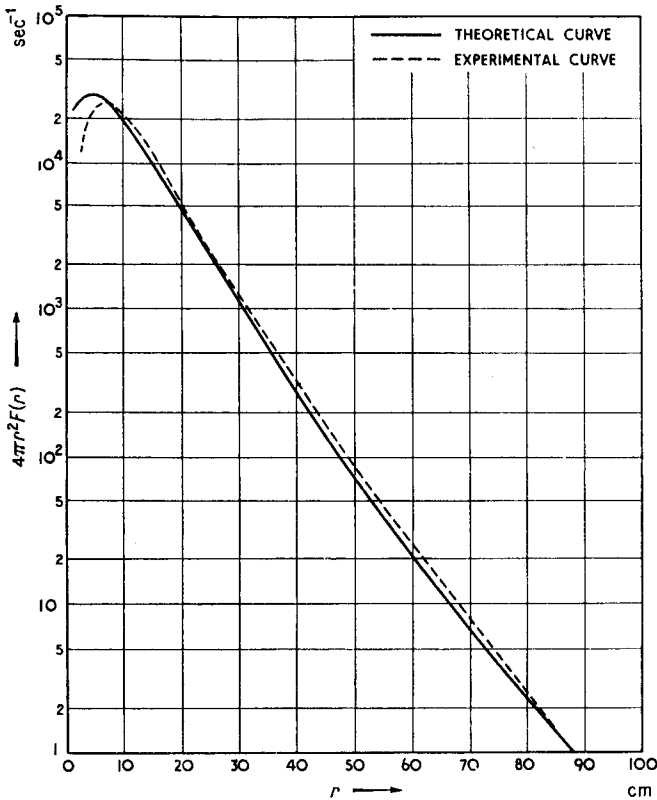


FIG. 4.10.2. Observed and calculated values of $F(r)$, the flux of indium resonance neutrons at a distance r from a point fission source in water. (CERTAINE and ARONSON.⁽¹⁶⁾)

takes account of the anisotropy of elastic scattering. Calculations are available for penetration into water^(3, 16) and for pure hydrogen.⁽¹⁷⁾ Fig. 4.10.1 shows results obtained for the energy spectrum of the neutron flux due to a point fission source in water at various distances from the source. The curves show the expected hardening of the spectrum which takes place as the distance increases.

Fig. 4.10.2 shows the distribution of indium resonance energy neutrons (1.44 eV) from a point fission source in water, as calculated by the moment method.⁽¹⁶⁾ There is good agreement with the experimental curve which is also shown.

MISCELLANEOUS TOPICS

4.11. THE ESCAPE OF NEUTRONS FROM A SURFACE

There are two features of neutron escape from a surface which are of interest in shielding: the angular distribution of the emergent current due to sources inside the medium, and the fraction of neutrons reflected back from a surface on which neutrons are falling.

Angular distribution of emergent current. Only thermal neutrons will be considered in this connection, since the problem of fast neutrons cannot be dealt with satisfactorily in any simple manner.

If thermal neutrons are diffusing in a medium which is finite in extent, some will eventually find their way to the surface and escape. A neutron which suffers a collision close to the boundary will have a maximum possible chance of escape if its direction after the collision is along the outward normal to the surface, and consequently the angular distribution of the escaping neutrons is peaked in this direction. For the simplified case of thermal neutrons diffusing in a non-capturing and isotropically scattering medium, the emergent angular distribution from a plane surface is given by FERMI's approximate formula⁽¹⁸⁾

$$f(\theta) = \frac{\cos \theta + \sqrt{3} \cos^2 \theta}{\pi \left[1 + \frac{2}{\sqrt{3}} \right]}, \quad (4.11.1)$$

where θ is the angle from the normal, and $f(\theta)$ is the number of neutrons emerging per unit area of surface per unit solid angle at angle θ . Thus $f(\theta)$ represents the angular distribution of neutron current. As given in equation (4.11.1) it is normalized to unit current leaving the plane per unit area; the function is shown in Fig. 4.11.1. An exact calculation of $f(\theta)$ by PLACZEK,⁽¹⁹⁾ using transport theory, is in such good agreement with FERMI's formula that the two distributions could not be distinguished if drawn on the same figure. PLACZEK's results, given in reference (19), refer to the angular distribution of the emergent neutron flux, and should be multiplied by $\cos \theta$ in order to obtain values proportional to the current $f(\theta)$. It should be noted that the commonly used approximation $f(\theta) \approx (1/\pi) \cos \theta$ overestimates f by a factor of $(1 + 2/\sqrt{3})$ for $\theta \approx \pi/2$; for $\theta \approx 0$ the approximate formula underestimates f by a factor 1.27.

Albedo. The term *albedo* is used in neutron physics to describe the reflective properties of a medium for neutrons. If the medium is bounded by a surface on which neutrons are incident, the albedo β is equal to the fraction of incident neutrons which are reflected. In the case of a semi-infinite non-capturing medium β must be unity, since any neutron will, if it diffuses for long enough, eventually recross the boundary. Conversely, in a purely capturing medium $\beta = 0$. The albedo therefore depends on the ratio, N , of the total to the capture cross-section: $N = \Sigma_t/\Sigma_c$. It must also depend on the angle of incidence, the angular distribution of scattering inside the medium and, in the case of non-thermal neutrons, on the slowing-down properties of the medium.

It was shown by FERMI⁽¹⁸⁾ that the albedo of an infinitely thick, isotropically scattering medium, bounded by a plane on which collimated thermal neutrons were incident at an angle θ (from the normal) is given approximately for large N by

$$\beta(\theta) = \frac{\sqrt{N} - 1}{\sqrt{N} + \sqrt{3} \cos \theta}. \quad (4.11.2)$$

More exact calculations have been carried out by HALPERN, LUENEURG, and CLARK,⁽²¹⁾ for three different types of incident angular distribution. For

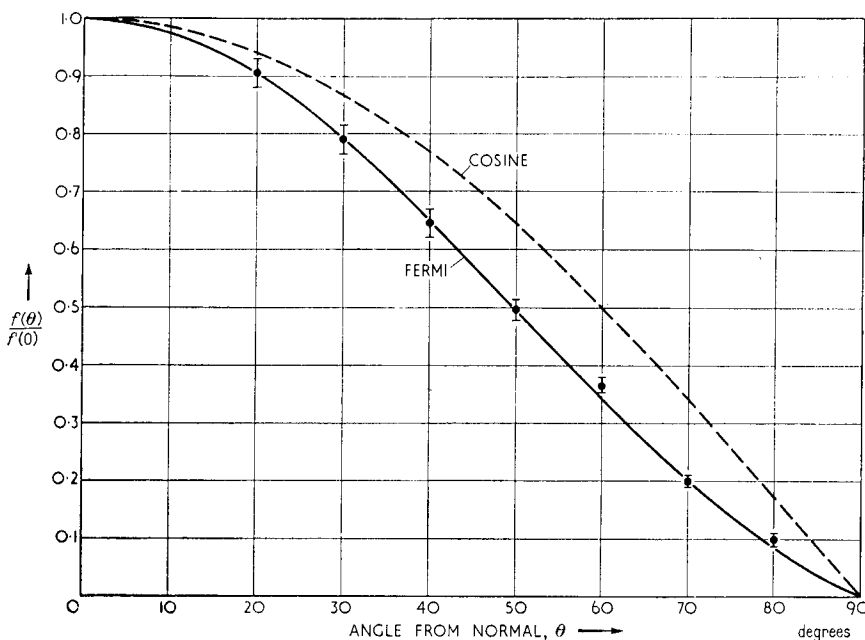


FIG 4.11.1. The angular distribution of the neutron current emerging from the plane surface of a half space (see text).

The solid line shows FERMI's function $f(\theta)$ in terms of the value of f at $\theta = 0$. The experimental points are due to HOFFMAN and LIVINGSTON.⁽²⁰⁾ The corresponding ratio for a cosine distribution is also shown for comparison.

large N their results differ from the simpler and more phenomenological treatment of FERMI by about 6 per cent in the coefficient of $\sqrt{N}$:

$$\left. \begin{array}{ll} \text{For normal incidence:} & \beta = 1 - 2.91 N^{-\frac{1}{2}} \\ \text{For a cosine distribution:} & \beta = 1 - 2.48 N^{-\frac{1}{2}} \\ \text{For a uniform distribution:} & \beta = 1 - 2.31 N^{-\frac{1}{2}} \end{array} \right\} \quad (4.11.3)$$

Albedo of a moderating medium. In the case of fast neutrons the albedo problem involves the determination not only of the fraction which re-emerges, but also the energy spectrum. If the assumption is made that only thermal

capture is possible, then the neutrons may be divided into three groups: (a) those that escape with energies above thermal, (b) those that first become thermal, and later escape by diffusion, and (c) those that are captured when thermal.

An expression may be obtained⁽²²⁾ for the energy spectrum and total fraction of those in group (a) by means of age theory, using a distributed source of fast neutrons given by the first collisions of the incident beam. It is convenient to use the following notation:

Σ_{rem} is the macroscopic removal cross-section for the fast neutrons entering the medium (as defined by (4.8.6));

$\lambda = \frac{2}{3} l_{\text{tr}}$, where l_{tr} is the transport mean free path for neutrons slowing down in the medium (assumed constant);

τ is the neutron age, as defined by the usual age formula (4.7.4);

$J(\tau) d\tau$ is the fraction of the incident neutrons which emerge with ages between τ and $\tau + d\tau$.

Case (i): *Parallel beam of incident neutrons entering a plane surface at an angle θ from the normal.*

Let

$$\alpha = \Sigma_{\text{rem}} \sec \theta.$$

Then

$$J(\tau) = \frac{\alpha}{(\alpha\lambda - 1)} \left[\alpha e^{\alpha^2\tau} \text{erfc}(\alpha\sqrt{\tau}) - \frac{1}{\lambda} e^{\tau/\lambda^2} \text{erfc}\left(\frac{\sqrt{\tau}}{\lambda}\right) \right], \quad (4.11.4)$$

where

$$\text{erfc}(x) = 1 - \text{erf}(x),$$

and

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

is the "normal error" function, tabulations of which are readily available. The fraction of the neutrons emerging with age less than τ_1 is given by $F(\tau_1)$, where

$$F(\tau_1) = \int_0^{\tau_1} J(\tau) d\tau = 1 - \frac{\alpha}{(\alpha\lambda - 1)} \left[\lambda e^{\tau_1/\lambda^2} \text{erfc}\left(\frac{\sqrt{\tau_1}}{\lambda}\right) - \frac{1}{\alpha} e^{\alpha^2\tau_1} \text{erfc}(\alpha\sqrt{\tau_1}) \right] \quad (4.11.5)$$

If τ_1 is given the value appropriate for neutrons just becoming thermal ($=L_s^2$) then $F(\tau_1)$ is the fraction of neutrons that emerge from the medium before being thermalized, that is, group (a) above. It may be noted that $F(\tau_1)$ tends to unity as τ_1 tends to infinity, as is to be expected on the assumption of no capture and continuous slowing down without thermalization.

Fig. 4.11.2 shows the dimensionless quantity $\lambda^2 J(\tau)$ plotted as a function of $\sqrt{\tau}/\lambda$ for various values of $\alpha\lambda$. Fig. 4.11.3 shows $F(\tau)$ as a function of $\sqrt{\tau}/\lambda$ for the same values of $\alpha\lambda$.

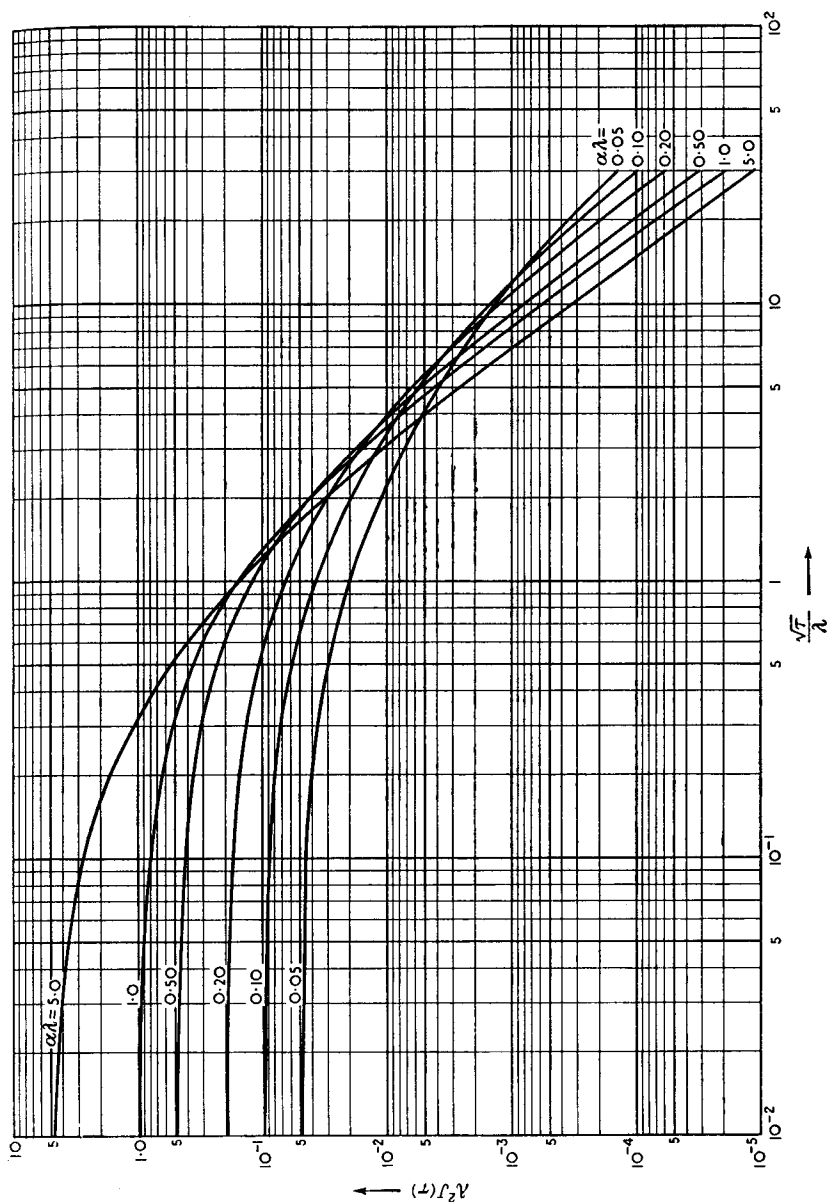


FIG. 4.11.2. Curves giving $J(\tau)$, the fraction of neutrons reflected by a moderating medium between ages τ and $\tau + d\tau$. The symbols α and λ are defined on the previous page.

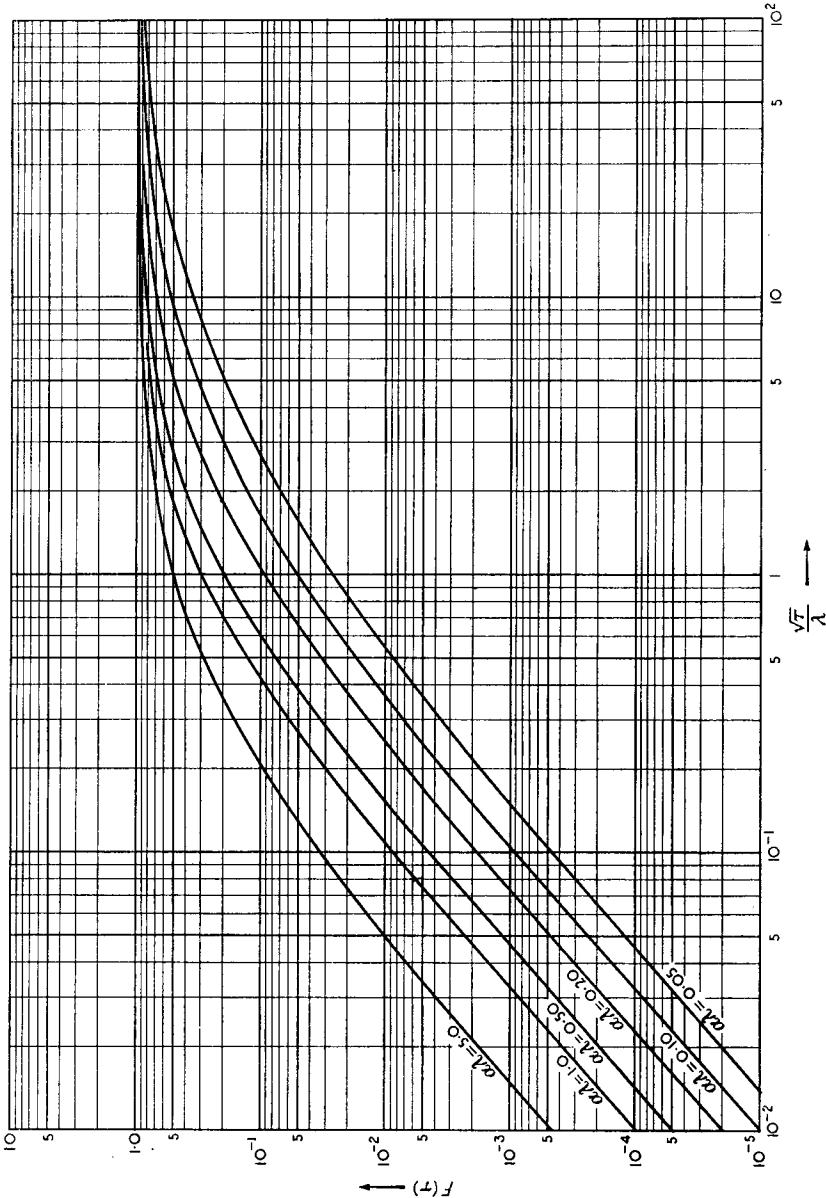


Fig. 4.11.3. Curves giving $F(\tau)$, the fraction of neutrons reflected by a moderating medium with age less than τ .

Case (ii). *Isotropic flux of neutrons incident on a plane surface.* The expression for $J(\tau)$ in this case cannot be given in a closed form, but may be written

$$J(\tau) = 2\Sigma_{\text{rem}} \int_1^\infty \frac{1}{s^2(1 - s\lambda\Sigma_{\text{rem}})} \left[\frac{1}{\lambda} \exp(\tau/\lambda^2) \operatorname{erfc}\left(\frac{\sqrt{\tau}}{\lambda}\right) - s\Sigma_{\text{rem}} \exp(\Sigma_{\text{rem}}s^2\tau) \operatorname{erfc}(\Sigma_{\text{rem}}s\sqrt{\tau}) \right] ds; \quad (4.11.6)$$

and, as before,

$$F(\tau_1) = \int_0^{\tau_1} J(\tau) d(\tau)$$

The exact determination of the fraction of neutrons in groups (b) and (c) is difficult. An approximate estimate may be obtained by means of diffusion theory, using a source distribution which falls off exponentially in the medium with a relaxation length equal to the slowing down length, and normalized to a total source strength S equal to $(1 - F(\tau_1))$, where τ_1 is the age of thermal neutrons. Then the fraction of neutrons emerging from a plane surface (group (b)) is given by

$$\beta_b = \frac{SL}{(L_s + L)(1 + 2D/L)},$$

where L is the thermal diffusion length in the medium,

D is the diffusion coefficient for thermal neutrons, and

L_s is the slowing down length. [$L_s^2 = \tau_1$]

The fraction of neutrons in group (c) is equal to $(S - \beta_b)$.

4.12. EFFECT OF DUCTS AND VOIDS IN SHIELDS

In practice, the shield of every reactor is pierced with a number of empty channels, which may vary in size from the quite tiny annular clearance gaps round "experimental plugs" to the large voids needed for the entry of the coolant of a gas-cooled reactor. For reasons which will be obvious from the preceding discussion, an exact calculation of the number of neutrons leaking down such a channel is extremely difficult. One is therefore left with the two alternatives of investigating each case of interest experimentally, or of using some approximate semi-empirical method of calculation. The former is naturally preferable if a vital point in the design of a reactor is involved; but for the more routine type of shielding calculation the information given below will usually give a sufficiently good indication. Most of the experimental information to be described has been obtained using thermal neutrons, which are easily observed, and which behave as a homogeneous group whose behaviour is simple to compare with theory. In what follows we shall make the implicit assumption that the intensity transmitted through the main body of the shield can be neglected in comparison with the intensity transmitted down the duct.

Leakage of Neutrons down Empty Straight Cylindrical Ducts

Consider first the duct shown in Fig. 4.12.1. If we assume initially that the walls of the duct are non-reflecting, that there is no appreciable leakage of neutrons into the duct from the walls and that the source is isotropic, then

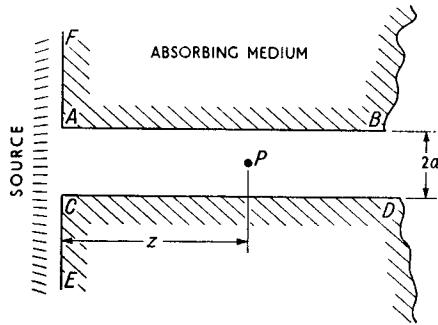


Fig. 4.12.1. Duct geometry. The source of neutrons is assumed to be uniform and of a strength such that a net current J' neutrons $\text{cm}^{-2} \text{sec}^{-1}$ enter the duct.

the flux of neutrons at the point P on the axis due to those neutrons which have streamed down the duct from the opening AC is

$$\begin{aligned}\phi &= \frac{J'}{2} \ln \left(1 + \left(\frac{a}{z} \right)^2 \right) \\ &\approx J' \frac{a^2}{2z^2},\end{aligned}\tag{4.12.1a}$$

for $a \ll z$.

Note that equation (4.12.1a) does not have any application for very small z/a , owing to the idealized assumptions regarding the source that are implicit in its derivation.

If the incident current entering the duct has a cosine distribution (which, as already explained, approximates to the practical case) then the flux at P is given by

$$\begin{aligned}\phi &= 2J' \left[1 - \left(1 + \left(\frac{a}{z} \right)^2 \right)^{-1/2} \right] \\ &\approx J' \frac{a^2}{z^2}\end{aligned}\tag{4.12.1b}$$

for $a \ll z$;

so that the approximate formula for large z/a is increased by a factor of 2 over the isotropic case. For a Fermi distribution (equation 4.11.1), which is more sharply peaked, the correction factor is 2.54. These scaling factors would apply to the whole of the following discussion regarding straight ducts (including annuli).

We notice that for a cosine source and non-reflecting walls the ratio of $\phi(z)$ to the flux at the entrance to the duct $\phi(0)$ is given by

$$\frac{\phi(z)}{\phi(0)} = \left[1 - \left(1 + \left(\frac{a}{z} \right)^2 \right)^{-1/2} \right] \quad (4.12.1c)$$

$$\approx \frac{a^2}{2z^2} \quad \text{for } a \ll z \quad (4.12.1d)$$

The ratio $\phi(z)/\phi(0)$ roughly corresponds to the quantity measured in the actual experiments described below.

For completeness, we also give expressions for the current at P , which is directed along the axis of the duct. For an isotropic source it is

$$j = J' \left[1 - \left(1 + \left(\frac{a}{z} \right)^2 \right)^{-1/2} \right], \quad (4.12.1e)$$

and, for a cosine source,

$$j = J' \left[1 - \left(1 + \left(\frac{a}{z} \right)^2 \right)^{-1} \right]. \quad (4.12.1f)$$

Following SIMON and CLIFFORD,⁽²⁴⁾ the effect of scattering from the wall of the duct may be evaluated in terms of its albedo β (see Section 4.11). In order to simplify the calculation it is necessary to make some assumptions regarding the angular distribution of the neutrons scattered back into the duct. If the scattered neutrons are isotropically distributed the flux is given by

$$\phi_{\text{isot}}(z) = \phi[\beta/(1 - \beta) + 1], \quad (4.12.2)$$

where ϕ is as in (4.12.1). If the scattered neutrons have a distribution varying as the cosine of the angle from the normal to the duct walls, and provided z is not too small, then the flux $\phi_{\text{cos}}(z)$ is

$$\phi_{\text{cos}}(z) = \phi[1 + (4a\beta/z(1 - \beta))]. \quad (4.12.3)$$

In the general case of a scattering law which is intermediate between an isotropic and a cosine-law distribution the flux $\phi_{\text{gen}}(z)$ is given by

$$\phi_{\text{gen}}(z) = B\phi_{\text{isot}}(z) + (1 - B)\phi_{\text{cos}}(z), \quad (4.12.4)$$

where B is an arbitrary constant. SIMON and CLIFFORD show that neutrons which re-enter the duct after being once scattered in the walls give rise to a variation of intensity along the duct similar to that expected on the cosine assumption.

The observed variation of flux with distance (Fig. 4.12.2) indicates that for thermal neutrons the distribution of radiation reflected from the walls is probably nearer cosine-law than isotropic. On the basis of isotropic scattering the intensity would have been expected to be proportional to z^{-2} for all z greater than about $3a$. On the other hand (and in agreement with experiment), the cosine law of reflection would lead to a z^{-3} dependence for moderate z/a (where reflection from the walls is important), and a z^{-2} dependence for large z/a . The region in which the changeover from inverse cube to inverse square

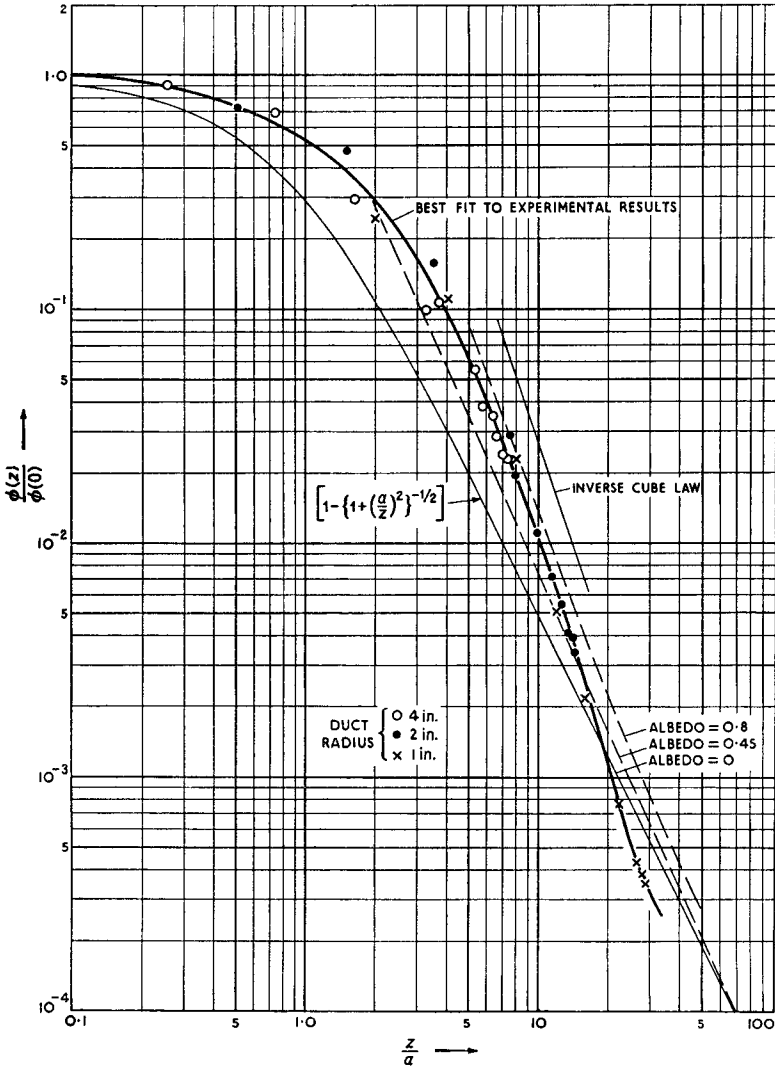


Fig. 4.12.2. Variation of thermal neutron flux along the axis of a steel-walled cylindrical air duct through a water shield, when one end of the duct is in contact with an extended uniform plane source of thermal neutrons.

The abscissa is the distance along the duct from the source, measured in units of the duct radius. Also shown are curves of $\phi(z)/\phi(0)$, where $\phi(0)$ is calculated from equation (4.12.1b) (i.e. neglecting any backscattering from the duct walls) and $\phi(z)$ is calculated from equation (4.12.3), assuming a cosine source and values of the albedo equal to zero, 0.45, and 0.8; and a line proportional to $(a/z)^3$, but of arbitrary magnitude. The steel lining of the duct reduces the albedo to an estimated value of 0.55. (HORTON and HALLIDAY.⁽²⁹⁾)

variation would be expected to take place depends on the albedo of the walls. Roughly speaking, the transition occurs when $(z/a) \sim (4\beta/(1 - \beta))$. For the steel-lined water ducts to which Fig. 4.12.2 applies the albedo for thermal neutrons is estimated to be 0.55; so that $(4\beta/(1 - \beta))$ is about 5. It will be noticed that this latter quantity is sensitive to the exact value of β that is used, and is subject to considerable error owing to the difficulty of calculating the albedo for a composite duct wall. With this reservation, the flux in these ducts would be expected to fall off as z^{-2} only for $(z/a) \gg 5$. The experimental points given in the figure do not extend very far into the region in which the inverse square law should hold; but the approximately inverse cube dependence in the region around $z = 5$ to 10 is clearly shown.

Thermal neutrons. Fig. 4.12.2 shows the variation of the flux of thermal neutrons along a steel-lined duct passing through a water shield. The source of thermal neutrons in this case extended over the whole of the shield face, as shown in Fig. 4.12.1, and the measurement included the effect of neutrons incident on the surfaces *FA* and *CE* which entered the duct after transmission through the shield. This contribution to the total flux is expected to be fairly small for $(z/a) \ll 1$, and also for large z/a , but possibly not for intermediate values. Owing to uncertainties regarding the exact effect of the source conditions the measured attenuation given in Fig. 4.12.2 cannot be compared with the predicted attenuation to a high degree of accuracy; but with this limitation it appears that the observed attenuation at large distances approximates to the geometrical attenuation, as would follow from the cosine assumption (equation 4.12.3). Within the accuracy permitted by the observations it appears that this equation also remains satisfactory for (z/a) values down to 4; this will be appreciated by interpolating values for $\beta = 0.55$ (the calculated albedo of the duct walls) from the curves shown in Fig. 4.12.2. For $4 \leq z/a \leq 30$, therefore, one concludes that for shielding calculations the assumption of cosine-law reflection from the walls is an adequate working hypothesis; and that any departures from pure cosine scattering that may exist can be taken care of by a safety factor of the order of two.

Fast neutrons. Experimentally⁽²⁶⁾ it is found that the dependence of the intensity of fast neutrons on the duct length and diameter can be satisfactorily explained on the basis of line-of-sight analysis, with no allowance for scattering from the walls. Thus equations (4.12.1) adequately summarize the data for cylindrical ducts. However, it should be borne in mind that while the probability is small for a fast neutron to be scattered back into the duct with an energy which still places it in the fast category, the probability for it to return with some lower energy is nevertheless appreciable (see Section 4.11). Fast neutron (e.g. threshold) detectors give no information regarding these backscattered neutrons, but an estimate of their probable contribution to the biological dose could be made with the albedo formulae given in the previous section. These formulae apply to a plane surface, and (for geometrical reasons) overestimate the albedo at a concave surface such as the wall of a duct; consequently they can safely be used in this application. Just as for thermal neutrons, the contribution from these intermediate energy neutrons scattered from the wall is expected to become small for large z/a .

Straight Annular Ducts

Thermal neutrons. With a slight modification, SIMON and CLIFFORD's treatment of cylindrical ducts also describes the streaming of thermal neutrons down annular ducts of varying sizes, under conditions which are typical of those encountered in practice. (Fig. 4.12.3.) We denote the outer radius of the annulus by b and the inner radius by c . We have already noted (Fig. 4.11.1) that the angular distribution of neutrons emerging from a source such as a graphite reflector is not greatly different from a cosine distribution, and we shall make use of this approximation in the analysis which follows. By analogy with equations (4.12.1b) and (4.12.3) we would expect the flux at a point distant z from the mouth of the annulus at which the source is situated to be given approximately (for $z \gg (b - c)$) by

$$\begin{aligned}\phi_{\text{ann}}(z) &= J' \frac{(\text{effective source area})}{\pi z^2} [1 + F(z, b, c, \beta)], \\ &= J' \frac{\delta(b^2 - c^2)}{z^2} [1 + F(z, b, c, \beta)],\end{aligned}\quad (4.12.5a)$$

where J' has the same meaning as in equations (4.12.1); δ is the fraction of the total source area $\pi(b^2 - c^2)$ which is visible from the point P in the annulus (Fig. 4.12.3); and the function $F(z, b, c, \beta)$ expresses the contribution due to scattering from the walls. From the results already discussed in connection with cylindrical ducts, we would expect this function to become small compared with unity when z is much greater than the width of the annulus. It can be shown from simple geometrical considerations that, for a point distant R from the axis of the plug, δ is given by

$$\begin{aligned}\delta = \frac{b^2}{\pi(b^2 - c^2)} &\left\{ \left(1 - \frac{c^2}{b^2}\right) \cos^{-1} \left(\frac{c}{R}\right) + \cos^{-1} \left(\frac{c}{b}\right) \right. \\ &\left. - \frac{c}{b} \sqrt{1 - \frac{c^2}{b^2}} \right\} \quad (b \geq R \geq c)\end{aligned}\quad (4.12.6)$$

The results for cylindrical ducts also suggest that a fairly small error—probably not more than a factor of two—is introduced by writing $\phi(0) = 2J'$ for a cosine emitter (i.e. by neglecting the wall-scattering term in equation (4.12.3) at $z = 0$). If we assume that the same treatment is permissible in this case, then we may tentatively write

$$\begin{aligned}\frac{\phi_{\text{ann}}(z)}{\phi(0)} &= \left\{ \frac{(b^2 - c^2)}{2z^2} \right\} [1 + F(z, b, c, \beta)] \\ &\quad (z \gg (b - c))\end{aligned}\quad (4.12.5b)$$

There is a clear analogy between the first part of this equation and equation (4.12.1d), as will be seen by substituting $z/\sqrt{(b^2 - c^2)}$ for z/a , and replacing $\phi(z)/\phi(0)$ by $\phi_{\text{ann}}(z)/\phi(0)$. With these changes, we may expect similar attenuation laws to apply to both cylinders and annuli.

The validity of this treatment has been studied by HORTON, HALLIDAY, LAKEY, and HARRISON, who used annular ducts through a barytes concrete shield. As in the work on cylindrical ducts mentioned above, the measurements included the effect of neutrons incident on the surfaces QR , ST , UV (Fig. 4.12.3); however, in this case the disturbing effect of leakage from the sides of the duct was minimized by the presence of the steel thermal shield shown in the figure. It will be seen from Fig. 4.12.4 that a single curve expresses the whole of the experimental results, and that this curve is strikingly similar to

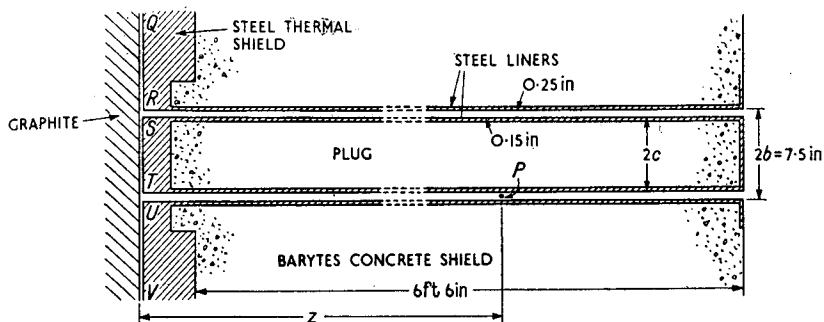


Fig. 4.12.3. Arrangement for annular duct experiment, the results of which are shown in the following figure. The plug was filled with barytes concrete.

the one obtained for cylindrical ducts. Although the materials used for the two experiments were not the same, it so happens that the albedos of the two duct walls are not very different (calculated values: ≈ 0.51 for the steel-lined barytes concrete in the annulus experiment, ≈ 0.55 for the steel-enclosed water for the cylindrical ducts). Thus from the similarity of the two results we may conclude that the dependence on z and β of the function F in equation (4.12.5) must be similar in form to the second term in equation (4.12.3).

Bent Cylindrical Ducts

We will now consider the behaviour of neutrons in bent cylindrical ducts, consisting of two or more straight sections of equal diameter inclined at angles θ_1 , θ_2 , etc. It is found experimentally⁽²⁹⁾ that the transmission of thermal neutrons round a bend, such as that shown in Fig. 4.12.5, is proportional to $\text{cosec } \theta$ (except for $\theta \approx 0$). The fluxes at the points Q , S are given to an accuracy sufficient for shielding purposes (~ 20 per cent) by the expressions:

$$\phi(z_2) = \frac{\phi_{AB} K a^2 \beta_{AB}}{2z_2^2} \text{cosec } \theta_1, \quad (4.12.7)$$

$$\phi(z_3) = \frac{\phi_{JK} K a^2 \beta_{JK}}{2z_3^2} \text{cosec } \theta_2, \quad (4.12.8)$$

where ϕ_{AB} , ϕ_{JK} are the mean values of the flux in the vicinities of the surfaces AB , JK ; β_{AB} and β_{JK} are the albedos of the duct walls in the same regions;

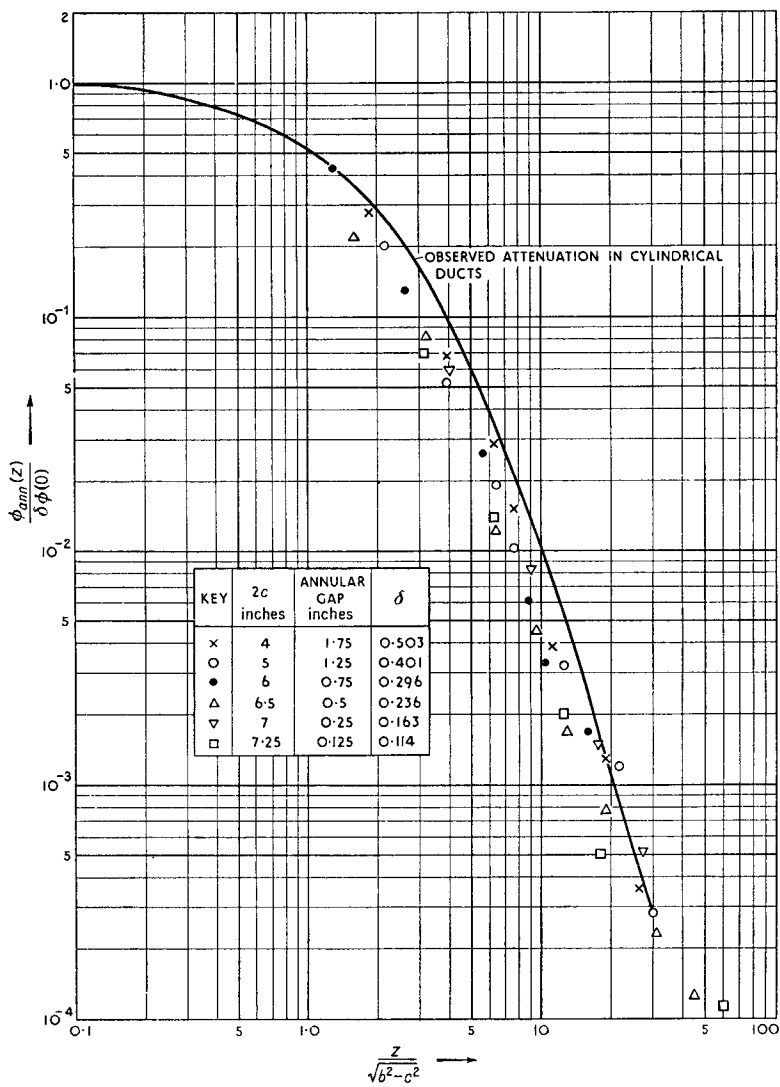


Fig. 4.12.4. Attenuation of thermal neutron flux in steel-lined annuli of inner diameter $2c$ and outer diameter $2b$ passing through a barytes concrete shield (as in Fig. 4.12.3).

The quantities z , δ , $\phi_{ann}(z)$, and $\phi(0)$ are defined in the text. For comparison with the results the attenuation in cylindrical ducts, as given by Fig. 4.12.2, is also shown. (HORTON, HALLIDAY, LAKEY, and HARRISON.⁽³⁰⁾)

the distances z_2, z_3 are measured as shown in Fig. 4.12.5; K is an empirical constant (discussed below); and a is the radius of the duct, which for simplicity is assumed to be constant throughout its length.

Thermal neutrons. In the course of an extensive series of thermal neutron measurements using air-filled steel ducts in a water tank (effective albedo ≈ 0.55) HORTON and HALLIDAY⁽²⁹⁾ have found empirically that the products $K\beta_{AB}, K\beta_{JK}$ in the above equations are both approximately equal to $\frac{1}{3}$. Their

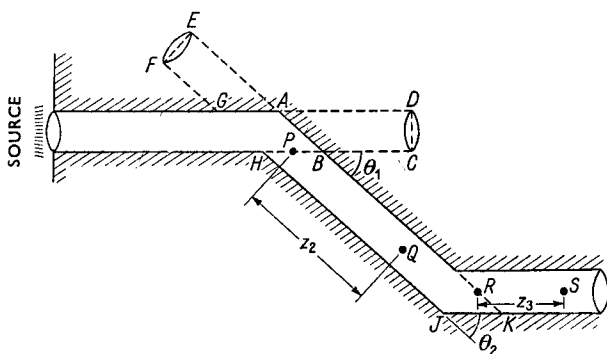


Fig. 4.12.5

results (Fig. 4.12.6) cover a range of duct sizes and double as well as single bends, and can all be interpreted in terms of a single value of K and the appropriate value of the albedo. Moreover, they find that the fall-off of flux with distance along the second and third legs of the duct follows very closely the same law as for straight cylindrical ducts, provided the distances z_2, z_3 are measured as shown in Fig. 4.12.5.

Fast neutrons. The information regarding fast neutrons is less extensive than for thermal neutrons. SHORE and SCHAMBERGER,⁽²⁶⁾ in a series of measurements on ducts through water, have shown that as the angle of the bend increases the number of scattered fast neutrons decreases more rapidly than $\text{cosec } \theta$, though for thermal neutrons their measurements confirm the cosecant dependence mentioned above. Thus, as far as shielding calculations are concerned, it appears as though the application of the thermal neutron formula to fast neutrons should result in an error which is on the safe side.

The same authors have also studied the relative effectiveness of the different portions of the region GAB (Fig. 4.12.5) in scattering fast neutrons into the second leg of the duct. Using a 45° bend, they replaced the water with air first in the volume $GFEA$ and then in the volume $ADCB$. Emptying the first volume did not appear to make very much difference to the number of scattered neutrons, but emptying the second volume reduced the fast neutron flux in the second leg by a factor of about three. Thus a moderate reduction in the neutron intensity can be obtained merely by extending the first leg for a short distance beyond its junction with the second leg.

Helical ducts. Little generalized information is available regarding the attenuation of neutrons by helical ducts, though it is known that ducts in which

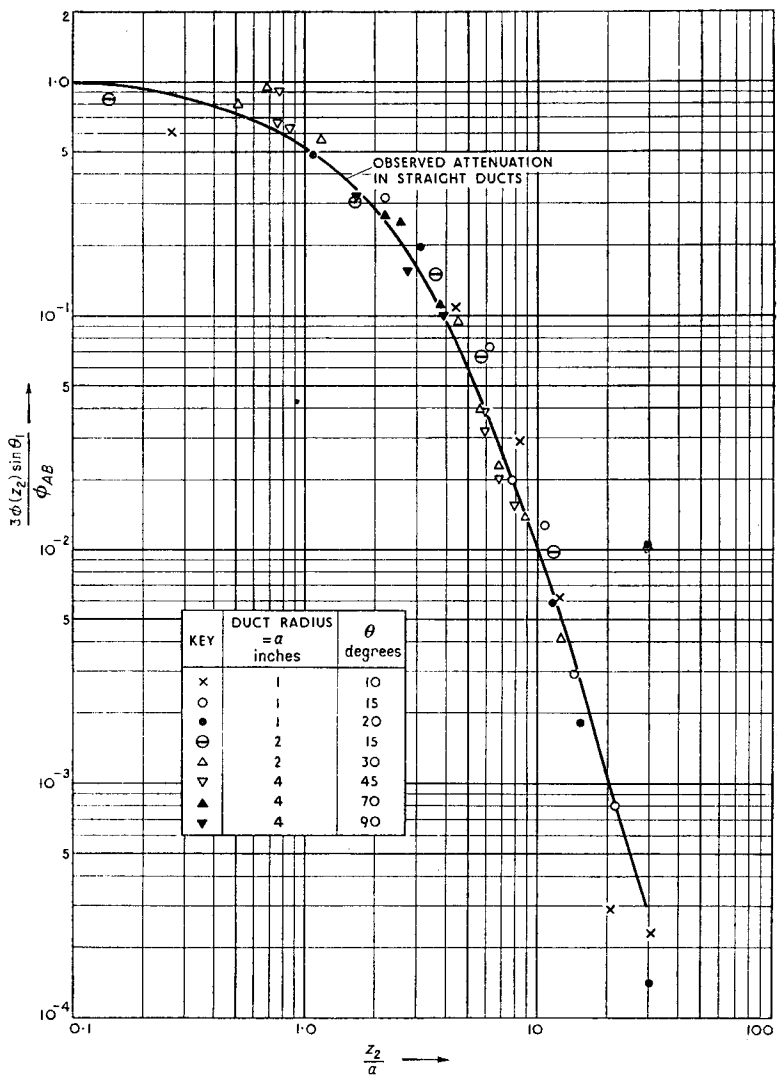


Fig. 4.12.6. Attenuation of thermal flux along the second leg of a bent cylindrical steel-lined duct in a water shield (effective albedo of walls ≈ 0.55). The factor 3 in the ordinate is the empirical value of the product $[K\beta_{AB}]^{-1}$. The distance z_2 and the angle θ_1 are defined as in the previous figure. Also shown for comparison is the curve of attenuation in straight ducts reproduced from Fig. 4.12.2. (HORTON and HALLIDAY,⁽²⁹⁾)

the “corkscrew” makes at least one-half of a turn, and in which the mean diameter of the corkscrew is appreciably larger than the diameter of the pipe, are characterized by extremely good attenuation properties.⁽³¹⁾ Consequently such ducts are widely used as a means of introducing samples into reactors during operation. A rough estimate of their performance can be made by treating them as equivalent to a series of straight sections of a length equal to the maximum chord which can be drawn inside the duct.

Stepped Holes

Those holes through reactor shields which must be closed when the reactor is operating are usually fitted with plugs of the stepped, or “gun-barrel,” type shown in Fig. 4.12.7. The neutrons streaming down the first clearance gap

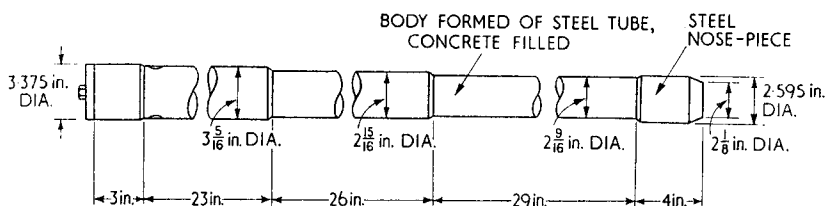


Fig. 4.12.7. Detail of a plug for a typical stepped hole through a reactor shield. All dimensions are in inches. The stepped hole liner into which the plug fits has a diameter larger by $\frac{1}{16}$ inch.

The example shown is from an early design. Current practice would favour a single central step of larger size, and an increased gap between the plug and the hole liner, in order to eliminate the need for accurate machining.

are scattered by the material forming the step, with the result that the intensity of radiation in the second gap is very much less than if there had been no step. The calculation of how much radiation is transmitted by such a stepped annular cavity is obviously a difficult matter. It is known empirically that the step need not be very large (say 0.5 inch), provided that the gaps round the plugs are less than about $\frac{1}{8}$ inch. Since dimensions of this kind are easily incorporated into a working design, there has been little incentive to place the calculation on a more reliable footing. However, it is worth noting in passing that it is considerably cheaper to construct a straight plug or hole-liner than one which is stepped; so that the general drive towards reducing construction costs of reactors will necessitate a careful examination of whether more than one step need be used, or indeed whether a step is necessary at all.

In an effort to acquire fundamental data on the behaviour of stepped ducts, SCHAMBERGER *et al.*⁽³²⁾ have studied the transmission of fast neutrons by a stepped pair of rectangular air slots through a water shield; the arrangement used is shown schematically in Fig. 4.12.8. The slot thickness t could be varied between 0.5 and 1.5 inches, while the dimension of the slots in the direction perpendicular to the plane of the paper was 34 inches, so that the problem was in some respects a two-dimensional one. For $(d/t) > 1$ the fast neutron intensity falls very rapidly with increasing d/t , and there is little advantage in having values of d/t much greater than 2. (See Fig. 4.12.9.) It was found

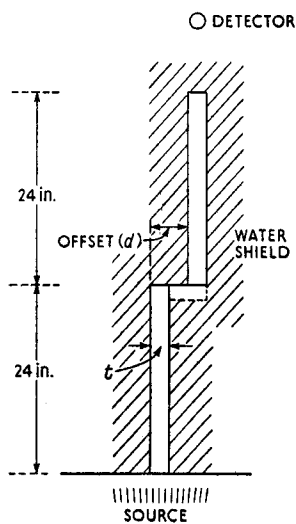


Fig. 4.12.8. Experimental arrangement used by SCHAMBERGER *et al.* to study the effect of stepped ducts.

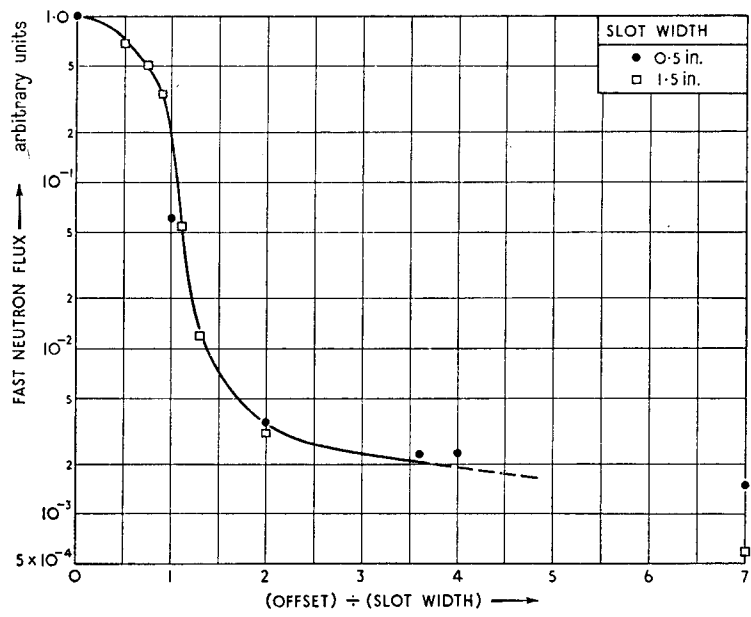


Fig. 4.12.9. Variation of the fast neutron intensity at the detector (see previous figure) with d/t . (SCHAMBERGER *et al.*⁽³²⁾)

that a small connecting channel between the two slots (as shown dotted in Fig. 4.12.8) made little difference to the transmitted fast neutron flux. The measurements also indicated that there would be very little advantage in having more than one step. If a single step is used, symmetry considerations show that the most efficient location is halfway along the duct, and SCHAMBERGER'S results confirm that the efficiency falls off quite rapidly as the step is moved from this central position. The work also gives a certain amount of empirical information regarding the thermal neutron fluxes associated with the streaming of fast neutrons down air-slots surrounded by water; such information should be of value for the design of some types of beam trap.

Corollaries

We may make some inferences from the above discussion. When ducts are filled with a material which is a poor fast neutron shield and which differs from the main material of the shield (e.g. the sodium coolant of a reactor), the fast neutron intensity transmitted by the duct should be given approximately by the intensity in an otherwise identical air-filled duct, multiplied by the transmission of the material filling the duct, as calculated using the removal cross-section technique.

Secondly, owing to the low albedo for gamma radiation, it is to be expected that geometrical attenuation formulae will be adequate for predictions of gamma intensity inside straight or annular ducts. However, caution should be used in oversimplifying the analysis for points outside the duct; in general a ray analysis, with allowance for "narrow-beam" attenuation by any interposed gamma-shielding material, does not reproduce at all satisfactorily the observed gamma-ray intensities in the vicinity of a duct.⁽¹²⁾ That is to say, multiple scattering must be taken into account.

A topic standing rather by itself is the reported "streaming" of neutrons down structural materials which happen to have a very low scattering cross-section at certain particular energies. Thus 25 keV neutrons have been found to stream through iron to a detectable extent,⁽³⁴⁾ though not when nickel is added to the iron (as in stainless steel). The explanation given for the difference in behaviour of the two materials is that the addition of the nickel fills up the cross-section "valley" exhibited by iron at this energy. The neutron energies at which this phenomenon has been observed are low enough for the subsequent moderation and absorption of the neutrons to present no difficulty.

Streaming of fast neutrons along straight steel beams immersed in a good moderator (such as water) is easily understood by analogy with a "narrow-beam geometry" attenuation experiment (page 32). A fast neutron which is scattered out of the steel into the water is rapidly degraded in energy and thermalized, and only "uncollided" neutrons make any appreciable contribution to the streaming current. Thus in such a case the factor which determines the magnitude of the streaming current must be the total cross-section for the particular neutron energy; the experimental results confirm this expectation. However, this simple method of estimating streaming would not apply to an extended

mass of steel (or any other medium of low moderating power) since build-up would then become important.

Effect of Random Distribution of Small Voids throughout Shield Volume

From the point of view of shielding theory it is interesting to consider the transmission of a shield consisting of a random collection of lumps of shielding material enclosed in some kind of containing vessel, with the space between the lumps filled with air or some other material of low density (such as a coolant gas). Such an arrangement, which is known as a "pebble shield," has the advantage of allowing coolant to penetrate the shield volume, without any great loss of shielding efficiency. Although not common, it is used in the reflector region of the MTR reactor (Fig. 6.6.6).

In the crudest approximation the transmission would be expected to be roughly that of a shield of the same average density as the pebble shield. However, on detailed examination it is found that statistical variations in the paths taken by the radiation through the solid material of the shield cause the transmission of the shield to be actually somewhat greater. SMITH⁽³⁸⁾ has considered the case of radiation which travels in straight lines and which is exponentially attenuated. His analysis should therefore apply to fast neutrons and to hard gamma rays. He finds that the ratio f of the thickness actually necessary for a given reduction in intensity to the thickness expected on the basis of the average density approximation is

$$f = 1 + 0.5Sv^2, \quad (4.12.9)$$

where v is the fraction of the shield volume occupied by voids, and S is the average distance between voids, in units of the relaxation length for the radiation in the material of the pebbles. The quantity f is called the "channelling effect factor." A similar problem has also been studied by KOUTS.⁽³³⁾

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Figs. 4.10.1 and 4.10.2 are reproduced by courtesy of U.S.A.E.C.

CHAPTER FIVE

USEFUL MATHEMATICAL FORMULAE

5.1. EXPONENTIAL INTEGRALS AND SECANT INTEGRAL FUNCTIONS

As we have explained in the previous chapters, the intensity of gamma rays at a distance r from an isotropic source of strength S photons sec^{-1} , situated in an extended absorbing medium, is equal to $Se^{-\mu r}/4\pi r^2$ photons $\text{cm}^{-2} \text{sec}^{-1}$, if build-up effects are neglected. These effects may be taken into account by using Taylor's approximation (equation 2.7.5), which amounts to an adjustment of the exponent μ . A similar combination of inverse square attenuation and exponential absorption also applies, under certain circumstances, to neutrons. Thus a class of problem of importance in shielding theory is the determination of the intensity of radiation due to various configurations of extended sources and of shields which attenuate the radiation in this exponential manner. Solutions of some of the simpler cases can be expressed in terms of standard exponential integrals and secant integral functions; the former occur in the solutions for the intensity from a plane source shielded by a plane slab shield, and the latter when the source is cylindrical. Definitions of these functions are given below, together with some of their principal properties.

Exponential Integrals (Figures 5.1.1 and 5.1.2)

Definition:

$$E_n(x) = \int_1^\infty \frac{e^{-xt}}{t^n} dt = x^{n-1} \int_x^\infty \frac{e^{-t}}{t^n} dt. \quad (\text{for } n \geq 0) \quad (5.1.1)$$

This family of functions has been described and tabulated by PLACZEK⁽¹⁾ and others.⁽²⁾ For our purposes the most important is $E_1(x)$; this function is also commonly written in the alternative notation adopted by JAHNKE and EMDE:⁽³⁾ $-Ei(-x)$.

$$E_1(x) \equiv -Ei(-x) = \int_x^\infty \frac{e^{-t}}{t} dt; \quad (5.1.2)$$

$$E_n(x) = Q_n(x)e^{-x} + \frac{(-x)^{n-1}}{(n-1)!} E_1(x), \quad (\text{for } n > 1) \quad (5.1.3)$$

with
$$Q_n(x) = \frac{1}{(n-1)!} \sum_{m=0}^{n-2} (n-m-2)!(-x)^m. \quad (5.1.4)$$

$E_1(x)$ is shown in Fig. 5.1.1 and $E_n(x)$ for $6 \geq n \geq 2$ in Fig. 5.1.2. Useful properties of these functions are:

$$E_n(x) = \int_x^\infty E_{n-1}(x') dx', \quad (5.1.5)$$

$$\frac{dE_n(x)}{dx} = -E_{n-1}(x) \left[\text{whence } \frac{d}{dx} (E_1(x)) = \frac{-e^{-x}}{x} \right], \quad (5.1.6)$$

$$E_n(x) = \frac{1}{(n-1)} (e^{-x} - xE_{n-1}(x)) \quad (\text{for } n > 1). \quad (5.1.7)$$

For small x the following expansion is useful:

$$E_n(x) = \sum_{\substack{m=0 \\ m \neq (n-1)}}^{\infty} \frac{(-x)^m}{m!(n-1-m)} + (-1)^n \frac{x^{n-1}}{(n-1)!} (\ln gx - A_n), \quad (5.1.8) \\ (n > 0)$$

where

$$g = 1.7811 \dots \quad \text{and} \quad A_1 = 0, \quad A_n = \sum_{m=1}^{n-1} \frac{1}{m}. \quad (n > 1).$$

In particular

$$E_1(x) = -\ln(gx) + x - \frac{x^2}{2.2!} + \frac{x^3}{3.3!} - \dots, \quad (5.1.9)$$

$$E_2(x) = 1 + x \ln(gx) - x - \frac{x^2}{2!} + \frac{x^3}{2.3!} - \dots \quad (5.1.10)$$

For large x we have the asymptotic expansion

$$E_n(x) = \frac{e^{-x}}{(x+n)} \left[1 + \frac{n}{(x+n)^2} + \frac{n(n-2x)}{(x+n)^4} + \frac{n(6x^2 - 8nx + n^2)}{(x+n)^6} \dots \right]. \quad (5.1.11) \\ (n > 0)$$

In all the formulae given above it is assumed that x is real and positive and n a positive integer. The function $-E_1(-x)$ is also useful, and we therefore give this a special notation, $\bar{E}_1(x)$.

Thus

$$\bar{E}_1(x) \equiv -E_1(-x) \equiv \bar{E}i(x) = \int_{-\infty}^x \frac{e^t}{t} dt. \quad (5.1.12)$$

This function is shown in Fig. 5.1.1. For small x

$$\bar{E}_1(x) = \ln(gx) + x + \frac{x^2}{2.2!} + \frac{x^3}{3.3!} + \dots, \quad (5.1.13)$$

and for large x there is the asymptotic expansion

$$\bar{E}_1(x) = \frac{e^x}{(x-1)} \left[1 + \frac{1}{(x-1)^2} + \frac{(2x+1)}{(x-1)^4} + \frac{(6x^2+8x+1)}{(x-1)^6} + \dots \right]. \quad (5.1.14)$$

The Secant Integral Function (Figures 5.1.1 and 5.1.3)

Definition:

$$\sec i(x, \psi) = \int_0^\psi e^{-x \sec \theta} d\theta. \quad (5.1.15)$$

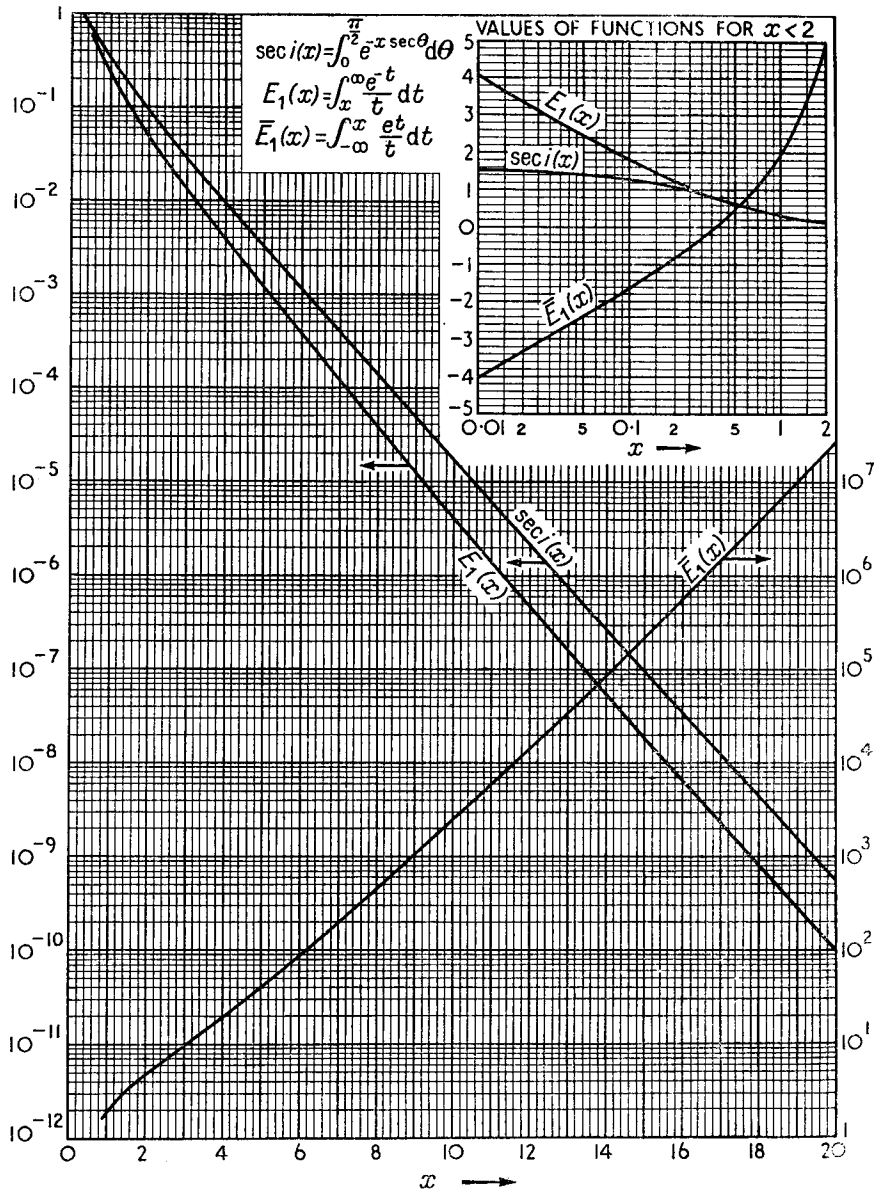


Fig. 5.1.1. Exponential integrals and the secant integral function $\sec i(x)$.
(Reproduced from WENDE,⁽⁴⁾ with permission)

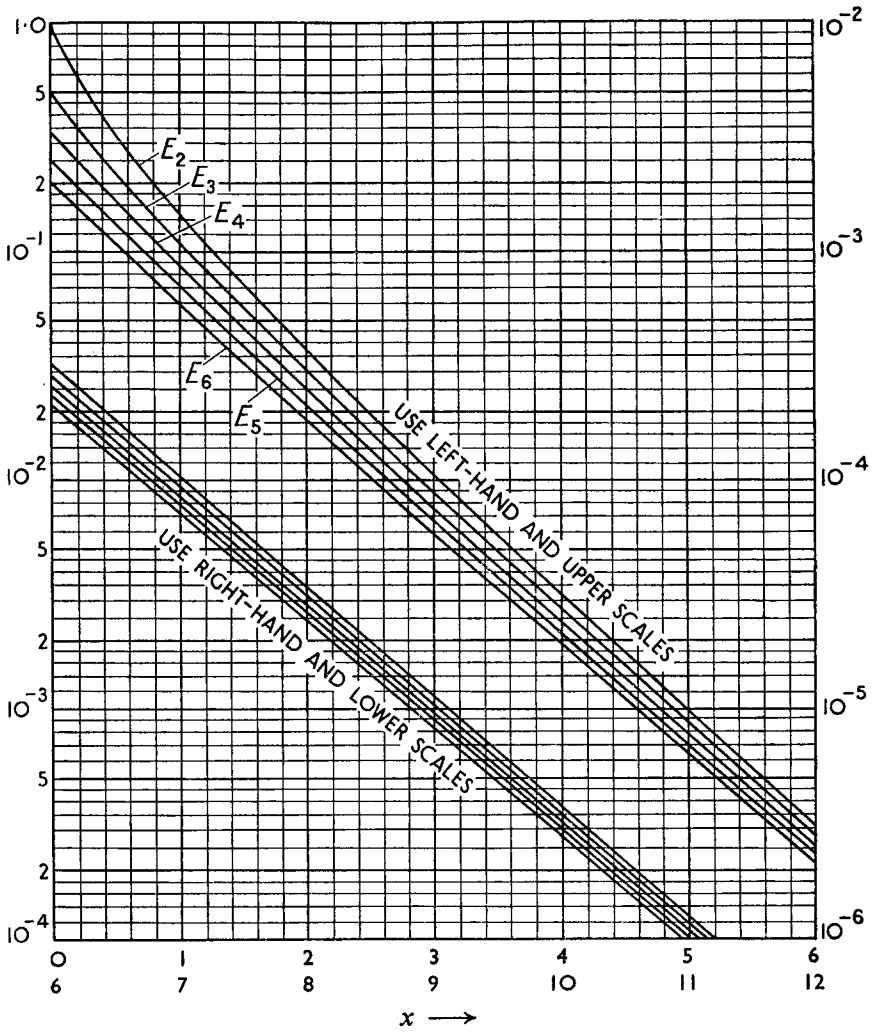


Fig. 5.1.2. Exponential integrals (equation 5.1.1) (PLACZEK⁽¹⁾).

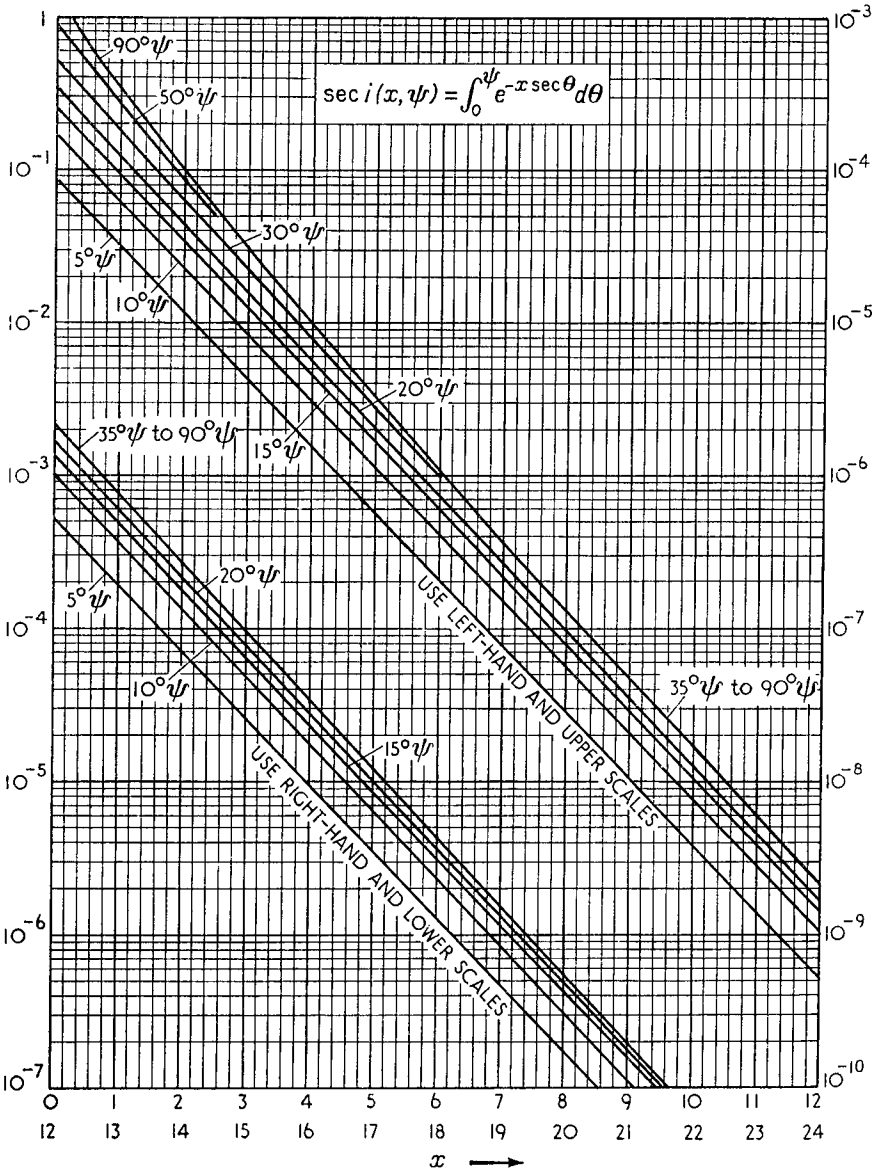


Fig. 5.1.3. Secant integral functions $\sec i(x, \psi)$ (equation 5.1.15)

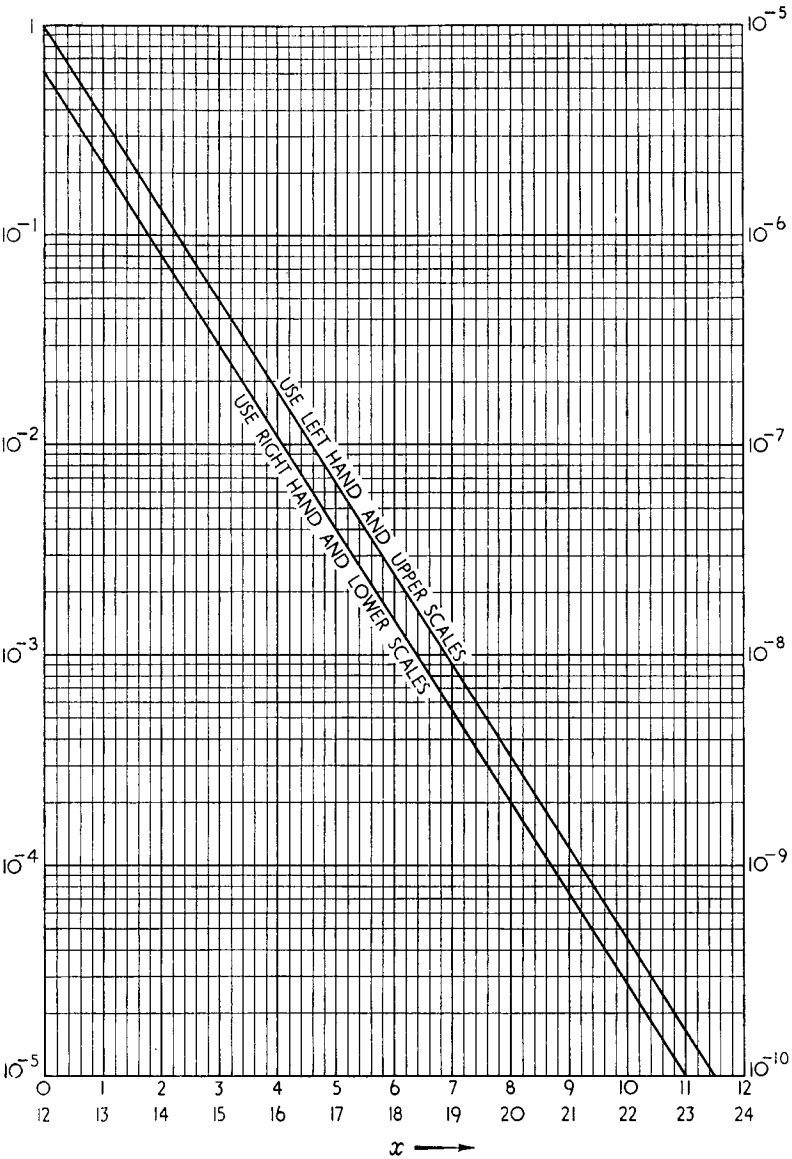


Fig. 5.1.4. The function e^{-x} .

For the special case of $\psi = \frac{\pi}{2}$, the notation $\sec i(x)$ is used, i.e.

$$\sec i(x) = \int_0^{\pi/2} e^{-x \sec \theta} d\theta. \quad (5.1.16)$$

Curves of $\sec i(x, \psi)$ against x are given in Fig. 5.1.3 for various values of ψ .

For small x

$$\sec i(x) = \frac{\pi}{2} + \left(\ln \left(\frac{x}{2} \right) \right) \left(x + \frac{x^3}{12} + O(x^5) \right) - 0.4228x + O(x^3). \quad (5.1.17)$$

For large x

$$\sec i(x) = \left[\frac{\pi}{2x} \right]^{1/2} e^{-x} \left(1 - \frac{5}{8x} + \frac{129}{128x^2} - \dots \right). \quad (5.1.18)$$

5.2. SOLUTIONS FOR STANDARD CASES INVOLVING PLANE AND SLAB SOURCES†

In the following pages we shall derive expressions for the flux and current of gamma rays resulting from various configurations of source and shield (build-up will be neglected). The current j across a given surface is defined as the quantity of radiation (number of quanta) crossing unit area of the surface per unit time; it is a vector quantity directed along the normal to the surface and is usually measured in units of $\text{cm}^{-2} \text{sec}^{-1}$. Calculations of the total heat energy carried into the shield by gamma radiation require a knowledge of the net current into the shield.

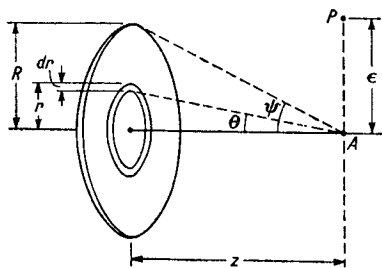


Fig. 5.2.1.

The flux ϕ at a given point is the quantity of radiation passing per unit time (irrespective of direction) through a sphere of unit cross-sectional area centred at the point; it is the quantity which determines the heat liberated per unit volume and the biological dosage at the point in question, and is also measured in units of $\text{cm}^{-2} \text{sec}^{-1}$. In the special case of collimated radiation, the current through a surface placed perpendicular to the beam is numerically equal to the flux.

(a) Current and Flux on the Axis of an Unshielded Isotropic Disc Source

Consider a plane disc of radius R emitting isotropically (into the whole of the 4π solid angle) S photons per second per cm^2 of surface (Fig. 5.2.1). The disc is assumed to be thin compared with the mean free path (in the disc) of the emitted radiation. These conditions will be satisfied in practice, for example, by any activity deposited on the floor of an empty cylindrical storage tank for radioactive effluent.

† More extensive compilations will be found in refs. (4) and (8).

At a point A , on the axis of the disc and distant z from it, the flux is

$$\phi = S \int_0^R \frac{2\pi r \, dr}{4\pi(r^2 + z^2)} = \frac{S}{2} \int_0^\psi \tan \theta \, d\theta = \frac{S}{2} \ln (\sec \psi), \quad (5.2.1)$$

where ψ is half the angle subtended at A by the disc.

Alternatively this may be written

$$\phi = \frac{S}{4} \ln \left(1 + \frac{R^2}{z^2} \right). \quad (5.2.2)$$

The current at A through unit area normal to the z direction is

$$\begin{aligned} j &= S \int_0^R \frac{2\pi r \cos \theta}{4\pi(r^2 + z^2)} \, dr = \frac{S}{2} \int_0^\psi \tan \theta \cos \theta \, d\theta \\ &= \frac{S}{2} (1 - \cos \psi) = \frac{S}{2} \left(1 - \frac{z}{\sqrt{R^2 + z^2}} \right). \end{aligned} \quad (5.2.3)$$

As $\psi \rightarrow \frac{\pi}{2}$ the energy transported outwards across each unit area of the plane through A approaches half the energy output of the source per unit area, a result which is physically obvious.

(b) *Flux and Current Due to a Plane Isotropic Thin Disc Source Embedded in an Infinite Absorbing Medium*

Using the same notation as before, the flux on the axis at A is now

$$\phi = S \int_0^R \frac{2\pi r}{4\pi(r^2 + z^2)} e^{-\mu\sqrt{r^2 + z^2}} \, dr,$$

where μ is the linear absorption coefficient for the radiation in the absorber, defined in Section 2.2. By writing $t = \mu\sqrt{r^2 + z^2}$, and noting that $\mu^2 r \, dr = t \, dt$, we have

$$\begin{aligned} \phi &= \frac{S}{2} \int_{\mu z}^{\mu z \sec \psi} \frac{e^{-t}}{t} \, dt \\ &= \frac{S}{2} (E_1(\mu z) - E_1(\mu z \sec \psi)). \end{aligned} \quad (5.2.4)$$

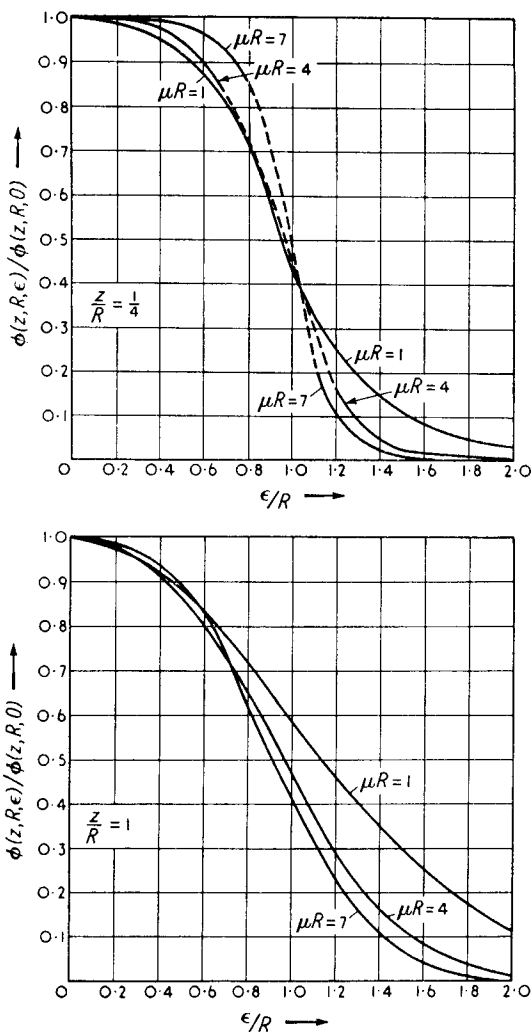
The current at A is given by

$$\begin{aligned} j &= \frac{S}{2} \int_{\mu z}^{\mu z \sec \psi} \mu z \frac{e^{-t}}{t^2} \, dt \\ &= \frac{S}{2} (E_2(\mu z) - \cos \psi E_2(\mu z \sec \psi)). \end{aligned} \quad (5.2.5)$$

Simple solutions are also obtained for non-isotropic sources whose emission per unit solid angle in the direction θ is proportional to $\cos^n \theta$, where n is a positive integer. The case $n = 1$ (cosine emitter) involves the function E_2 in the expression for the flux and E_3 in that for the current; in general an exponent n is associated with solutions in terms of E_{n+1} and E_{n+2} respectively.

For points not on the axis of the disc the solutions cannot be expressed in terms of tabulated functions. However, SMITH and STORM⁽⁶⁾ have published results obtained from an expansion in series, and these are shown in Fig. 5.2.2. The flux $\phi(z, R, \epsilon)$ at the point P (Fig. 5.2.1) is given in terms of the flux at $A = \phi(z, R, 0)$, which in turn may be obtained from equation (5.2.4).

The curves show the trends which one would expect from physical considerations. When z/R and ϵ/R are both small, the flux at P does not depend sensitively on the exact value of ϵ , but as ϵ increases a sudden drop occurs when the edge of the disc is approached. Increasing z/R removes the "plateau" at small values of ϵ , and spreads out the region over which the edge effects are felt.



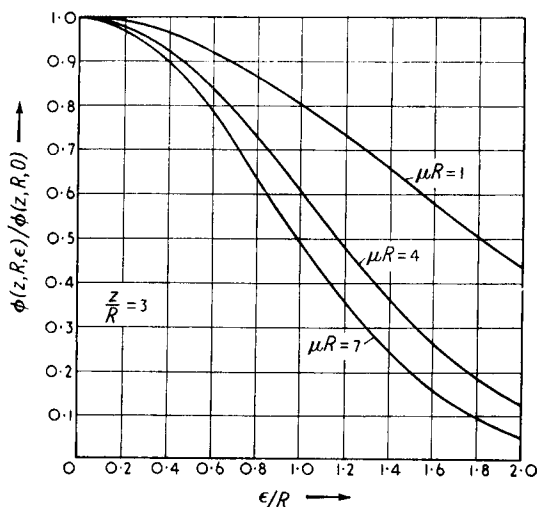


Fig. 5.2.2. Flux at the point P (Fig. 5.2.1) when the disc source is embedded in an infinite shielding medium (see text) (SMITH and STORM⁽⁶⁾).

(c) *Infinite Thin Plane Isotropic Source in an Absorbing Medium*

If we make R/z tend to infinity in equations (5.2.4) and (5.2.5) (i.e. $\psi \rightarrow \frac{\pi}{2}$) we obtain the solution for an infinite plane. The flux is

$$\phi = \frac{S}{2} E_1(\mu z) \quad (5.2.6)$$

and the current

$$j = \frac{S}{2} E_2(\mu z). \quad (5.2.7)$$

When the point A is many mean free paths from the plane ($\mu z \gg 1$),

$$j \simeq \phi \simeq \frac{S}{2} \frac{e^{-\mu z}}{\mu z} \quad (\text{see equation 5.1.11}).$$

The approximate equality of current and flux means physically that under these conditions the flux at A is highly anisotropic, most of the uncollided photons being directed more or less along the direction of the axis.

When A is close to the plane,

$$\phi \simeq \frac{S}{2} \ln \left(\frac{0.561}{\mu z} \right)$$

and

$$j \simeq \frac{S}{2}.$$

The fact that ϕ tends to infinity as A is brought up to the source plane has no physical significance, though it sometimes causes misunderstanding. It is due

to the idealized assumption of an infinitely thin source plane of finite strength per unit area.

(d) *Fluxes and Currents at the Surface of an Absorbing Slab of Finite Thickness but of Infinite Lateral Extent which Contains a Uniform Distribution of Isotropic Sources throughout its Volume*

Let the medium be a slab of thickness t bounded on one side by the plane through CAB (Fig. 5.2.3) perpendicular to the z direction. Let the source

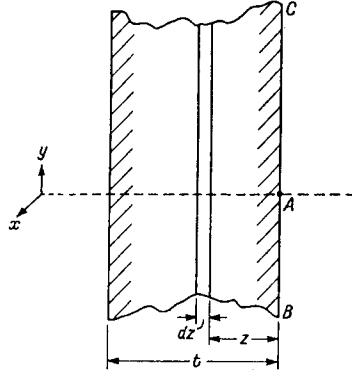


Fig. 5.2.3.

strength be S photons per unit volume per second. The flux at A due to the thin elementary slab of thickness dz shown in the diagram is, by equation (5.2.6), equal to

$$d\phi = \frac{S}{2} E_1(\mu z) dz.$$

The flux at A due to the whole slab is therefore

$$\begin{aligned} \phi &= \frac{S}{2} \int_0^t E_1(\mu z) dz = \frac{S}{2\mu} \int_0^{\mu t} E_1(u) du \\ &= \frac{S}{2\mu} (E_2(0) - E_2(\mu t)), \text{ from equation (5.1.5),} \\ &= \frac{S}{2\mu} (1 - E_2(\mu t)). \end{aligned} \quad (5.2.8)$$

Similarly it may be shown that the current across the CAB face is

$$\begin{aligned} j_z &= \frac{S}{2\mu} (E_3(0) - E_3(\mu t)) \\ &= \frac{S}{4\mu} (1 - 2E_3(\mu t)). \end{aligned} \quad (5.2.9)$$

It should be noted that in this particular case of an infinite slab the expressions for flux and current are constant everywhere outside the slab (assuming that the outer medium is non-absorbing). If another similar slab is introduced on the other side of A so that the point is embedded in the medium, considerations of symmetry show that ϕ is doubled and j_z is zero.

If the thickness t of the slab tends to infinity, ϕ and j_z tend to the limits

$$\phi = \frac{S}{2\mu}, \quad (5.2.10)$$

$$j_z = \frac{S}{4\mu} \quad (5.2.11)$$

in the case of a semi-infinite medium, and $\phi = \frac{S}{\mu}$ and $j_z = 0$ in the complete infinite medium. The result (5.2.10) should be committed to memory, because it is so frequently needed in calculations of biological dose. In rough calculations the difference between this expression and the exact value given by equation (5.2.8) can be ignored provided that $\mu t > 1$, since the difference between the two values is then less than 15 %.

Consider now the case of a point in an infinite absorbing medium which is distant z from a source slab of thickness t as already described. It may be shown that the flux is given by

$$\phi = \frac{S}{2\mu} (E_2(\mu z) - E_2(\mu[z + t])); \quad (5.2.12)$$

it will be seen that $\phi \rightarrow 0$ as $t \rightarrow 0$, as is physically obvious. If, in the case of a very thin source slab, we make use of formula (5.2.6), and wish to compare this with the exact expression (5.2.12) we must modify the source terms accordingly. Let there be source strength S_0 per unit area of slab; then the appropriate source per unit volume in equation (5.2.12) is S_0/t . Thus

$$\phi = \frac{S_0}{2\mu t} \left[(E_2(\mu z) - E_2(\mu(z + t))) \right],$$

which for small t may be expanded as

$$\frac{S_0}{2} \left(E_1(\mu z) - \frac{t}{2z} e^{-\mu z} + \frac{\mu t^2}{6z} e^{-\mu z} \left(1 + \frac{1}{\mu z} \right) - O(t^3) \dots \right).$$

Thus the expression (5.2.6) is a reasonable approximation provided that

$$\left| \frac{te^{-\mu z}}{2zE_1(\mu z)} \right| \ll 1.$$

For large z this implies $\mu t \ll 1$, and for small z

$$\left| \frac{t}{z \ln \mu z} \right| \ll 1.$$

(e) *Fluxes and Currents at the Surface of an Absorbing Slab of Finite Thickness but of Infinite Lateral Extent, which contains a Distribution of Isotropic Sources whose Strength varies Linearly with the Distance from one Bounding Surface*

The geometry is identical with that shown in Fig. 5.2.3, but the source strength is now assumed to be $S = a + bz$. The distribution of capture gamma-ray sources in the reflector of a reactor can in many cases be represented approximately by such a linear relation. (See, for instance, Fig. 6.2.4.)

The flux and current at the point A are equal to

$$\phi = \frac{1}{2\mu} \left(a + \frac{b}{2\mu} (1 - e^{-\mu}) - \left(a + \frac{bt}{2} \right) E_2(\mu t) \right), \quad (5.2.13)$$

$$j = \frac{1}{2\mu} \left(\frac{a}{2} + \frac{b}{3\mu} (1 - e^{-\mu}) - \left(a + \frac{2bt}{3} \right) E_3(\mu t) \right). \quad (5.2.14)$$

For a semi-infinite medium ($t \rightarrow \infty$) the solutions tend to the values

$$\phi = \frac{1}{2\mu} \left(a + \frac{b}{2\mu} \right), \quad (5.2.13a)$$

$$j = \frac{1}{2\mu} \left(\frac{a}{2} + \frac{b}{3\mu} \right). \quad (5.2.14a)$$

(f) *As for Case (e), but with a Source Strength Equal to $S_0 e^{kz}$*

This case arises when calculating the dose at the surface of a reactor shield due to capture gamma-rays, whose source distribution follows the approximately exponential attenuation of the neutrons. We must distinguish three cases: k greater than, equal to, or less than μ ; this last case includes the possibility of k being negative, so that the source strength can be allowed to increase or decrease with increasing distance into the shield as measured from the point A in Fig. 5.2.3; μ is, of course, an essentially positive number.

The solutions are given in the following table.

Table 5.2.1

	Flux	Current
$k > \mu$	$\frac{S_0}{2k} \left[e^{kt} E_1(\mu t) + \bar{E}_1((k - \mu)t) + \ln \left(\frac{\mu}{k - \mu} \right) \right]$	$\frac{S_0}{2k} \left\{ e^{kt} E_2(\mu t) - 1 + \frac{\mu}{k} \left(e^{kt} E_1(\mu t) + \bar{E}_1[(k - \mu)t] + \ln \left(\frac{\mu}{k - \mu} \right) \right) \right\}$
$k = \mu$	$\frac{S_0}{2\mu} (e^{\mu t} E_1(\mu t) + \ln(g\mu t))$	$\frac{S_0}{2\mu} \left\{ e^{\mu t} (1 - \mu t) E_1(\mu t) + \ln(g\mu t) \right\}$
$k < \mu$	$\frac{S_0}{2k} \left[e^{kt} E_1(\mu t) - E_1((\mu - k)t) + \ln \left(\frac{\mu}{\mu - k} \right) \right]$	$\frac{S_0}{2k} \left\{ e^{kt} E_2(\mu t) - 1 + \frac{\mu}{k} \left(e^{kt} E_1(\mu t) - E_1[(\mu - k)t] + \ln \left(\frac{\mu}{\mu - k} \right) \right) \right\}$

In the above table $g = 1.7811 \dots$

(g) *Flux and Current on Axis of Disc Source of Radius R Distant z from Point of Observation, the Source being Shielded by a Slab of Thickness $t < z$*

The flux at A (Fig. 5.2.4) is

$$\phi = \frac{S}{2} (E_1(\mu t) - E_1(\mu t \sec \psi)). \quad (5.2.15)$$

The current at the same point is

$$j = \frac{S}{2} (E_2(\mu t) - \cos \psi E_2(\mu t \sec \psi)). \quad (5.2.16)$$

Although these relations are formally identical with (5.2.4) and (5.2.5), note that we now have *two* variables, t and ψ , which are independent.

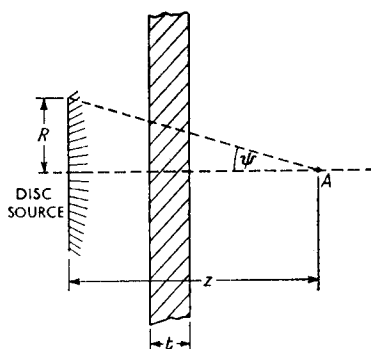


Fig. 5.2.4.

5.3. LINE SOURCES AND CASES WHICH CAN BE CONSIDERED AS SUPERPOSITIONS OF LINE SOURCES

(a) *Unshielded Line Source*

A long thin pipe containing radioactive material may be satisfactorily represented by a uniform isotropically-emitting line source, provided that the diameter of the pipe is much less than the mean free path of the radiation in the pipe, and provided that the diameter is also small compared with the distance from the pipe to the point of observation.

In the arrangement shown in Fig. 5.3.1 the flux at the point A due to the line source LM is

$$\phi = \frac{S}{4\pi h} (\psi_1 + \psi_2), \quad (5.3.1)$$

where h is the perpendicular distance from A to LM , S is the number of photons emitted per unit length of the line source per second, and ψ_1, ψ_2 are in radians. The current through A in the direction perpendicular to LM is

$$j = \frac{S}{4\pi h} (\sin \psi_1 + \sin \psi_2). \quad (5.3.2)$$

(b) *Line Source Shielded by a Slab of Thickness t (shown dotted in Fig. 5.3.1)*

The flux at A is

$$\phi = \frac{S}{4\pi h} (\sec i(\mu t, \psi_1) + \sec i(\mu t, \psi_2)), \quad (5.3.3)$$

where $\sec i(x, \psi)$ is the function shown in Fig. 5.1.3.

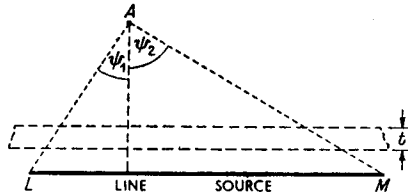


Fig. 5.3.1.

For an infinitely long source this becomes

$$\phi = \frac{S}{2\pi h} (\sec i(\mu t)), \quad (5.3.4)$$

where $\sec i(x)$ is the function shown in Fig. 5.1.1.

The current in the latter case may be written

$$j = \frac{S}{2\pi h} (\mu t K_1(\mu t) - \mu t \sec i(\mu t)), \quad (5.3.5)$$

where K_1 is the modified Bessel function of the second kind.

(c) *The Fluxes and Currents due to certain other Source Geometries*

These can in some cases be derived by regarding the source as an assembly of line sources. An obvious example is that of a thin rectangular source. Another case is that of the unshielded tubular source shown in Fig. 5.3.2.

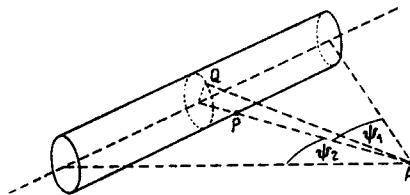


Fig. 5.3.2.

Let the radius of the tube be R . The flux at A due to an elementary strip which passes through Q and subtends an angle $d\theta$ at the axis is, by equation (5.3.1),

$$d\phi = \frac{S}{4\pi} \frac{(\psi_1 + \psi_2) R d\theta}{(\text{distance } AQ)},$$

where S is the source strength per unit area of the cylinder. The total flux due to the tube (which is assumed to be non-absorbing) is then

$$S(\psi_1 + \psi_2) \left(\frac{1}{4\pi} \int_0^{2\pi} \frac{R}{(AQ)} d\theta \right) = S(\psi_1 + \psi_2) I \left(\frac{h}{R} \right), \quad \text{say,} \quad (5.3.6)$$

where h is the perpendicular distance from A to the axis of the tube, and I is the function plotted in Fig. 5.3.3. For $(h/R) > 5$, less than 5 per cent error is made by assuming that all the emission is concentrated at the axis of the tube.

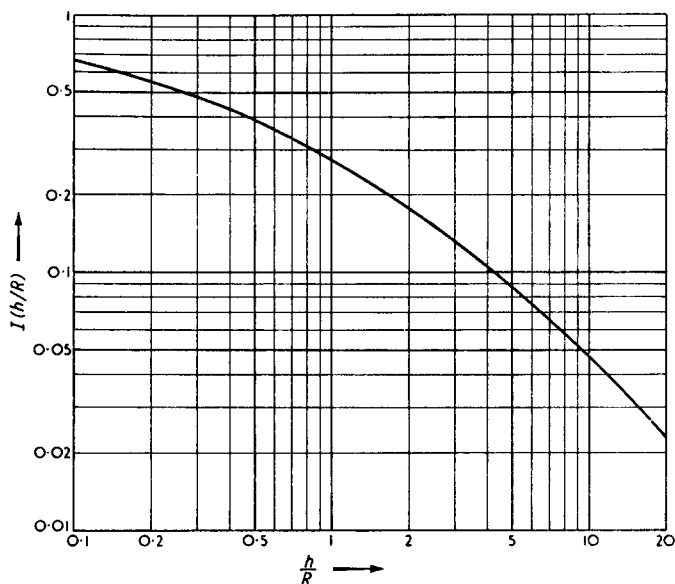


Fig. 5.3.3. The integral I determining the flux due to a tube source (equation 5.3.6) (ANDREWS⁽⁶⁾).

5.4. CYLINDRICAL SOURCES

(a) *Infinite Non-absorbing Cylinder*

The current at the surface of a cylinder which contains a uniform distribution of isotropic sources of strength S per unit volume is given by

$$j = \frac{\pi R^2 S}{2\pi R} = \frac{SR}{2} = \frac{SD}{4}, \quad (5.4.1)$$

where D is the diameter.

(b) *Infinite Absorbing Cylinder*

The current at the surface has been evaluated numerically by FIELD.⁽⁷⁾ If we write

$$j_{\text{attenuated}} = j_{\text{unattenuated}} f(\mu, D). \quad (5.4.2)$$

where $j_{\text{unattenuated}} = SD/4$, then the function $f(\mu, D)$ is as shown in Fig. 5.4.1. When $\mu D < 0.14$ the effect of self-absorption is very small. For $\mu D > 6$,

$f(\mu, D)$ varies as μ^{-1} and the cylinder behaves very like an infinite plane slab (for which $j = \frac{S}{4\mu}$, by equation 5.2.11.) For most shielding calculations the approximation of an infinitely long cylinder is adequate near the middle of a cylinder whose length is greater than 3 diameters.

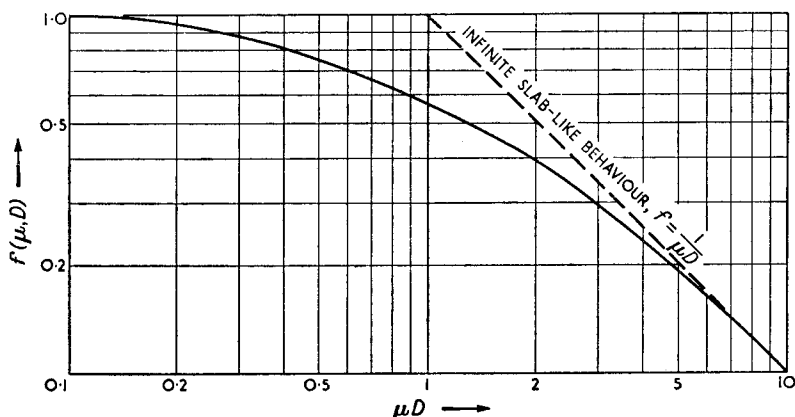


Fig. 5.4.1. The function $f(\mu, D)$ giving the current at the surface of an infinite absorbing cylinder (FIELD⁽⁷⁾).

5.5. SPHERICAL SOURCES

(a) Non-absorbing Sphere

The flux at a distance d from the centre of a sphere of radius R which contains a uniform distribution of isotropic sources of strength S per unit volume is given by

$$\phi = \frac{S}{4d} \left(2Rd + (R^2 - d^2) \ln \left(\frac{R+d}{R-d} \right) \right) \quad \text{for } d \leq R, \quad (5.5.1)$$

$$\phi = \frac{S}{4d} \left(2Rd - (d^2 - R^2) \ln \left(\frac{R+d}{d-R} \right) \right) \quad \text{for } d \geq R. \quad (5.5.2)$$

The current is given by

$$j = \frac{S}{3}d \quad \text{for } d \leq R \quad (5.5.3)$$

and

$$j = \frac{SR^3}{3d^2} \quad \text{for } d \geq R. \quad (5.5.4)$$

(b) Absorbing Sphere

The flux at the surface of an absorbing sphere of radius R is given by

$$\phi = \frac{S}{2\mu} \left(1 - \frac{1}{2R\mu} (1 - e^{-2\mu R}) \right). \quad (5.5.5)$$

The probability f of escape from an absorbing sphere of radius R is shown in Fig. 5.5.1. This quantity is related to the current j at the surface by the relation:

$$j = fS \left(\frac{\text{volume}}{\text{surface area}} \right) = \frac{fSR}{3} \quad (5.5.6)$$

The current at a distance d from the centre is given by

$$j = \frac{SR^2}{4\mu d^2} \left(1 - \frac{1}{2R^2\mu^2} + e^{-2R\mu} \left(\frac{1}{R\mu} + \frac{1}{2\mu^2 R^2} \right) \right) \quad \text{for } d \geq R. \quad (5.5.7)$$

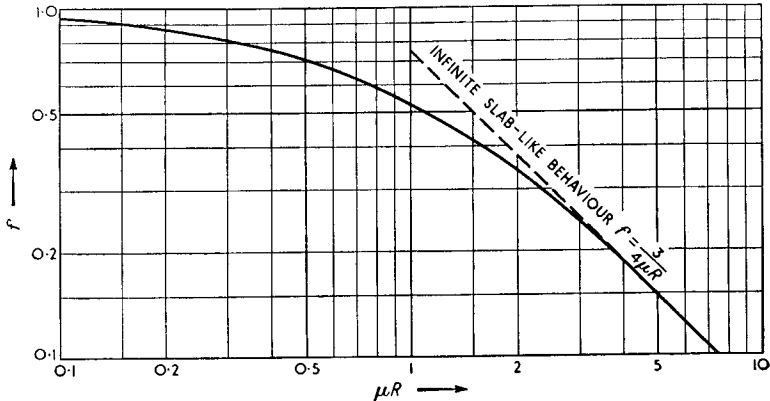


Fig. 5.5.1. Fraction f of radiation escaping from an absorbing sphere of radius R containing a uniform distribution of isotropic sources.

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CHAPTER SIX

THE SHIELDING OF REACTORS AND RADIOACTIVE MATERIALS

6.1. INTRODUCTION

IN THE earlier chapters of this book we have emphasized the basic physics of shielding, and, to avoid confusing the reader, have ignored the practical difficulties which often complicate the task of designing a reactor shield. It is now necessary to consider some of the engineering factors which also influence the design, and which at times conflict with the physics requirements. In the course of this chapter it will become obvious that some quantities of importance to the engineering of the shield cannot in practice be evaluated with great accuracy, in some cases because the physical facts are not available, in others because the geometrical layout is so complicated that detailed calculation would make unjustifiable demands on scientific manpower. The designer then has to rely on his scientific knowledge and intuition for evolving some approximate method of reaching a reliable conclusion.

There is no hard and fast method of deciding upon a design, but in general terms the following steps have to be carried out for all except the simplest and lowest-powered reactors:

(i) The required performance must be decided and the level of radiation that can be permitted at the surface of the shield must be specified. For a power reactor this level will probably be one maximum permissible level (or MPL, see Chapter 1); it could conceivably be greater, at points where the operating staff rarely went; but under no circumstances should the dose at any accessible part of the surface exceed 1000 MPL.

Somewhat greater dose-rates can be tolerated if portions of the shield are completely inaccessible during operation (e.g. the underside of a ship-borne reactor, or the charging room of the Windscale piles (Fig. 6.1.2)). Here the only requirement is that the dose-rate after shutdown should be tolerable. Owing to the fact that the gamma radiations emitted by the fission products after shutdown are both less intense and softer than the gamma radiation emitted during operation, it turns out that such a "partial shield" need be only about half the thickness of the full shield. The partial shield must be a neutron shield as well as a gamma shield, in order to reduce the activity induced in materials situated outside the shield which may later have to be manipulated or maintained (e.g. the hull of a ship). The exact amount of neutron shielding required depends on factors such as the nature of the materials exposed to the neutrons, how soon after shutdown it is necessary to approach them, and what operations have to be carried out; but in many practical cases this condition regarding neutrons also points to a partial shield of about half the full thickness.

The level of radiation specified for a research reactor is usually lower than

for a power reactor, in order to reduce the “background” of radiation to a level at which it would not interfere with experimental work. There is no point in reducing the radiation below 10^{-3} MPL because the radiation would then be no more intense than the irreducible background of naturally occurring radiation (Table 1.5.1); in fact, there is usually little point in shielding a research reactor to better than 0.1 MPL, because where good shielding is provided the radiation level in the reactor building is apt to be determined more by what is scattered from external experimental beams than by what leaks out through the shield. Any points which need to be particularly well shielded can be enclosed in a “castle” of small shielding blocks.

It must also be decided whether the design of remote handling devices sets any limitation on the thickness of the shield; whether part of the shield is to be demountable, and if so whether there are any limitations on the residual activity; whether the shield must be as light as possible regardless of expense (as it obviously must be for a submarine reactor); whether it must be as cheap as possible; or whether the use of the more expensive materials needed for the thinner shields can be justified by advantages in operation or in construction (e.g. smaller area of foundation). A reactor shield is a costly item, and the cost goes up by about 15 per cent for every extra factor of ten in attenuation. It is therefore always desirable to make a careful assessment of the acceptable dose-rate at the surface, rather than to assume unquestioningly that it is everywhere equal to one biological maximum permissible level or (in the case of a research reactor) to some lower “instrument tolerance level.” The attenuations required are commonly of the order of 10^8 or 10^9 , for both neutrons and gamma radiation.

(ii) A list is then made of the various possible materials from which the main portion of the shield (termed the “bulk shield” or the “biological shield”) could be constructed, and which satisfy all other requirements. Chemical compatibility with reactor materials must be satisfactory: for instance, a water-metal shield would probably be considered unsatisfactory for a sodium-cooled reactor. As far as nuclear requirements are concerned almost any hydrogen-containing mixture could be used, but in practice the choice is limited to the various kinds of concrete (Section 6.6), sandwiches of water or some other hydrogenous material and a metal such as lead or iron (Section 6.8), or (for small low-power research reactors) water alone (page 320). If organic materials are used for neutron shielding, they should have a high hydrogen content and low residual activity, and should be resistant to fire, durable, non-toxic, non-odorous, dimensionally stable, resistant to radiation damage, and easy to apply.⁽¹⁾ The choice of possible materials will be influenced to some extent by the ratio of the gamma and neutron intensities which have to be shielded. It may also be restricted by requirements of reactor safety: for instance, it must not be possible for the shield to melt during a reactor runaway, and to add reactivity by falling into the core. This consideration applies particularly to fast reactors.

(iii) The next step is to establish whether the heat release in the bulk shield is within acceptable limits. Most of the energy carried by the neutrons and gamma rays escaping from the reflector is converted into heat within the first

foot or so of the shield. For high-power reactors the energy dissipation may be sufficient to produce unacceptably large temperature gradients and thermal stresses in a shield made of a material with a low thermal conductivity, such as concrete. It is necessary to decide whether the material to be used for the bulk shield is capable of withstanding these stresses, or whether (as is usually the case) provision must be made for absorbing most of the heat energy in an additional "thermal shield", from which the heat can be easily conducted away. (By a *thermal* shield we mean a *heat* shield, but as it necessarily has to absorb thermal neutrons efficiently, the use of the one word thermal in two different senses fortunately does not cause confusion.) If the radiations are sufficiently intense it may even be necessary to provide some protection for materials which are normally regarded as good conductors of heat. Thus in a pressurized water reactor it is usual to provide thermal shields inside the pressure vessel on whose integrity the safety of the system depends.

The possible effects of radiation damage must also be examined, and an assessment made of the expected rate of gas evolution from the bulk shield, if it is made of water or an organic material (Section 6.8). The latter is an important practical point, since methods of venting the evolved gases must be devised.

(iv) For each combination of a possible bulk shield and a possible thermal shield the thickness of bulk shield necessary to reduce the core gamma rays to the desired level is calculated. A check is then made to see whether the gamma dose from neutron capture in the thermal shield is less than or greater than the unavoidable contribution from the core. In the latter case boron is added, if possible, to the thermal shield, until the thermal shield contribution is negligible; stage (iii) would then have to be repeated.

(v) The methods described in Chapter 4 are then used to verify that the thicknesses calculated in stage (iv) attenuate the neutrons sufficiently. This part of the calculation is less satisfactory than the others, and an adequate safety factor should be included. The contribution to the transmitted gamma-ray dose-rate from neutron capture in the bulk shield can now be calculated. For some commonly used reactor shields it is small compared with the other sources of gamma radiation; but where this is not the case the addition of boron to the bulk shield should be considered.

(vi) Having arrived in this way at a series of possible bulk shields, attention must be given to the activity, layout, and shielding of the coolant circuit. Whenever possible the heat exchangers should be placed in a separate shielded compartment into which entry is possible after the reactor has been shut down; i.e. there would be a "partial shield" between the reactor vessel and the heat exchanger compartment, the shielding provided for the latter also being part of the reactor shield. This layout is the obvious choice for a propulsion reactor, and is also used (for example) in the Dounreay reactor experiment sketched in Fig. 6.1.1.

Owing to the relatively large diameter of a heat exchanger shield it contributes a very substantial fraction of the total shield weight. For a propulsion reactor it is therefore most necessary to plan the layout of the coolant pipes in the heat exchanger compartment with great care. They should be made to contribute

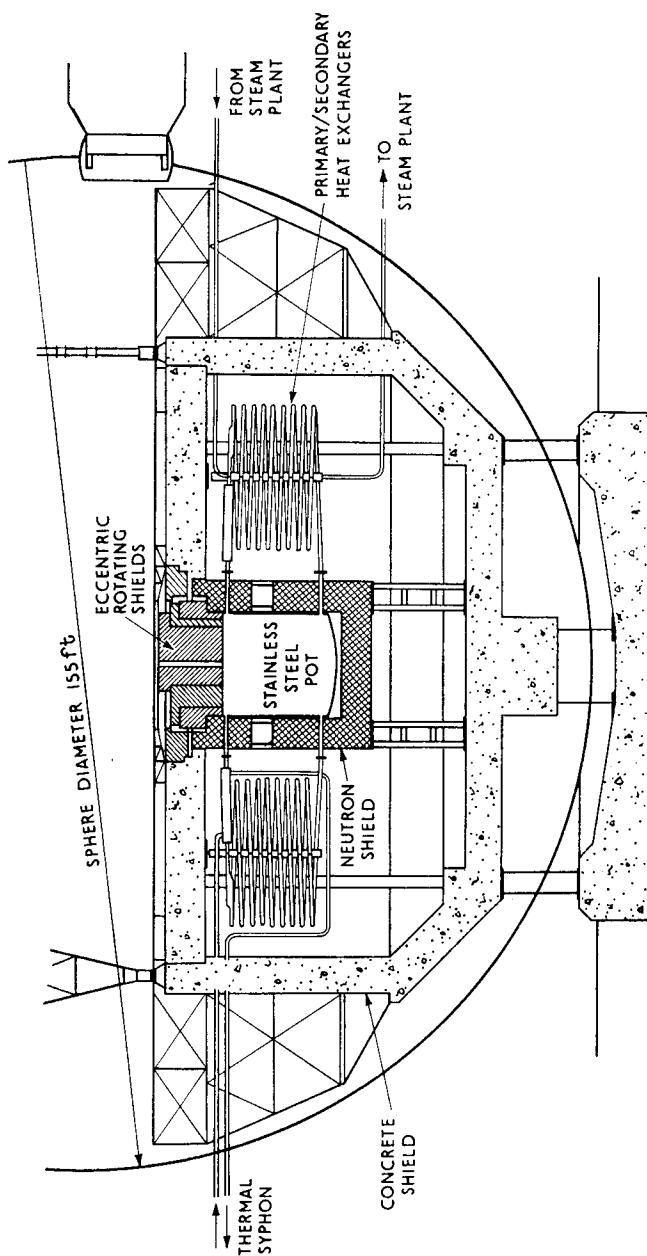


Fig. 6.1.1. Section through Dounreay reactor, showing the boron-graphite neutron shield between the core and the heat exchanger compartment, the "secondary" shield for the latter, and the containing sphere. (After KENDALL and FRY⁽²⁾.)

as much as possible to the shielding of the reactor core itself, and should also be arranged in such a way that the most active portions of the coolant circuit are placed furthest from the outside of the shield, so that they are shielded to some extent by the remainder of the plant. It may also be necessary to incorporate localized shields for these very active parts. At this stage the designer may find himself considerably limited in his freedom of choice by the necessity of adopting a pipe layout which is suitable from the point of view of

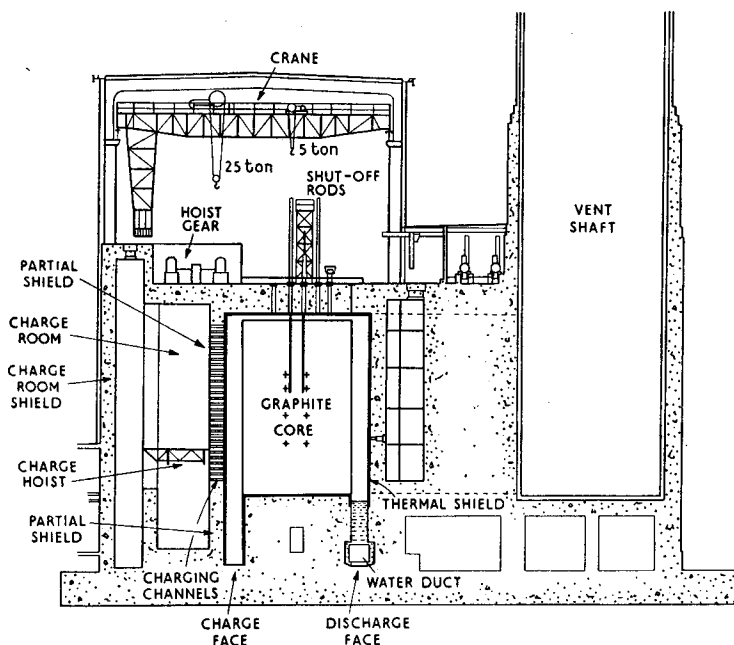


Fig. 6.1.2(a). Section through Windscale reactor, showing the partial shield between the core and charge room. Access to the charge room is prohibited when the reactor is operating. (After HINTON.⁽³¹⁾)

thermal expansion. The layout in the heat exchanger compartment is also intimately bound up with the way in which the coolant ducts penetrate the partial shield between the reactor and the heat exchanger compartment. Here again, it is necessary to allow sufficient clearance for thermal expansion—and this must be done in such a way that there is no large escape of radiation. Because of the many conflicting considerations which are involved, a satisfactory solution demands the closest collaboration between the mechanical engineer and the designer of the shield. Even so, it is unlikely that they will reach one solution which is obviously the best. For a mobile reactor, where the parasitic weight must be a minimum, it is more likely that calculations will have to be carried out for a large number of possible arrangements before the most satisfactory layout can be selected.

(vii) Having assembled all the necessary information by means of these

calculations, a choice is made on the basis of criteria such as weight, cost, or ease of construction. For a mobile reactor the final stage would be to test the proposed shield in an actual mock-up experiment, because only in this way can the necessity for safety factors be avoided, and the last few tons of shield weight be eliminated.

(viii) Finally, when the shield has been completed and the reactor is operating, it is necessary to carry out a radiation survey to verify the correctness of the

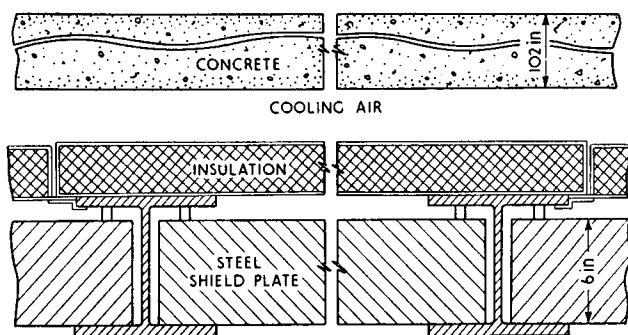


Fig. 6.1.2(b). Detail of arrangement of Windscale thermal shield, showing the 6-in. steel plates separated from the concrete of the bulk shield by insulation and an air-gap. (After ILIFFE.⁽⁴⁾)

calculations. The survey will be facilitated, and additional information will be more easily assembled, if a few holes and removable plugs are included in the shield design.

Occasionally it is found during radiation surveys that, owing to some defect of construction or design, there are local regions of high radiation intensity. A practice adopted in America is to ignore all peaks less than 7 times the normal level, if they do not cover more than 1 per cent of the shield area; otherwise the radiation level is reduced by adding "blisters" to the outside of the shield.⁽¹⁾

We shall begin our study of shield design by considering the process of energy absorption in a homogeneous shield which is not provided with a separate thermal shield. Three processes contribute to the heating of the shield: (i) absorption of gamma radiation generated in the core and reflector, (ii) transfer of kinetic energy from fast neutrons during slowing down, and (iii) absorption of gamma radiation produced during the neutron capture process in the shield.

HEAT GENERATION IN SHIELDS

6.2. HEATING OF THE SHIELD BY GAMMA RAYS FROM CORE AND REFLECTOR

Gamma Sources

Before the shield design is commenced the engineering and nuclear design of the reactor is usually in an advanced stage, and neutron flux distributions

in the core and reflector have already been calculated. For a thermal reactor (which is the only case we shall consider in any detail) the usual "two-group" treatment enables the gamma ray source distribution in the core and reflector to be estimated, provided the simplifying assumption is made that capture of thermal and resonance neutrons is the only important cause of gamma production (i.e. neglecting any contribution from inelastic scattering).

Consider first the case of a homogeneous reactor. If the thermal neutron flux at any point in the core is θ n cm⁻² sec⁻¹, then the number of thermal neutrons captured per cm³ per second by the p th constituent of the core is $\theta \Sigma_p$, where Σ_p is the macroscopic absorption cross-section of the p th component in cm⁻¹. In the same way, the number of fissions is $\theta \Sigma_{\text{fiss}}$ cm⁻³ sec⁻¹; and from this number the fission gamma source strength can be found at once, using the data of Section 2.9. Similar expressions give the rate at which thermal neutrons are captured in the moderator, in the canning material, and in the fertile material.

The abundance and energy distribution of the gamma rays from radiative capture are given for many elements in Table 3.5.1. Where experimental information is lacking, an upper limit to the total gamma ray energy can be found from the binding energy of the ingoing neutron, which for most elements is about 6–8 MeV, and which in many cases is known exactly from measurements of the threshold of the reverse (γ, n) process (Table 3.3.1). Any assumptions regarding the energy distribution of the photons should be such that the calculation gives a "pessimistic" (i.e. certainly safe) result. For instance, if the total shield thickness is required, the binding energy could be assumed to be emitted in quanta of the energy to which the proposed shield is most transparent (see Figs. 2.6.4 and 2.6.5). Thus, if the binding energy is 7 MeV, and lead is the gamma-ray shielding material, it would be safe to assume that the binding energy is liberated as two 3.5 MeV photons; but if it is intended to use ordinary concrete, it would be better to assume that the energy is liberated in the form of a single penetrating photon carrying the whole of the 7 MeV. It will be noticed from Fig. 2.6.5 that barytes concrete has an absorption coefficient which is constant to within $\pm 10\%$ over the range 2.5 MeV to 10 MeV; so that calculations of attenuation by this material are not very sensitive to these arbitrary assumptions. For calculations of shield heating it may be more pessimistic to assume that the energy is emitted in the form of relatively soft quanta. In any case, it is always desirable to establish how far the result depends on the assumptions used.

In the majority of reactors resonance capture makes a contribution to the rate of loss of neutrons which cannot be ignored. The resonance capture density is calculable from the reactor equations, but there is little direct experimental information regarding the spectral distribution of the resulting gamma radiation. It is certainly to be expected that there would be differences between the thermal and resonance neutron capture gamma spectra for those elements which show a strong ground-state transition when capturing thermal neutrons.⁽⁸⁴⁾ There might also be differences even when the thermal capture spectrum is continuous, though natural uranium (which is in this category) shows very similar spectra for all of the five lowest resonances. Although data are scanty,

the binding energy is always known, and for the purposes of calculation a suitably pessimistic resonance capture gamma spectrum can be assumed.

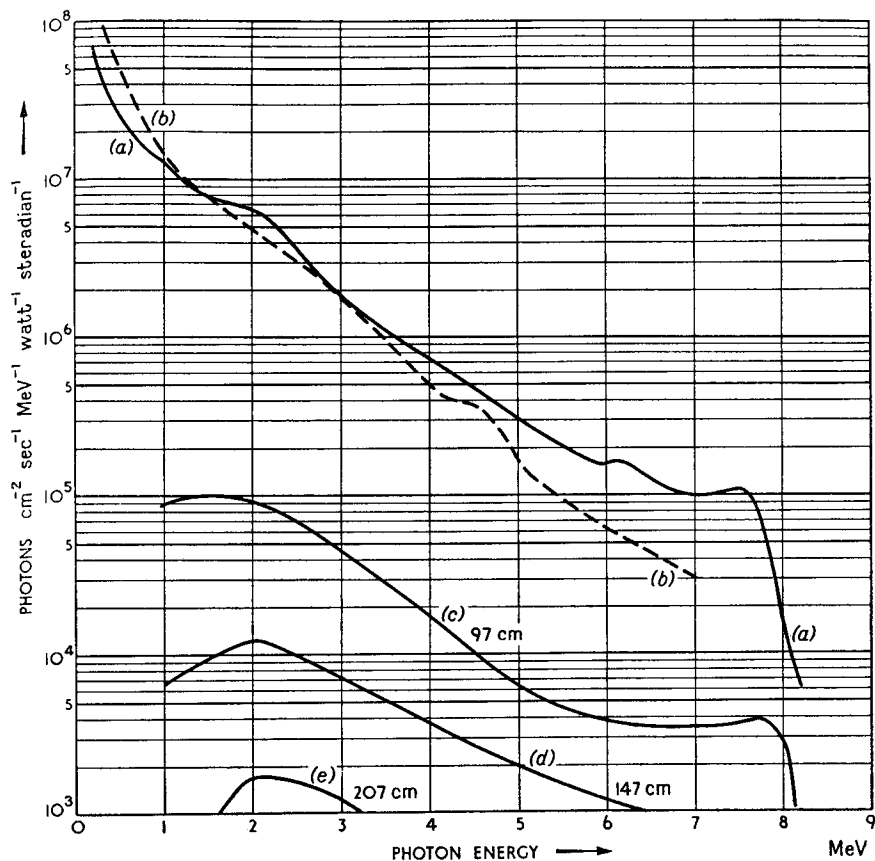


Fig. 6.2.1. Gamma-ray spectra of three reactor systems.

Curve (a): Absolute values of the intensity and spectral distribution at the edge of the core of the Oak Ridge Bulk Shielding Reactor.⁽⁵⁾

Curves (c)(d)(e): Absolute values observed with varying amounts of water between core of Bulk Shielding Reactor and detector (MAIENSCHIEIN and LOVE⁽⁵⁾).

Curve (b): Los Alamos water-boiler with U^{235} inserted in experimental channel (normalized to curve (a) in the region of 1 to 3 MeV) (MOTZ⁽⁶⁾). The Los Alamos fast reactor gave an almost identical result.

There are a number of other sources of core gamma radiation, such as fission product decay, coolant activity, activation of structural materials, and bremsstrahlung. When the reactor is operating these are unimportant in comparison with capture and fission radiation, though they have to be considered when the reactor is shut down, or when designing the shielding for any portions of the primary coolant circuit which lie outside the main shield.

By summing the contributions from the various components of the reactor, the gamma ray source strengths are obtained in the form: photons with energies in a given energy interval ΔE emitted per cm^3 per second. We shall use the symbol $\Xi(E)\Delta E$ to denote the source strength in the core or reflector, it being understood that this quantity is a function of position.

For heterogeneous reactors the above simple scheme must be modified to take into account the depression of neutron flux in absorbers such as the fuel elements; formally similar expressions can be used if Σ_a is replaced by a suitable effective value, chosen to simulate correctly the rate of neutron absorption. Depending on the algebraic methods used for the reactor calculations, and on the geometry of the core, it may in some cases be more convenient to express the photon source strength as photons per given energy interval per second per fuel element, rather than per unit volume, and to estimate the fraction of the photons which escape from the parent fuel element. For cylindrical fuel elements in which the depression of flux is small this fraction can be estimated from Fig. 5.4.1. Resonance absorption in U^{238} is a special case, since an appreciable proportion of capture takes place in the surface layer of the fuel element, from which at least half of the photons can escape.

Before considering methods of calculating how much of the gamma ray energy reaches the shield, it is helpful to examine the experimental results given in Fig. 6.2.1. This shows the intensity and energy distribution of the gamma radiation observed in three reactor systems. Although there are substantial physical differences between the three systems, the shapes of the distributions are very similar, and are characterized by a roughly exponential decrease with increasing energy, such that there are about one hundred times as many photons at 1 MeV as there are at 7 MeV. The exponential character of the curve is due in part to the somewhat similar form of the fission gamma ray spectrum (Fig. 2.9.1), but also to the effect of the multiple scattering of gamma radiation in the core of the reactor. It will be seen from the curves that a large part of the energy escaping from the reactor core into the shield is concentrated in the low-energy region. Some of these low-energy gamma rays are merely due to the degradation of higher energy radiation, and are therefore automatically taken into account by the factors describing build-up in the core. However, there remains an uncertainty that stems from our imperfect knowledge of the lower end of the various component gamma-ray source spectra, and this imposes a limitation on the accuracy of heating calculations, especially when there is little absorbing material between the core and the shield.

Gamma Leakage

Returning to the methods of calculation, we shall assume that the distribution, in position and quantum energy, of the gamma-ray sources in the core and reflector has been determined. The next step is to calculate (a) what fraction of the hard gamma radiation, and (b) what fraction of the total gamma-ray energy escape into the shield. The former is required for calculating the total thickness of shield, and the latter for the heating effect of the radiation. The complicated geometry of most reactors prevents a general solution from

being given, and it will usually be found necessary to resort to numerical methods of calculation. Whatever method of obtaining a result is used, there are a number of general points that are worth discussing, since they often lead to confusion.

It is necessary to distinguish between flux and current of gamma radiation. Flux is the fundamental quantity needed for calculation of heat emission per unit volume, or of biological dose-rate. Consider, for instance, the situation shown in Fig. 6.2.2. We require the heat deposition per unit volume at the point B , due to the source in the very small volume element dV at the point A ,

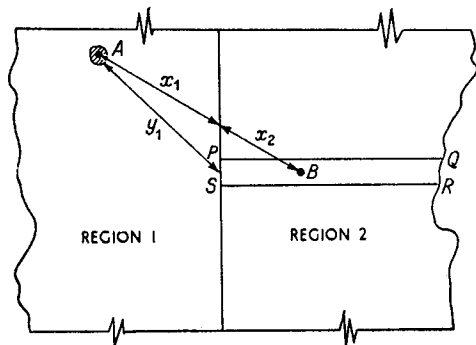


Fig. 6.2.2.

which emits $\Xi dV dE$ photons per second, lying within the energy interval E to $(E + dE)$ MeV. In the diagram region 1 is the source and region 2 is the shield. The number per second of “uncollided” photons threading a spherical volume element of unit cross-sectional area placed at B would be

$$d\phi_{\text{uncoll}} = \frac{\Xi dV dE}{4\pi(x_1 + x_2)^2} e^{-x_1\mu_1} e^{-x_2\mu_2} \quad (6.2.1)$$

where μ_1, μ_2 are the linear “narrow-beam” absorption coefficients in the two media for the energy E (Fig. 2.6.4). The energy deposition per unit volume at B due to photons in this narrow energy-band is therefore

$$\rho \cdot \chi_{a,2} \cdot E \cdot d\phi_{\text{uncoll}} \cdot B(x_1, x_2) \text{ MeV sec}^{-1} \text{ cm}^{-3}, \quad (6.2.2)$$

where ρ is the density of the material and $\chi_{a,2}$ is the mass energy absorption coefficient (Fig. 2.6.6) in the second medium. The energy build-up factor is written in the form $B(x_1, x_2)$ because it depends to some extent on the order and nature of the two media, and is not necessarily equal to the product of the build-up factors in the two media (see discussions on pages 53 and 244). If we now suppose that the first medium is an extended gamma-emitter, we see at once that the quantity needed for calculating the heat liberated per unit volume per second is the total energy threading unit area placed normally to the direction of the photons, i.e. the *flux*. By a double integration over the whole volume of the core and over the whole gamma spectrum, we can determine the total heat released per unit time in our spherical element of shield; and, by

summing for all points in the shield we can, for instance, determine the heat released in the cylinder of unit cross-section $PQRS$, shown in Fig. 6.2.2.

In the particular case of infinite slab geometry shown in the figure, symmetry considerations show that this total heat release must also equal the energy current across the unit area PS , which, if backscattering is neglected, is

$$\int_{\text{all energies}} \int_{\text{Region 1}} \frac{\cos \psi}{4\pi y_1^2} B(y_1) \Xi E e^{-y_1 \mu_1} dV dE \text{ MeV cm}^{-2} \text{ sec}^{-1}, \quad (6.2.3)$$

where ψ is the angle between AP and the normal to PS , $B(y_1)$ is the energy build-up factor (Fig. 2.7.3c) in the first medium, and the integral extends over the whole of the gamma-emitting region 1. Thus, for this case of infinite slab geometry, the use of the current provides in a single step the total heat transported into unit area of shield, a quantity which would otherwise have required a lengthy calculation. The current is therefore the first quantity which should be calculated, in order to establish the order of magnitude of the heat energy entering the shield. It may be found that this amount of heat is too small to be troublesome, in which case there is no point in carrying out the more laborious flux calculations outlined above. This would be so, for instance, if the total gamma-ray energy entering a concrete shield were found to be much less than 30 milliwatts per cm^2 (see Section 6.6).

A calculation of the current does not provide any information regarding the variation of heat release with depth (which determines the maximum temperature rise in the shielding material); so that if the preliminary estimate shows that heating is likely to be important, it is usually necessary to calculate the fluxes. However, it may be sufficient in certain cases to establish rough upper and lower limits within which the heating effect at a given depth probably lies. If the total gamma-ray energy current into the shield from the core and reflector, due to photons in the energy interval ΔE , is $j_0(E) \cdot \Delta E \text{ MeV cm}^{-2} \text{ sec}^{-1}$, then at a depth z into the shield the outward current of gamma energy probably lies between

$$j_1(z, E) \cdot \Delta E = j_0(E) \cdot \Delta E \cdot B_s e^{-\mu_s z} \text{ MeV cm}^{-2} \text{ sec}^{-1} \quad (6.2.4)$$

which is the value it would have if the incident photons were initially all directed normally into the shield, and

$$j_2(z, E) \cdot \Delta E = j_0(E) \cdot \Delta E \cdot B_s F \text{ MeV cm}^{-2} \text{ sec}^{-1} \quad (6.2.5)$$

which is the value it would have if the incident photons were isotropically distributed over the outward 2π solid angle (see equation 5.2.7). In these expressions μ_s is the linear absorption coefficient in the shield for photons of energy E , B_s is the energy build-up factor, and $F \equiv E_2(\mu_s z)$, where E_2 is the exponential integral function defined on page 212. The upper and lower limits for heat release per unit volume may be obtained by differentiating these expressions with respect to z .

The simple methods of calculating current, by means of expressions such as those given in Chapter 5, do not give any explicit information regarding the angular distribution of the escaping radiation. This may lead to difficulty

when attempting to deduce heating effects from calculations of current. As an illustration, consider how the DIDO reactor might possibly be treated. This reactor is shown in section in Fig. 6.2.3. It is provided with two reflectors, an

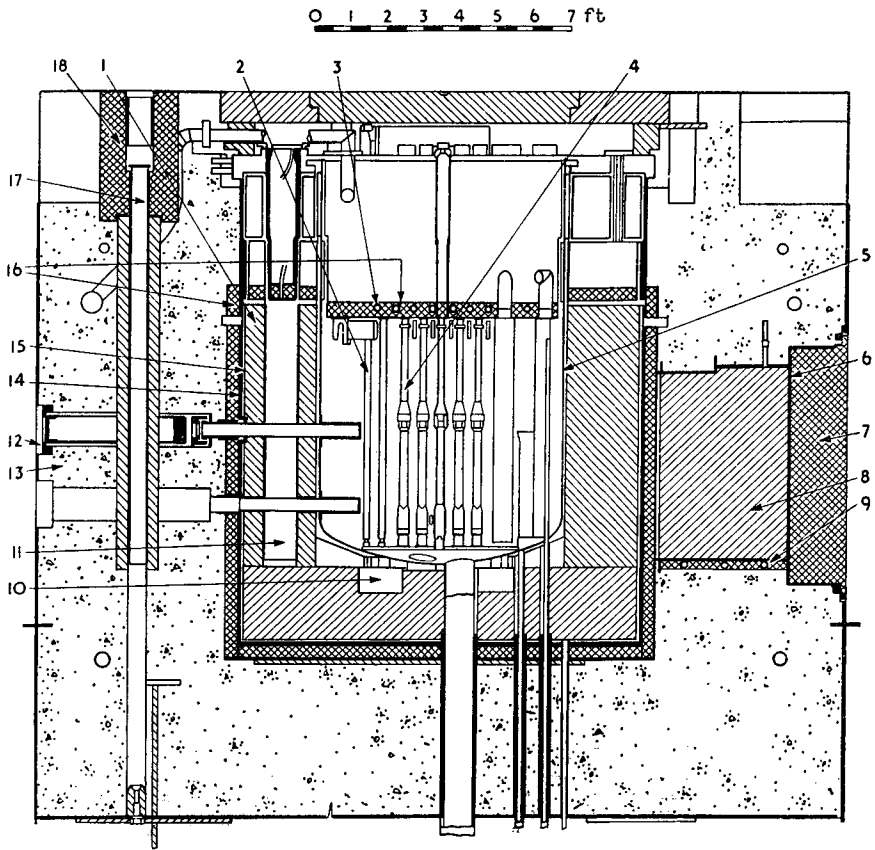


Fig. 6.2.3. Elevation through DIDO. For clarity some mechanical details have been omitted.

KEY:

- | | |
|---|--|
| 1. Graphite reflector | 11. Vertical experimental facility |
| 2. Vertical experimental facility | 12. Horizontal experimental facility |
| 3. Cooling coils for lead shield (shown only on section above tank) | 13. Barytes concrete bulk shield |
| 4. Fuel element | 14. Steel tank |
| 5. Aluminium tank containing moderator (heavy water) | 15. Boral lining in steel tank |
| 6. Thermal column boral lining | 16. Lead shield |
| 7. Thermal column shield | 17. Storage channel for highly active fuel elements |
| 8. Thermal column | 18. Lead shield to prevent gamma rays "shining" through shield when transferring irradiated fuel elements to the storage channel |
| 9. Thermal column cooling coils | |
| 10. Horizontal experimental facility | |

inner one of heavy water, and an outer one of graphite, separated by an aluminium tank. Two quantities needed for the reactor design are the heating in the graphite and the heating in the shield. The following indirect step-by-step method seems at first sight to be one way of obtaining the outgoing current from the core into the shield: (i) calculate the gamma current over the boundary between the core and the first reflector; (ii) use the result of (i) as the source term, and calculate the attenuation introduced by the first reflector; (iii) use the calculated current out of the first reflector as the source term for the second, and so on. This method has the difficulty that in stages (ii) and (iii) it is necessary to make an assumption regarding the angular distributions of the radiation escaping across each boundary. The effect of build-up makes it practically impossible to calculate these angular distributions, so that any assumptions are necessarily arbitrary. A repeated application of some erroneous assumption at each boundary can lead to a considerable error in the result. Although for the calculation of the overall shield thickness quite gross errors are allowable, this is not the case for shield and reflector heating calculations, where accuracies considerably better than a factor of two are desirable. The necessity for these arbitrary assumptions can usually be avoided by using first principles, as outlined above, to calculate flux as a function of position in the shield, and hence to determine the heat release per unit volume due to the core and reflector gamma radiation.

In order to simplify the calculations of either flux or current, it is useful to divide the gamma rays into several energy groups. The number of groups will depend on the accuracy required, on the quality of the information used for the source strength calculations, and also on how the attenuation coefficients used vary with quantum energy. The gamma energy escaping into the shield is calculated for each group, and the effect of the whole source spectrum is obtained by summation.

In many cases it is useful to represent the spatial variation of the gamma source density by means of approximate functions, such as a sum of a number of exponential terms. It may be sufficient to use a simple linear function: for instance, the thermal neutron flux (which determines the neutron-capture rate) along a diameter of a typical natural-uranium-graphite reactor (BEPO) can be approximated by a linear fall-off in the reflector and (very crudely) by a constant value in the core (Fig. 6.2.4). In this particular case, although the current of gamma rays escaping into the shield is not very sensitive to the value chosen for the core region (owing to the shielding effect of the three feet of graphite forming the reflector) it would be best to choose the value at the core-reflector boundary, since the escaping current of gamma rays is influenced mainly by the source density in the outer regions of the reactor. A graphite moderated reactor is usually a large structure, with a lateral extent considerably greater than the mean free path of photons in the material; and in these circumstances the analysis can be further simplified by making use of the expressions given in Chapter 5 for gamma currents escaping from infinite plane slabs.

The BEPO reactor which we have used as an illustration happens to be a particularly simple case. For other reactors it often happens that simple analytical expressions for the escaping radiation cannot be derived, either

because the reactor is not large compared with the mean free path of the photons, or because the actual geometry of the core is difficult to handle mathematically. In such cases it is helpful to use the "lumped source" method. For this, the sources in the reactor are considered as lumped into a number of point sources, from which the contribution at any point of interest is evaluated numerically using expression (6.2.2). The choice of the number and position of the lumped sources has to be a compromise between accuracy and labour of calculation.

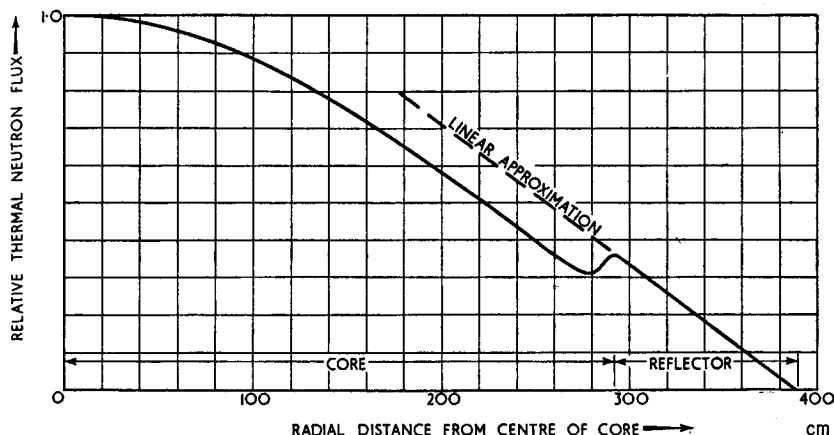


Fig. 6.2.4. Radial variation of thermal neutron flux in BEPO.

In general, the sources should not be separated by much more than one mean free path for the particular energy of gamma radiation considered. The most important sources are usually those located within one or two mean free paths of the surface of the source medium which faces the receiving point; so that it would pay to have a relatively fine net in this region, and a coarser net for the regions farther from the receiving point, where the attenuating effect of the medium is more pronounced.

Strictly speaking, these methods of calculation give the energy current out of the reflector only when there is no material nearby to scatter back some of the gamma energy into the reflector. Bringing up the shield will perturb the net energy current. However, inspection of Table 2.8.2 shows that albedos for gamma rays are small, of the order of 10%, so that the error introduced by neglecting back-scattering from the shield is insignificant in the context of a shielding calculation. In this respect gamma rays behave very differently from neutrons.

Choice of Build-up and Attenuation Data

The analytical expressions given in Chapter 5 do not include the energy build-up factor. Provided configurations such as that shown in Fig. 2.7.6 are not involved, this factor may be estimated approximately by inspection of Fig. 2.7.3. If more accuracy is required, TAYLOR's method should be used (equation 2.7.5). To give an example of its use: if the source density at a depth d in the reflector can be written in the form $\Xi(d) = \alpha + \beta d$ (where α and β are

constants), then the energy current j out of the reflector is given (for infinite slab geometry) by the following modification of equation 5.2.14(a).

$$i = E_0 A_1 \left\{ \frac{1}{2\mu_r(1+a_1)} \left[\frac{\alpha}{2} + \frac{\beta}{3\mu_r(1+a_1)} \right] \right\} + E_0 A_2 \left\{ \left(\frac{1}{2\mu_r(1+a_2)} \right) \left[\frac{\alpha}{2} + \frac{\beta}{3\mu_r(1+a_2)} \right] \right\} \quad (6.2.6)$$

In this expression A_1 , A_2 , a_1 , and a_2 are the energy-build-up constants derived by TAYLOR and listed in Table 2.7.3, μ_r is the linear absorption coefficient in the reflector, and E_0 is the energy of the photons emitted by the source.

The choice of attenuation and build-up constants in calculations affecting only the reflector usually presents no difficulty, since reflectors are almost invariably simple substances for which reliable data are available. There are, however, difficulties in deciding what values to use for the core, owing to the fact that it may be heterogeneous, and that in any case it contains both light and heavy elements. Regarding the choice of the mean "narrow-beam" absorption coefficient, it will be noticed that the mean free path of gamma rays is usually comparable with, or larger than, the size of the inhomogeneities in the core; in these circumstances it is reasonable to assume that the core behaves as a homogeneous gamma absorber, with a mean linear absorption coefficient $\bar{\mu}_c$, as defined on page 36.

The build-up factor for the core is not so easily specified. An upper limit can always be obtained by using the value appropriate to the pure moderator; and for graphite reactors, in which a relatively small fraction of the core is occupied by uranium, this simple treatment is probably good enough. In water-moderated reactors, however, the fuel elements tend to be packed together much more tightly, and build-up in water itself is more important than in graphite, so that a better approximation is needed. Arguments similar to those given on page 53 suggest that the build-up factor applicable to a given thickness t of core should be close to the build-up factor for a thickness $t\bar{\mu}_c/\mu_u$ of uranium, where μ_u is the linear absorption coefficient of uranium, and $\bar{\mu}_c$ is the mean absorption coefficient of the core. That is to say, one should use the uranium build-up data calculated for a depth equal to the same number of mean free paths as are included in a thickness t of core. When using TAYLOR'S approximate representation (page 50) one therefore needs values of the constants A_1 , A_2 , a_1 , and a_2 appropriate to uranium; if these are not available, figures for the same depth (in mean free paths) of lead may be used without serious error.

Units and Conversion Factors

Gamma currents are most conveniently expressed in units of $\text{MeV cm}^{-2} \text{sec}^{-1}$. The factors which convert these units to those used in orthodox engineering are as follows:—

$$\begin{aligned} 1 \text{ MeV cm}^{-2} \text{sec}^{-1} &= 1.602 \times 10^{-6} \text{ ergs cm}^{-2} \text{sec}^{-1} \\ &= 1.602 \times 10^{-13} \text{ joules cm}^{-2} \text{sec}^{-1} \\ &= 1.602 \times 10^{-13} \text{ watts cm}^{-2} \end{aligned}$$

The engineering unit usually used in heat calculations in Great Britain and America is the British Thermal Unit (BThU) equal to 1.055 kW-sec. Thus

$$\begin{aligned} 1 \text{ MeV cm}^{-2} \text{ sec}^{-1} &= 1.52 \times 10^{-16} \text{ BThU cm}^{-2} \text{ sec}^{-1} \\ &= 1.41 \times 10^{-13} \text{ BThU ft}^{-2} \text{ sec}^{-1} \\ &= 5.08 \times 10^{-10} \text{ BThU ft}^{-2} \text{ hr}^{-1} \end{aligned}$$

6.3. HEATING BY NEUTRONS

Neutrons entering the Shield as Thermal Neutrons

The kinetic energy of thermal neutrons is negligible, and they heat the shield only because of the binding energy released when they are captured (Section 3.5). Some of the capture gamma radiation which happens to be emitted in the direction of the core will succeed in escaping from the shield, so that the total energy available for shield heating is somewhat less than the figure obtained by simply multiplying the number of incident neutrons by their average binding energy in the capturing medium. As already explained in Section 4.3, the thermal neutron flux $\phi(z)$ at a depth z below the surface of the shield, due to neutrons which entered the shield as thermal neutrons, may be represented to a sufficiently good approximation by the equation

$$\phi(z) = \phi_0 e^{-z/L} \quad (6.3.1)$$

where ϕ_0 is the flux at $z = 0$ and L is the diffusion length given by equation (4.2.10). The number of neutrons captured per unit volume per unit time at a depth z is $\phi(z)l_c^{-1}$, where l_c is the capture mean free path in the shield. We shall assume for simplicity that one photon of a particular energy Γ is emitted per capture; any more complicated case can be dealt with by the superposition of the effects of the several energy groups given in Table 3.5.1.

The fraction of the gamma-ray energy returned from a depth z to the surface on which the neutrons were incident is given, from equation (5.2.7), by $\frac{B_s}{2} E_2(\mu z)$, where B_s is the energy build-up factor in the shield.

On manipulation of the algebra,⁽⁴⁾ we find that the photon energy escaping across the front face *per neutron incident on the shield* is given by $f_1 \Gamma$, where

$$\begin{aligned} f_1 = \frac{\bar{B}_s}{2} \mu L \left\{ \frac{1}{\mu L} \left[1 - e^{-(1+\mu L)t/L} \right] + \left(\frac{t}{L} + 1 \right) e^{-t/L} E_1(\mu t) \right. \\ \left. - E_1 \left[(1 + \mu L)t/L \right] - \ln \left(\frac{1 + \mu L}{\mu L} \right) \right\} \quad (6.3.2a) \end{aligned}$$

In this expression $\bar{B}_s$ is a suitable average value of the build-up factor (see below); alternatively, build-up may be included by TAYLOR's method.

For a shield which absorbs thermal neutrons heavily, $\mu L \ll 1$, and $f_1 = 0.5$. This result is physically obvious: the neutrons are captured close to the inner surface of the shield, and since the gamma rays are emitted isotropically, one half enter and one half leave the shield (in this case $\bar{B}_s = 1$).

The major contribution to f_1 comes from neutrons which are captured within a distance of the order of one diffusion length from the surface of the shield. In some materials (e.g. concrete), this represents only a fraction of a mean free path for the higher energy capture gamma radiations. In such a case a rough value of $\bar{B}_s$ can be determined without difficulty; it will not differ greatly from unity, and will in fact be close to the value appropriate to a depth of the order of μL mean free paths.

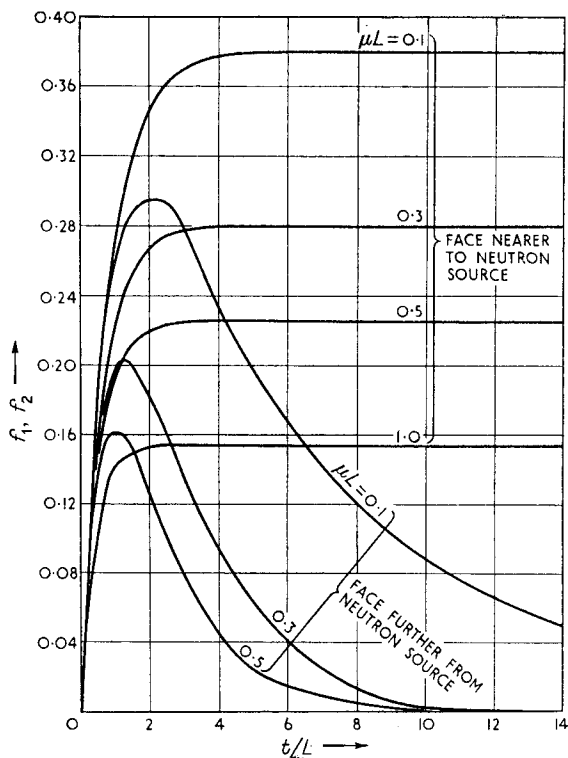


Fig. 6.3.1. Number f of capture gamma photons per incident neutron which escape across the front and rear faces of an infinite plane slab shield of thickness t . It is assumed that the neutron flux is attenuated exponentially with a relaxation length L , and that one photon is emitted for every neutron captured.

Build-up of gamma radiation is neglected. (LIFFE.⁽⁴⁾)

The complementary function f_2 , which determines the gamma energy ($f_2 \Gamma$) which escapes across the shield face remote from the reactor per incident neutron (not, it should be noted, per neutron captured) is given⁽⁴⁾ by the equations:

$$f_2 = \frac{\mu L}{2} e^{-t/L} \left\{ \frac{1}{\mu L} \left[e^{(1-\mu L)t/L} - 1 \right] - \left(\frac{t}{L} - 1 \right) e^{t/L} E_1(\mu t) + \bar{E}_1 \left[(1 - \mu L)t/L \right] - \ln \left(\frac{1 - \mu L}{\mu L} \right) \right\}, \quad (6.3.2b)$$

$(\mu L < 1)$

$$f_2 = \frac{\mu L}{2} e^{-t/L} \left\{ \frac{1}{\mu L} \left[e^{(1-\mu L)t/L} - 1 \right] - \left(\frac{t}{L} - 1 \right) e^{t/L} E_1(\mu t) \right. \\ \left. - E_1 \left[(\mu L - 1) \frac{t}{L} \right] - \ln \left(\frac{\mu L - 1}{\mu L} \right) \right\}, \quad (\mu L > 1) \quad (6.3.2c)$$

where $\bar{E}_1(x)$ is the function defined in equation (5.1.13), and build-up has been neglected. Fig. 6.3.1 shows the functions f_1 and f_2 for values of μL between zero and 1.0, neglecting build-up.

Heating by Neutrons entering the Shield with Energies above Thermal

The standard two-group type of reactor calculation enables an estimate to be made of the total number of "fast" (i.e. non-thermal) neutrons escaping from the reflector into the shield. Although the energy assignment is without any exact meaning, the calculated value at least gives some qualitative information as to whether or not the fast neutrons make an important additional contribution to the total energy dissipated in the shield. The average kinetic energy of the "fast" neutrons is much less than 1 MeV, while the energy released when the neutrons are finally captured is usually of the order of 7 MeV (the principal exceptions being capture in hydrogen (2.2 MeV), boron* (2.79 MeV), and lithium* (4.78 MeV)). For most shielding materials, therefore, the energy given up by the neutrons while they are being slowed down to thermal energies is small in comparison with the heating due to the capture process. Although the total amount of heat carried into the shield is thus not directly affected very much by the energy distribution of the fast neutrons, the latter has an important indirect effect, since it controls the depth at which the fast neutrons reach thermal energies and are captured. The capture gamma radiation from neutrons absorbed at a considerable depth has a small probability of escaping back into the reactor, so that the factor f_1 (eq. 6.3.2a) for high-energy neutrons is much smaller than it is for neutrons which have been thermalized before they enter the shield. It would be wise to assume that the whole of the radiation resulting from the inelastic scattering and eventual capture of fast neutrons is available for heating the shield. Since methods of calculating the absorption of epithermal and fast neutrons in a shield have already been discussed at length in Chapter 4, we shall not deal further with the heating produced by neutron capture.

The calculation of the heat release due to elastic scattering is straightforward once the energy spectrum $\phi(E) dE$ of the fast neutrons in the shield is known. For isotropic scattering in the centre-of-mass system the heat release per unit volume of shield per unit time is

$$\sum_i \left(\int_{\text{Fast neutron energies}} E \delta_i \Sigma_i \phi(E) dE \right), \quad (6.3.3)$$

where Σ_i is the macroscopic scattering cross-section of the i th component

* Note that for boron the majority, and for lithium the whole of the reaction energy is carried off by short-range products, and does not appear as gamma radiation (Section 3.6). The energy given up in elastic scattering processes is also deposited locally.

(which is approximately equal to the total cross-section, since absorption is small for fast neutrons) and δ_i is the average fractional loss of energy when a neutron is scattered from nuclei of the i th component (Section 4.6). Methods of calculating $\phi(E)$ in shields containing hydrogen are discussed in Chapter 4.

6.4. THE DETERMINATION OF THE TEMPERATURE DISTRIBUTION IN THE SHIELD

Before the temperature distribution can be determined, a knowledge of the heating as a function of position is required. The calculation, although straightforward, is lengthy, owing to the necessity of taking into account the transfer of energy from point to point in the shield by emission and re-absorption of capture gamma radiation. Once the heat release as a function of position has

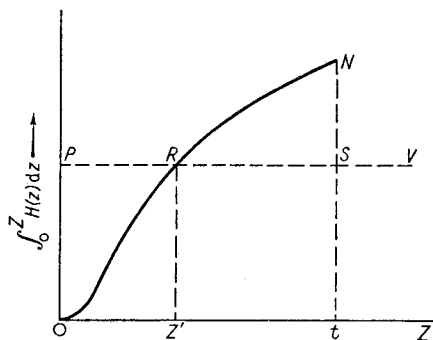


Fig. 6.4.1. Geometrical construction for determination of hottest point in a shield. (HALLIDAY⁽⁸⁾.)

been determined, a simple geometrical construction may be used to give the position of the hottest point of the shield. Consider the case of a cylindrical reactor which is so large that for our purposes its boundary may be treated as an infinite plane, and the shield as an infinite plane slab of thickness t . Then the heat release per unit volume $H(z)$ is a function only of z , the depth into the shield.

The quantity of heat $Q(Z)$ conducted across unit area of any given plane $z = Z$ is linked with the heat release per unit volume $H(Z)$ by the equation

$$\left(\frac{dQ}{dz}\right)_{z=Z} = H(Z),$$

so that

$$Q(Z) = \int_0^Z H(z) dz + \text{constant} \quad (6.4.1)$$

The quantity $\int_0^Z H(z) dz$ is plotted in Fig. 6.4.1. The equation linking Q with

the temperature T is

$$Q = -k \left(\frac{dT}{dz} \right)_{z=z'}, \quad (6.4.2)$$

where k is the thermal conductivity. From (6.4.2) we have

$$T(Z) = T_0 - \frac{1}{k} \int_0^Z Q(z) dz;$$

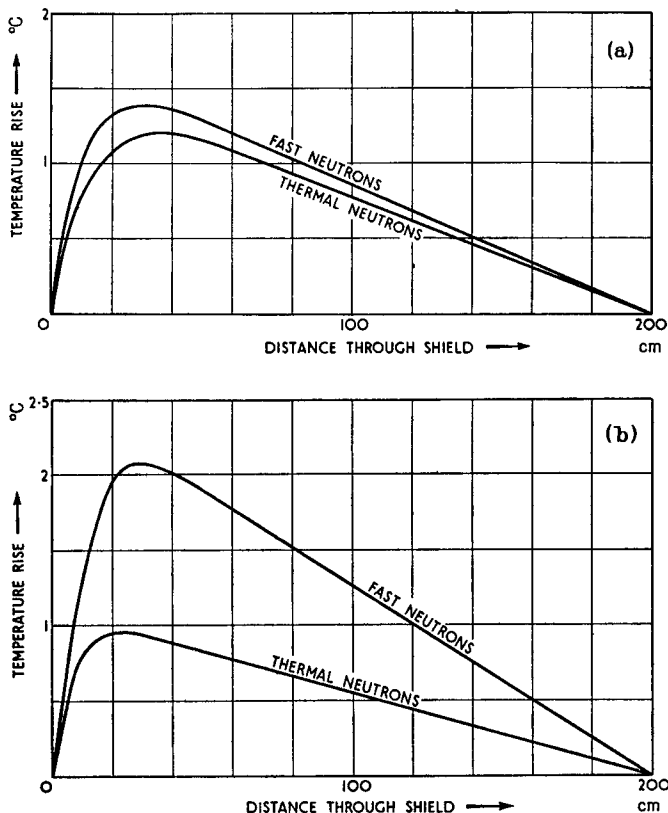


Fig. 6.4.2. Calculated temperature rise per incident milliwatt cm^{-2} in (a) ordinary and (b) barytes concrete shields, two metres in thickness.

The average kinetic energy of the fast neutrons was neglected in comparison with the average binding energy, which in both cases was taken to be 5.5 MeV; this corresponds roughly to 2.2 MeV binding energy in hydrogen and 8 MeV in other materials. In both cases an energy current of 1 mW cm^{-2} corresponds to a thermal neutron flux at the inner face of the concrete of about $2.3 \times 10^{10} \text{ n cm}^{-2} \text{ sec}^{-1}$.

Thermal conductivity of ordinary concrete = 0.00865 $\text{W/cm}^\circ\text{C}$ = 0.5 BThU/hr-ft- $^\circ\text{F}$

Thermal conductivity of barytes concrete = 0.00655 $\text{W/cm}^\circ\text{C}$ = 0.38 BThU/hr-ft- $^\circ\text{F}$
(HALLIDAY.⁽⁸⁾)

whence

$$\int_0^t Q(z) \, dz = k(T_0 - T_t), \tag{6.4.3}$$

where T_0 and T_t are the boundary values of T .

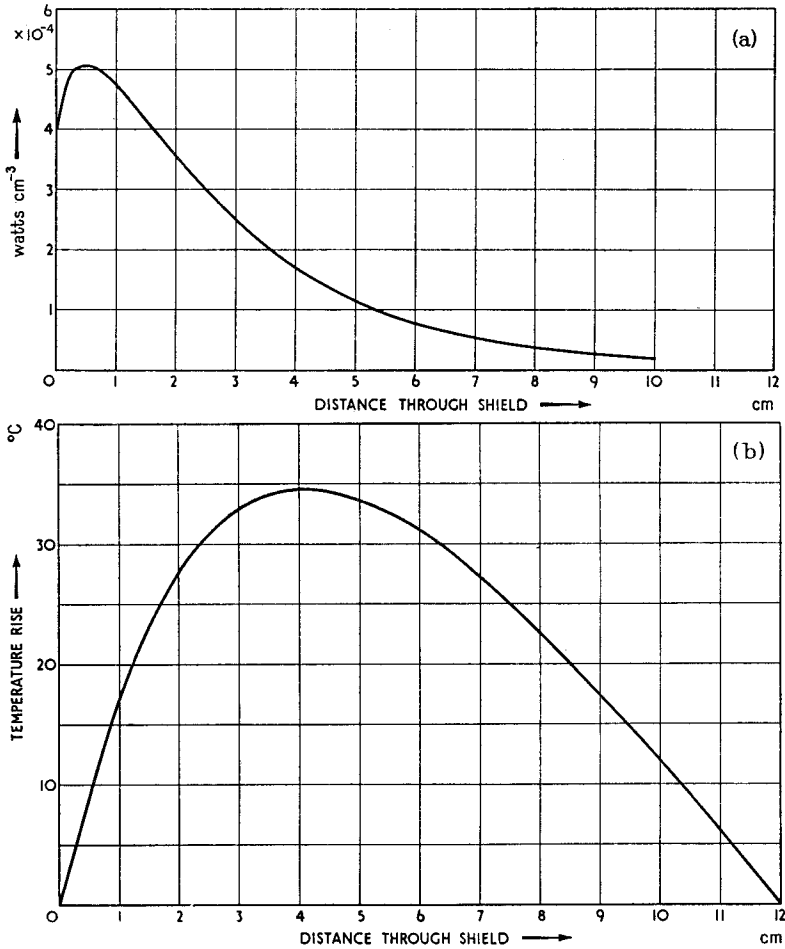


Fig. 6.4.3.

(a) Heat release per unit volume due to a flux of 10^{10} thermal neutrons $\text{cm}^{-2} \text{sec}^{-1}$ incident on an iron shield 12 cm thick.

(b) Temperature distribution, assuming equal temperatures on both faces, when the flux at the inner edge = 6.8×10^{10} thermal neutrons $\text{cm}^{-2} \text{sec}^{-1}$.

The thermal conductivity of iron was assumed to be $0.477 \text{ W/cm}^\circ\text{C} = 27.6 \text{ BThU/hr-ft}^\circ\text{F}$. The binding energy of a neutron in iron was assumed to be 7.8 MeV, and to simplify the calculation was assumed to be liberated in the form of a single quantum.

The constant in equation (6.4.1) can be eliminated by adjusting the Z axis in Fig. 6.4.1 until the area under the curve is $k(T_0 - T_t)$. For the particular case of $T_0 = T_t$, the total integral must vanish. The repositioned Z axis is then indicated by the line $PRSV$ in the figure. The ordinate, measured from P , is then equal to $Q(Z)$.

We find the hottest point $z = Z'$ by making use of the condition $Q(Z') = 0$. Thus Z' is given by the point R where the repositioned Z axis cuts the curve. When $T_0 = T_t$ the hottest point can therefore be determined very simply by choosing R so that the areas OPR , NRS are equal.

Figs. 6.4.2 and 6.4.3 give the results of approximate calculations of the temperature rise expected when neutrons are incident on concrete and iron shields. It will be seen that in concrete the maximum temperature rise occurs at about 30 cm from the inner face of the shield. A rough rule of thumb is that the temperature rise is of the order of 1.5°C per incident milliwatt per cm^2 .

As mentioned below (Section 6.6) it is thought that a temperature rise of 30°C is acceptable in a monolithic concrete shield, so that incident energy currents of ~ 20 milliwatts cm^{-2} , or $\sim 10^{11}$ MeV $\text{cm}^{-2} \text{sec}^{-1}$, can be tolerated. The maximum acceptable temperature rise in a heavy-walled steel pressure vessel, such as is needed for a pressurized water reactor, is also about 30°C , which corresponds to an incident total energy current due to photons and neutrons combined of about 10^{14} MeV $\text{cm}^{-2} \text{sec}^{-1}$.

6.5. THERMAL SHIELDS

If the investigation into heating effects, outlined above, is carried out for a high flux reactor it will probably be found that the proposed material for the bulk shield (if unprotected) would be thermally overstressed. It is then usually considered advisable to reduce the energy flow into the bulk shield to an acceptable level, by interposing between it and the reflector an auxiliary *thermal shield* (Figs. 6.1.2 and 6.2.3). This consists essentially of a slab of high density material of high thermal neutron capture cross-section. A material with these properties absorbs the core gamma radiation efficiently, as well as a large fraction of the gamma radiation associated with the capture of neutrons in the shield, and so concentrates much of the heat in a place from which it is easily conducted away.

The calculation of the heat released in the thermal shield itself follows the lines already indicated. The heating by gamma rays from core and reflector can be obtained once the gamma flux is known as a function of position (page 239), and the method of Section 6.3 can be applied to the heating due to capture radiation from the capture of low-energy neutrons. However, except when the thermal shield is in the form of a very thin plate (e.g. a boral sheet), it is necessary to know fairly precisely how the neutron capture density varies with the depth into the shield. This is not easily calculated, except when the neutrons escaping from the reflector are predominantly thermal; it is therefore best to rely whenever possible on experimentally determined curves of the fall-off of neutron capture density, taken with incident neutron energy spectra similar to those

expected in practice. An example of such a curve, for attenuation in steel, is given in Fig. 6.5.2.

It is reasonable to neglect any heating caused by epithermal and fast neutrons in thermal shields. The great majority have energies well below the inelastic scattering threshold (Section 3.4), so that there will be no appreciable contribution

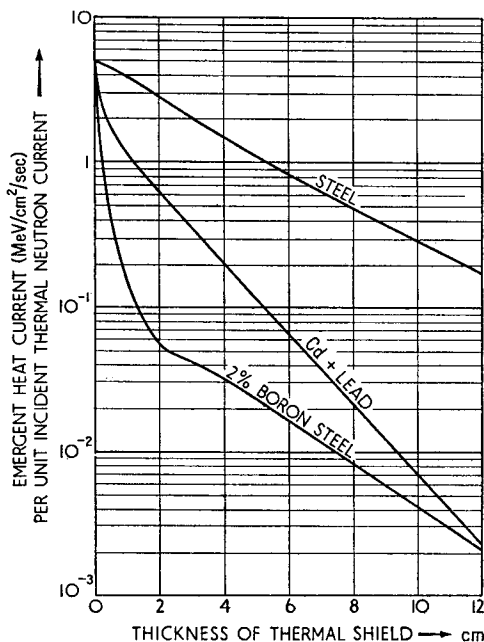


Fig. 6.5.1. Illustrating the effectiveness of three types of thermal shield as absorbers of the energy carried by a beam of thermal neutrons.

The figure shows the total energy current (which, except for very small shield thicknesses, is mainly in the form of capture gamma radiation) penetrating the three types of shield, for unit incident thermal neutron current. The calculation takes into account the isotropic emission of capture radiation, and also the binding energies and capture spectra in the three media.

from this process. Moreover, the energy given up in an elastic collision with a heavy element such as iron is relatively small (equation 4.6.11), and the number of such collisions is normally not large compared with the number of events in which low-energy neutrons are captured. Thus in this context we can assume that the heat evolution is determined by the neutron absorption per unit volume.

Materials for Thermal Shields

In addition to possessing high density, a high melting point, high atomic number, and a large neutron capture cross-section, a thermal shield should also be stable under irradiation, of good thermal conductivity, and cheap and easy to fabricate. *Iron* (in the form of steel or cast iron) was for long regarded as the obvious choice, since it possesses the required nuclear properties, and

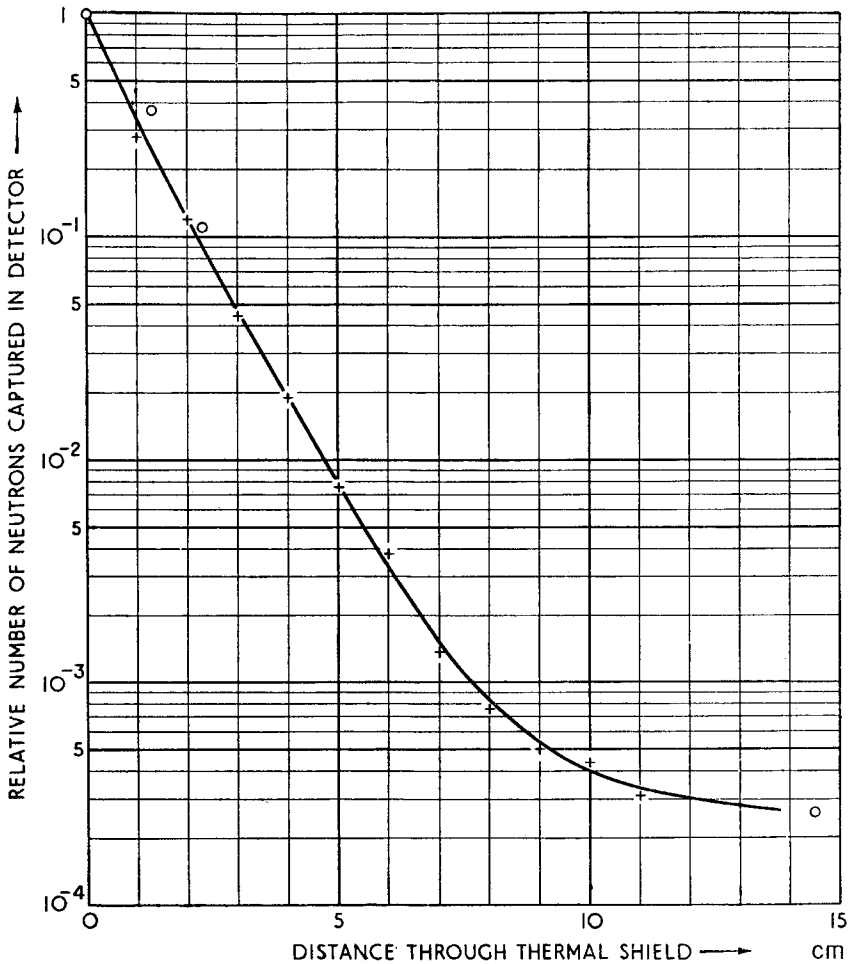


Fig. 6.5.2. Slow neutron absorption in the thermal shield of a graphite-moderated natural uranium reactor (BEPO). The measurements were taken by sectioning a portion of the thermal shield, and measuring the induced activity as a function of position.

Crosses: Reflector thickness 36 inches; Circles: Reflector thickness 46 inches. Calculated two-group currents for 36 inch reflector are in ratio: 1 thermal to 0.03 fast. The shield thickness was 15 cm. The levelling off in the curve as the rear of the shield is approached is partly due to back-scattering of slowed-down neutrons from the concrete bulk shield (see also Fig. 6.6.5) (HARRISON and PRICE).

is easily incorporated into the mechanical structure of the reactor. However, it also has two disadvantages: (i) Long-lived activities are induced in it by neutron irradiation (which may be an important matter if it is ever intended to replace the core) (see Section 6.9); (ii) it emits an extremely penetrating capture gamma radiation (7.7 MeV) which may in certain circumstances be the most important source of the gamma radiation at the surface of the main reactor shield. For instance, an iron thermal shield would be an important extra source of gamma radiation when the main shield was of ordinary concrete, and a considerable thickness of graphite was used as the reflector. Reactor designs vary so much that it is impossible to generalize, but calculations carried out on the lines already indicated will show how much of the hard gamma radiation entering the main shield would be due to neutron capture in an iron thermal shield.

Boron-loaded Iron or Steel

An obvious method for reducing the hard capture gamma production in an iron thermal shield is to mix the iron intimately with some material which has a high thermal neutron capture cross-section and which does not itself produce hard capture radiation; in practice this means using boron-loaded steel. Provided the boron and iron are intimately mixed, boron reduces both the number of neutrons giving hard capture radiation and the residual activity by the factor

$$R = \frac{\sigma_{\text{Fe}}N_{\text{Fe}} + \sigma_{\text{B}}N_{\text{B}}}{\sigma_{\text{Fe}}N_{\text{Fe}}}$$

where the N 's are numbers of nuclei per cm^3 and the σ 's are thermal neutron absorption cross-sections: $\frac{\sigma_{\text{Fe}}}{\sigma_{\text{B}}} = 0.0033$. As R is increased, the contribution of the thermal shield capture gamma radiation to the total dose at the surface of the main shield falls, and it is therefore possible to make the main shield thinner and cheaper; but a limit to what is worthwhile is reached when the surface dose due to other sources of radiation reaches one maximum permissible level. For instance, measurements taken near the outside of two typical graphite-moderated natural uranium reactors (Table 6.5.1) show that if the majority of the gamma radiation observed had been due to capture radiation from the steel thermal shields, it would have been worthwhile to have used borated steel giving R values of up to ~ 10 and ~ 50 respectively.

If boron is introduced by adding ferroboron to molten steel, the maximum R value that can be obtained (with material of the naturally occurring isotopic composition) is about 50, when the boron content of the steel is 3% by weight. Specimens containing more boron are brittle and impossible to machine (Table 6.5.2). Somewhat greater amounts can, however, be introduced by using a dispersion of boron carbide in steel. In this way steels containing up to 8% by weight of boron, and having improved mechanical properties, can be obtained. The effect of boron in reducing the energy current emerging from an iron thermal shield is shown in Fig. 6.5.1.

Besides reducing the rate of production of penetrating capture-radiation, the inclusion of boron also reduces the total heat generated, because the “ Q ” of the $B^{10}(n,\alpha)$ reaction—2.79 MeV—is much less than the binding energy of a neutron in most other materials. On the other hand, most of the energy appears

Table 6.5.1. Approximate biological doses in the concrete shields of two large graphite-moderated reactors, provided with steel thermal shields 6 inches thick

The values were taken at the point in the shield at which the total biological dose is equal to one maximum permissible level.

Type of concrete	Dose in MPL			Asymptotic 10-folding length for fast neutrons (inches)
	Gamma	Slow neutrons	Fast neutrons	
Barytes	1	0.04	~0.1	9
Ordinary	1	<0.001	~0.02	11

See also Fig. 6.6.5.

as kinetic energy of the short-range products, and cannot be radiated away from the point of origin, as is the case with the normal capture gamma radiation. Details of the reaction are given in Table 3.6.1.

*Table 6.5.2. Physical properties of boron steels**

Percentage of boron by weight	2	3	4	5
Thermal expansion over range 20–100°C (10 ⁻⁶ per degree Centigrade)	10.0	10.0	9.5	—
Specific gravity (20°C)	7.72	7.44	7.36	—
Specific heat (cals g ⁻¹ °C ⁻¹)	0.110	0.124	0.125	—
Thermal conductivity (c.g.s. units at 70–100°C)	—	~0.096	—	—
Machinability	Easy, similar to cast iron	Good, normal methods	Stiff, carbide tools needed	Unma- chinable
Available forms	Forged and rolled bar and plate, extruded tubes		Castings	

* These steels are covered by British Patent Application No. 34872/55. The data are reproduced by permission of Messrs Hadfields Ltd., Sheffield. The steels contain a maximum of 0.1% of cobalt and 0.5% of manganese.

The difficulties of manipulating steel with a high boron content have been partly responsible for the development of other types of thermal shield. On the whole the most promising approach appears to be to use separate materials for neutron absorption and for gamma ray attenuation, rather than to try and develop a mixed material combining all the required properties.

The best material for the gamma ray shield which follows the neutron absorber is lead. It has good thermal conductivity, it is easily fabricated by pouring, and it is easy to ensure good thermal contact between the lead and the cladding material which is exposed to the coolant. However, it is more costly than steel, there is a slight health hazard during the pouring operation, and in the event of a reactor runaway the shield might melt. An account of the technique of fabrication of lead shields is given in reference (1).

A large number of substances could be used as neutron absorbers, but in practice the choice rests between boron- and cadmium-bearing materials. Boron may be used in the form of boral, in an aluminium spray, or as boron-impregnated graphite. Cadmium can be alloyed with lead.

Boral^(9,10)

This is the name given to a mixture of boron carbide (B_4C) and aluminium powder, which has been compacted in an aluminium frame, clad with aluminium sheets, and hot-rolled to a final thickness of about 0.25 inch. The technique of fabricating the material, which was originally developed at the American Oak Ridge Laboratory, is described by McKINNEY and ROCKWELL⁽⁹⁾ and by KITZES and HULLINGS.⁽¹⁰⁾ The measured thermal neutron attenuation of a core 0.175 inch in thickness and containing 30% of boron carbide by weight is approximately 1000, which is well above the value normally required. The density is about 2.5 g cm^{-3} . The material can be produced in sheets with an area of up to 20 ft^2 . It can be welded by the heli-arc process, and is easily cut by shearing. It has an adequate thermal conductivity ($\approx 19 \text{ BThU/hr-ft-}^\circ\text{F}$), but some care is needed in design to ensure that for material containing a high percentage ($\sim 50\%$ by weight) of B_4C there is good thermal contact between the boral and the pipes carrying the coolant for the shield. A satisfactory solution is to bolt the boral to a steel plate which is cooled. If this method is adopted, the number and size of the bolts must be such that the area cut out from the thermal shield is kept to an absolute minimum. The stability of the material, under neutron irradiation, is quite adequate for most applications: a sample irradiated until the integrated neutron dose had reached $2.6 \times 10^{19} \text{ n cm}^{-2}$ showed no very marked change ($\sim 10\%$) in average tensile strength.⁽¹⁰⁾

The total cost of boral, including site fixing costs, is about £20 per square foot in Great Britain (1955); of this, production costs account for £7, and transport and fixing for the remaining £13. These figures are based on the construction costs of the DIDO reactor (Fig. 6.2.3). An alternative cheaper material, boroxal (a mixture of B_2O_3 and aluminium) has also been developed in America,⁽¹¹⁾ but since the main cost associated with this kind of thermal shield is site installation, rather than first cost, and since the properties of boroxal are somewhat inferior to those of boral, it is unlikely to be so widely used.

Boron-impregnated Graphite

Graphite combines good thermal conductivity with good neutron moderating properties; so that by impregnating it with a boron compound one obtains a material which is a possible thermal shield, and which in addition makes some contribution towards absorbing the epithermal neutrons. Samples containing 4% by weight of natural boron have been prepared which give an attenuation of thermal neutrons by a factor of 400 in a one inch thickness. An advantage of boron-impregnated graphite is the very low residual activity, even after exposure to intense and prolonged neutron irradiation. This material is used for the primary shield between the core and the heat exchanger compartment of the Dounreay fast reactor experiment.⁽²⁾

Cadmium

This is an excellent absorber of neutrons of energies less than 0.6 eV (see Fig. 3.3.2), but mechanical weakness and the difficulty of securing good thermal contact make the material in sheet form unsuitable as a thermal shield. These difficulties can be overcome by alloying the cadmium with lead. Metallographic examination has shown that good dispersion of the cadmium can be obtained. A well-dispersed sample containing 5% of cadmium, and $\frac{1}{8}$ inch in thickness, attenuates thermal neutrons by a factor of about 500. Cadmium is a cheaper absorber of low energy neutrons than boron. Its neutron capture gamma radiation is less energetic than the principal iron radiation, but is much more penetrating than that from boron. (See Fig. 3.5.5.)

Thermal Shields for Pressurized Water Reactors

A somewhat unusual application of the thermal shield is found in pressurized water reactors. Because of the high pressure at which they operate it is an engineering requirement that the pressure vessel should have the smallest possible diameter (normally less than 10 feet). This results in the use of thin reflectors and small cores. These two factors, coupled with the high power per unit volume which is attainable in such reactors, cause a very large energy current to emerge from the outer surface of the reflector. In practice it is possible for this current to exceed the limiting value beyond which nuclear heating of the thick-walled pressure vessel would cause undesirable thermal overstressing (page 251). Consequently, it is necessary to provide a thermal shield inside the pressure vessel. If made of steel, this may need to be a few inches in thickness; it would be built in laminations, in order to facilitate cooling, the innermost section being about one inch thick.

The difficulties of calculating the nuclear heating due to the absorption of low energy neutrons by such a highly absorbing laminated structure are undoubtedly severe, and they are accentuated by the flux peaking which normally exists in the reflector just outside the reactor core. It should be possible to obtain a direct measurement during the "zero-energy" experiments which normally form part of the development of any new reactor. Failing this, rough results may be obtained by calculation, using age theory (see, for instance, page 66 of reference 1; and also Section 4.7).

The highly corrosive nature of hot water, and the undesirability of introducing

any neutron "poisons" (such as boron or cadmium) into the reactor vessel, probably preclude the use of any material other than steel for such a thermal shield. Even if this were not so, the case against steel would be less strong than in some other situations, since (owing to the thinness of the water reflector) the irreducible fission gamma rays are much more important in relation to the thermal shield capture gamma radiation, and consequently it is much less essential to eliminate the latter.

BULK SHIELDS

6.6. CONCRETE SHIELDS

Concrete is by far the most widely used material for reactor shielding. Its popularity is due to its cheapness, to its satisfactory mechanical properties, and most of all to the fact that it possesses many of the physical attributes of an ideal reactor shielding material. It is a mixture of hydrogen and other light nuclei, and nuclei of fairly high atomic number. It is therefore efficient both in absorbing gamma rays and in slowing down fast neutrons by elastic and inelastic scattering; and the hydrogen contained in the water of hydration of the set cement is sufficient for the rapid thermalization of the intermediate energy neutrons. Its density can be controlled within wide limits, and it lends itself

Table 6.6.1. *Composition of typical Portland cements*⁽¹³⁾

Material	Per cent by weight of unhydrated cement
CaO	60-67
SiO ₂	17-25
Al ₂ O ₃	3-8
Fe ₂ O ₃	0.5-6.0
MgO	0.1-5.5
Na ₂ O + K ₂ O	0.5-1.3
SO ₃	1.0-3.0

Density of unhydrated cement: Normally
3.0-3.2 g cm⁻³

Density of hydrated cement: Normally
about 2.0 g cm⁻³

Water content of hydrated cement: 13-16%
by weight of hydrated cement

easily to monolithic construction, without the presence of inhomogeneities and voids. Its principal disadvantage is the low value of the thermal conductivity (Table 6.6.5) which makes it difficult to extract the heat evolved in the shield as a result of the attenuation of the radiation; for a high-flux reactor the resulting temperature gradients can be large enough to constitute an important design problem (p. 278).

A concrete consists of an *aggregate* (e.g. gravel, barytes, iron shot) bound together with a *cement*. Certain limitations of size are set on the aggregate to

ensure that the mixture is free from large voids and is of sufficiently high density; there are also chemical limitations to ensure that the aggregate does not interfere greatly with the setting properties of the cement. Even with these limitations, there is still considerable freedom in the choice of aggregate, and densities easily obtainable in the final concrete range from well below 1 g cm^{-3} (62 lb ft^{-3}) to about 5.4 g cm^{-3} (340 lb ft^{-3}) for iron shot concrete; the normal mixture of gravel and Portland cement averages about $2.2\text{--}2.4 \text{ g cm}^{-3}$ ($140\text{--}150 \text{ lb ft}^{-3}$).

Provided a normal concrete mix is used, as in general civil engineering practice, the cost is competitive with any other method of shielding. If, however, the

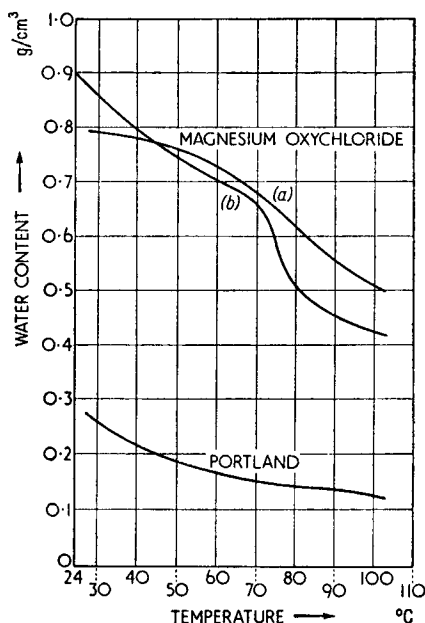


Fig. 6.6.1. Water content of hydrated Portland and magnesium oxychloride cements as a function of temperature. Curve (a) is for normal MgO cement; curve (b) for a cement which uses "previously set" cement as a constituent. (From PAVLISH and WYND.⁽¹⁴⁾)

composition is modified to include expensive ingredients, or if the method of construction needs complicated shuttering⁽¹¹⁾ or an unusual succession of work by different trades, then the overall cost of the placed concrete may be very greatly increased. It is against this background that the properties of the many possible variants mentioned below should be considered: the possible advantage as a shield should always be related to the extra cost that may be incurred.

Cements

The hydrated cement provides a large fraction of the hydrogen content of the shield. The overall shielding effectiveness of a concrete does not depend at all sensitively on the hydrogen content, provided it is above a certain minimum

value. A hydrogen content of the order of 0.5 per cent by weight of the final mixture is satisfactory in this respect, and this is a figure which can easily be reached without departing from normal engineering practice (see page 263).

By far the commonest cement is *Portland cement*, which consists of a mixture of calcium silicate and calcium aluminate, formed by calcining a mixture of chalk and clay. When hydrated it contains about 13–16 per cent by weight of combined water at room temperature.⁽¹²⁾ The water content decreases as the temperature of the cement is raised, a rise in temperature to 100°C resulting in the loss of more than half of the combined water (Fig. 6.6.1).

By using barium instead of calcium one obtains a *barium-Portland cement*—a mixture of the oxides of barium, aluminium, and silicon—which, on hydration, retains more water than normal Portland cement ($\sim 0.4 \text{ g cm}^{-3}$ at room temperature), and which is of higher density.⁽¹⁵⁾

High alumina cements contain more alumina than ordinary Portland cement. They are used industrially on account of their rapid hardening properties. The water content of the hydrated cement may be as high as 0.7 g cm^{-3} if the hydration is carried out below 30°C. If the cement is hydrated at a higher temperature, or if it is heated after hydration, different hydration products are formed, and the water content falls to about 0.5 g cm^{-3} ; however, much of the water of hydration is then firmly attached, and is not driven off unless the cement is heated above 200°C.⁽¹⁶⁾

Table 6.6.2. *Composition of high alumina cements*⁽¹³⁾

Material	Per cent by weight of unhydrated cement
CaO	36–44
SiO ₂	4–10
Al ₂ O ₃	35–44
Fe ₂ O ₃ + FeO	1–20
MgO	0.1–1.4
Na ₂ O + K ₂ O	0.1–1.0
SO ₃	0–1.2

Density of unhydrated cement $\approx 3.3 \text{ g cm}^{-3}$

Density of hydrated cement at room temperature $\approx 2.2 \text{ g cm}^{-3}$

Density of hydrated cement after heating for 15 hours at 100°C $\approx 2.6\text{--}2.7 \text{ g cm}^{-3}$

High alumina cement is used commercially for under-water work, where high early strength is required, and (in conjunction with a suitable material such as firebrick) as a refractory cement. Its main disadvantage is the large amount of heat released on hydration, which may introduce serious problems of stressing and heat removal in a large monolithic structure (Fig. 6.6.2).

Some attention has been given to the use of *magnesium oxychloride cement* as a possible shielding material, on account of its extremely high hydrogen content

(Fig. 6.6.1). The cement, which is made by the reaction between magnesium oxide and magnesium chloride solution, corrodes steel very rapidly, and for this reason has been little used for mass concrete work of the kind needed for a reactor shield. Although many other cements have been investigated, particularly by SNYDER, PAVLISH, and their collaborators,^(14,15,21,77) it appears unlikely that Portland cement will be supplanted by new materials to any large extent.

Aggregates

(a) *Local sand and gravel.* Most reactor shields are constructed from the local aggregate, since haulage of aggregate over any appreciable distance rapidly

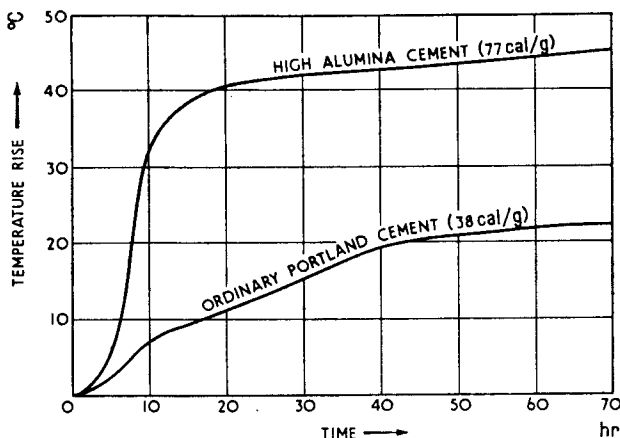


Fig. 6.6.2. The heat of hydration of Portland and high alumina cements, and the resulting temperature rise during the setting of concrete.⁽¹⁷⁾

puts up the cost of the placed concrete. According to the nature of the local material—flint, gravel, etc.—various kinds of “ordinary Portland concrete,” with densities of from 2 to about 2.4 g cm^{-3} , may be made. Because of this low density, the shield needs to be fairly bulky (it is normally between 6 and 9 feet thick). Usually this does not matter, but in some special circumstances the use of denser and more expensive aggregates results in a saving on the cost of the whole installation (page 275). If there is a choice, it is always preferable to choose the densest aggregate locally obtainable.

The gamma-shielding properties of a typical ordinary concrete are shown in Figs. 2.6.5 and 6.6.3. A representative chemical analysis and values for a few of the more important mechanical properties are given in Table 6.6.5. The same analysis is used in Table 6.6.3 to illustrate some of the nuclear properties of the material. One important conclusion to be drawn from the figures in the last column is that the hydrogen contributes a fairly small part of the total removal cross-section for fast neutrons. Consequently, even quite major changes in the water content of the concrete will have a minor effect on the fast neutron attenuation. Thus reducing the hydrogen content by half, from 1.0% to 0.5%, causes an increase of about 1.0 cm in the fast neutron relaxation length—a

Table 6.6.3. Composition and some nuclear properties of a typical ordinary Portland concrete

Nucleus	% wt. in concrete	Macroscopic absorption cross-section (cm ² g ⁻¹ of concrete)	Macroscopic activation cross-section (cm ² g ⁻¹ of concrete)	Half life of induced activity	Quantum energy of induced γ -radiation and abundance	Dose-rate at 1 cm from 1 gram concrete after prolonged irradiation in thermal neutron flux of 10 ¹⁰ n cm ⁻² sec ⁻¹ (mr hr ⁻¹)	Intensity (photons g ⁻¹ sec ⁻¹) of capture γ -rays in thermal neutron flux of 9 $\times$ 10 ⁸ n cm ⁻² sec ⁻¹				Macroscopic removal cross-section for fast neutrons (cm ² g ⁻¹ concrete)
							Photon energies (MeV)				
							1-3	3-5	5-7	>7	
H	1	2.0 $\times$ 10 ⁻³					17.8				6.02 $\times$ 10 ⁻³
O	52.9	—									2.17 $\times$ 10 ⁻²
Si	33.7	9.4 $\times$ 10 ⁻⁴					8.0	19.2	3.46	1.33	9.94 $\times$ 10 ⁻³
Al	3.4	1.7 $\times$ 10 ⁻⁴					0.20	1.19	0.328	0.54	1.02 $\times$ 10 ⁻³
Fe	1.4	3.8 $\times$ 10 ⁻⁴	4.2 $\times$ 10 ⁻⁷	47 d	50% 1.1 MeV 50% 1.3 MeV	0.75	0.27	0.8	0.745	1.69	0.28 $\times$ 10 ⁻³
Ca	4.4	2.9 $\times$ 10 ⁻⁴					1.26	1.51	2.54	0.06	1.06 $\times$ 10 ⁻³
Mg	0.2	3 $\times$ 10 ⁻⁶									0.06 $\times$ 10 ⁻³
C	0.1	2 $\times$ 10 ⁻⁷									0.05 $\times$ 10 ⁻³
Na	1.6	2.1 $\times$ 10 ⁻⁴	2.1 $\times$ 10 ⁻⁴	15.04 hr	100% 2.78 MeV 100% 1.38 MeV	1100	0.932	1.14	0.54		0.53 $\times$ 10 ⁻³
K	1.3	4.0 $\times$ 10 ⁻⁴	1.4 $\times$ 10 ⁻⁵	12.4 hr	20% 1.51 MeV	6.3	1.26	1.26	1.12	0.42	0.32 $\times$ 10 ⁻³
Co (from Fe)	3.5 $\times$ 10 ⁻⁶	1.3 $\times$ 10 ⁻⁶	1.3 $\times$ 10 ⁻⁶	5.3 yr	100% 1.17 MeV 100% 1.33 MeV	4.7					—
Total 0.0044							Total capture γ -rays				Total 0.0410
							30.2	25.0	8.74	4.04	

Calculated fast neutron removal length for density of $2.3 \text{ g cm}^{-3} = 10.6 \text{ cm}$ (observed value for a typical concrete = 12.2 cm).

Calculated average thermal neutron capture m.f.p. for density of $2.3 \text{ g cm}^{-3} = 114 \text{ cm}$.

Calculated diffusion length for thermal neutrons at 20°C (density 2.3 g cm^{-3}) = 10.3 cm.

Typical observed value for diffusion length of concrete containing 0.6% wt. of hydrogen = 7.0 cm;⁽¹⁸⁾ exact value is sensitive to composition.

Notes (1) The cross-sections in columns (3) and (4) of this table and of Table 6.6.4 are for neutrons of velocity 2200 m/sec. Values used are those given by HUGHES and HARVEY.⁽¹⁹⁾

(2) The only activities listed are those of γ -ray emitters with half-lives greater than 1 hour and less than 10^8 years.

change of only 9%. However, some light elements must always be present to prevent build-up of lower energy neutrons. The water content of ordinary concrete appears (from calculations based on Section 4.9 and from the available experimental evidence (e.g. Table 6.5.1)) to be already sufficient to reduce the dose from lower energy neutrons, near the outside of a reactor shield, to a value well below the fast neutron dose. Thus there appears to be no difficulty in preventing undue build-up of lower energy neutrons, and the water content need in practice be increased only if it is desirable to augment the fast neutron attenuation.

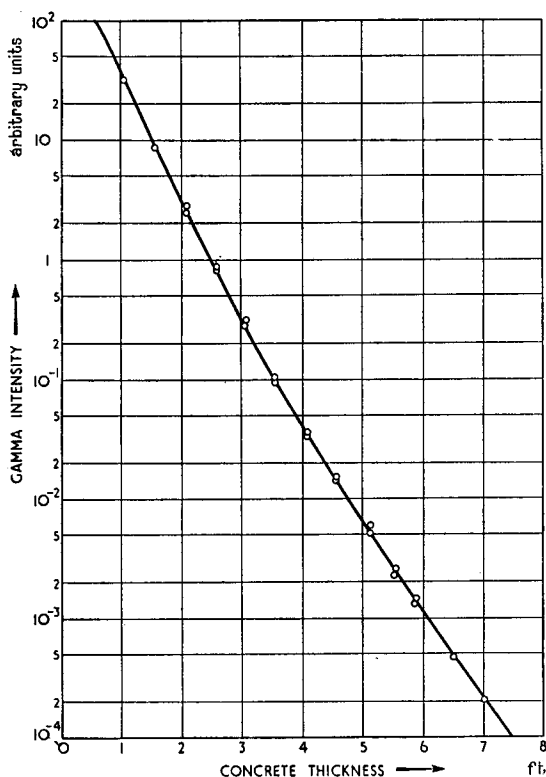


Fig. 6.6.3. Gamma intensity as a function of position in the ordinary Portland concrete shield (density 2.4 g cm^{-3}) of a graphite-moderated natural uranium reactor. The circles are experimental points. The bulk shield was provided with a 6-in. steel thermal shield (MURRAY and SKILLINGS).

The decision whether or not to increase the hydrogen content beyond the normal value is therefore one which should be taken on economic grounds. Increasing the hydrogen is most important when fast neutron attenuation is the limiting factor. By extrapolating from the data just given, we conclude that a further increase of 0.5% in the hydrogen content might then result in a saving of, at most, about 10% in the volume of concrete which has to be poured. Thus the extra hydrogen is justifiable only if it can be incorporated with very little extra cost.

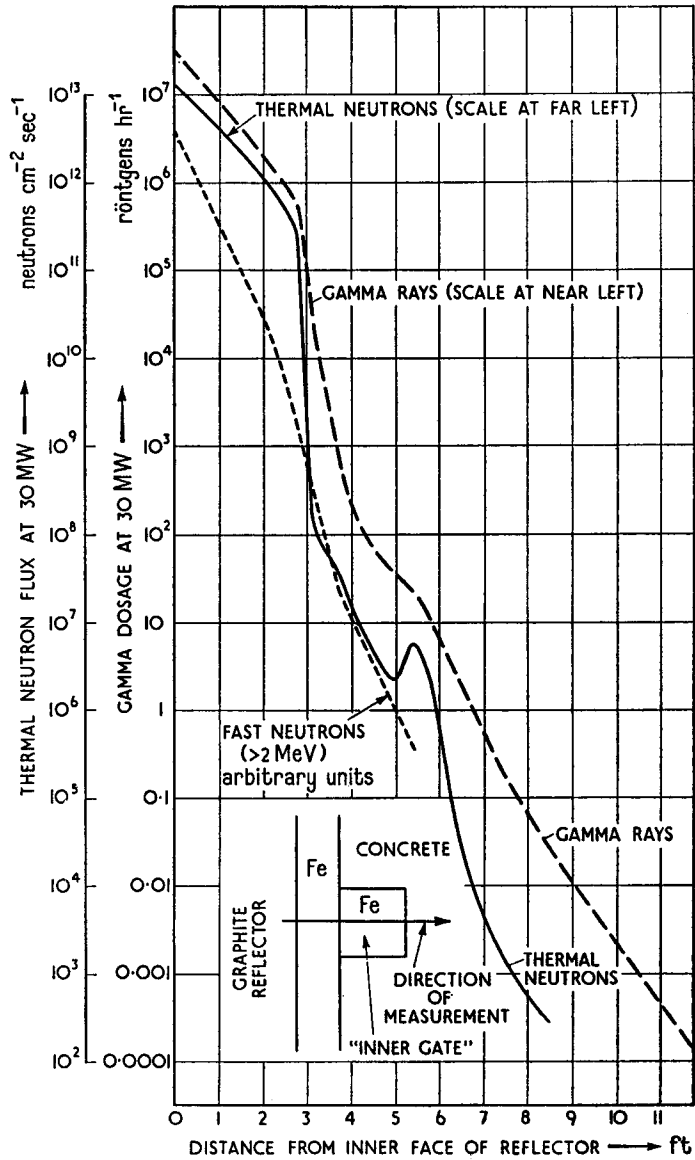


Fig. 6.6.4. Observed radiation intensities in the reflector and shield of the NRX heavy-water reactor. The bulk shield is made of ordinary concrete of density 2.35 g cm^{-3} . The subsidiary peak in the thermal neutron curve at 5.5 ft from the inner face of the reflector is due to the discontinuity between a highly absorbing medium for thermal neutrons (the iron of the "inner gate") and the less heavily absorbing concrete. (BELL, MILLAR, and ROBSON,⁽⁷⁸⁾)

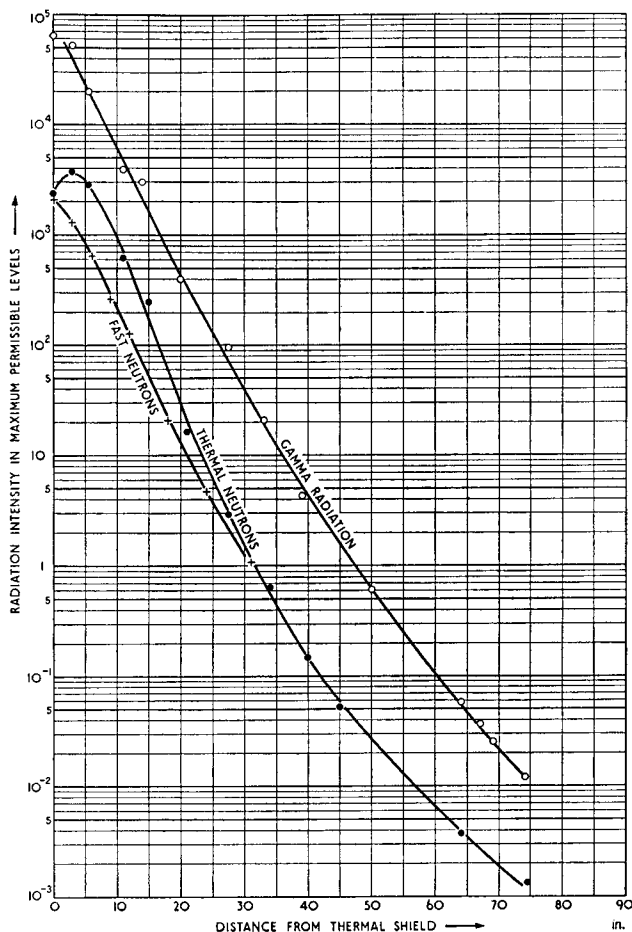


Fig. 6.6.5. Neutron and gamma intensities in the barytes concrete shield of the BEPO reactor, at a reactor power of 6.3 MW.

The reactor is fuelled with natural uranium and is graphite moderated. It has a graphite reflector 3 ft thick and a steel thermal shield 6 in. thick. The concrete has a density of 3.1 g cm^{-3} .

The measured fluxes were converted to MPL by assuming (a) 1 MPL = 7.5 mrem hr^{-1} ; (b) 1 MPL of thermal neutrons = $2000 \text{ n cm}^{-2} \text{ sec}^{-1}$; (c) 1 MPL of fast neutrons = $40 \text{ n cm}^{-2} \text{ sec}^{-1}$. The fast neutrons were measured with a proton recoil counter, which recorded the flux at energies above 0.5 MeV. (PRICE⁽⁸²⁾ and HARRISON, MILLS, and BENDELL.⁽⁸¹⁾)

(b) *Barytes aggregates*. Barytes, the naturally occurring form of barium sulphate, is a widely distributed mineral which is often associated with galena, one of the lead ores. Some, but not all, barytes deposits are of suitable strength and chemical purity to enable them to be used, in the form of gravel and sand, to make a concrete of high density and good mechanical properties (Table 6.6.4). The principal impurities which have a deleterious effect on the mechanical properties are opal and chalcedony. The relatively high densities of from 3.0 to 3.6 g cm⁻³ which may be obtained in practice, combined with the high atomic number of barium, makes barytes concrete particularly suitable as a gamma ray shield. Its properties as an attenuator of pile neutrons are not greatly different from those of ordinary concrete. Barytes has other industrial applications, for instance as a filler for high-grade paper; and this, combined with the additional transport which is usually necessary, may result in material costs eight times as great, and costs of placed concrete twice as great, as when using local materials. Barytes concrete was used for the shields of the MTR, DIDO, and BEPO reactors.

(c) *Iron-bearing aggregates*. The densest concretes are made by using as the aggregate the ore of a heavy metal or the metal itself.^(21,77) Considerations of cost and supply in practice limit the choice to iron in one or other of its forms. Magnetite and iron pyrites (naturally occurring ores) and ferrophosphorus (a byproduct of phosphorus manufacture) have been used to give densities of up to 4.8 g cm⁻³. The resulting concretes are excellent shielding materials, but the materials may be from ten to fifty times more expensive than those for ordinary concrete. The unusually high cost of the aggregate also necessitates more elaborate handling and storage arrangements, and the high density makes ordinary mixers unable to handle a full load and leads to difficulty in placing. These factors are all reflected in the very considerable cost of the complete shield.

Scrap iron of small size and compact shape may be used to give densities of 4.5 g cm⁻³. For special applications it may be economically justifiable to use the more expensive steel shot, in order to obtain the highest density and uniformity. The fast neutron relaxation length in steel-shot concrete is about 6 cm, the thermal neutron capture mean free path is about 7 cm, and the gamma-ray relaxation length at 7 MeV is also about 6 cm (which is less than half the value in ordinary concrete). Thus the neutron and gamma-ray attenuation properties are "well-balanced." The compact nature of the shield makes it a very suitable material for use in places where a thicker shield would complicate other features of the reactor design. For instance, it has been used for the top shield of the British DIDO reactor, where it enables shorter fuel element assemblies and fuel element handling flasks to be used. The resultant saving more than offsets the additional cost of the special concrete. The mechanical properties of the concrete are indicated in Table 6.6.5. In general it is somewhat inferior to ordinary concrete and has a tendency to crumble.

Equal volumes of scrap iron and limonite (see below) were used in the construction of the Brookhaven reactor shield, the limonite being introduced because of its effect in increasing the water content. The shield consisted of a 6-in. inner steel thermal shield, 51 inches of the high-density concrete, and finally a further 3 inches of steel. The average density of the concrete was 279 lb ft⁻³, and it contained 70.4 wt. % of iron and 0.59 % of hydrogen. The

Table 6.6.4. Composition and some nuclear properties of a typical barytes concrete

Nucleus	% wt. in concrete	Macroscopic absorption cross-section (cm ² g ⁻¹ of concrete)	Macroscopic activation cross-section (cm ² g ⁻¹ of concrete)	Half life of induced activity	Abundance and quantum energy of induced γ -radiation (MeV)	Dose-rate at 1 cm from 1 gram concrete after prolonged irradiation in thermal neutron flux of 10 ¹⁰ n cm ⁻² sec ⁻¹ (mr hr ⁻¹)	Intensity (photons g ⁻¹ sec ⁻¹) of capture γ -rays in thermal neutron flux of 9 $\times$ 10 ³ n cm ⁻² sec ⁻¹				Macroscopic removal cross-section for fast neutrons (cm ² g ⁻¹ concrete)
							Photon energies (MeV)				
							1-3	3-5	5-7	>7	
H	0.43	8.6 $\times$ 10 ⁻⁴					7.55				2.59 $\times$ 10 ⁻³
O	31	—									12.7 $\times$ 10 ⁻³
Si	0.74	2 $\times$ 10 ⁻⁵					0.187	0.417	0.08	0.0266	0.22 $\times$ 10 ⁻³
Al	0.57	3 $\times$ 10 ⁻⁵				4.8	0.0338	0.204	0.053	0.0906	0.17 $\times$ 10 ⁻³
Fe	8.78	2.4 $\times$ 10 ⁻³	2.6 $\times$ 10 ⁻⁶	47 d	50% 1.1 50% 1.3 100% 1.17 100% 1.33	31	2.11	5.02	4.65	10.55	1.76 $\times$ 10 ⁻³
Co (from Fe)	0.0022	8 $\times$ 10 ⁻⁶	8 $\times$ 10 ⁻⁶	5.3 yr				0.0266	0.335	0.006	—
Ca	4.53	3.0 $\times$ 10 ⁻⁴					1.295	1.55	2.63	0.062	1.09 $\times$ 10 ⁻³
Ba	41.93	2.1 $\times$ 10 ⁻³					15.1	14.2	2.66	0.186	5.2 $\times$ 10 ⁻³
Mg	0.38	6 $\times$ 10 ⁻⁶				90	0.0266	0.055	0.0124	0.0055	0.12 $\times$ 10 ⁻³
Na	0.13	1.7 $\times$ 10 ⁻⁵	1.7 $\times$ 10 ⁻⁵	15.04 hr	100% 2.78 100% 1.38		0.0765	0.0922	0.0444		0.04 $\times$ 10 ⁻³
S	9.94	9.1 $\times$ 10 ⁻⁴				265	1.545	6.49	7.38	0.65	2.73 $\times$ 10 ⁻³
Mn	0.07	1.0 $\times$ 10 ⁻⁴	1.0 $\times$ 10 ⁻⁴	2.58 hr	100% 0.822 20% 2.06 30% 1.77			0.24	0.266	0.24	0.01 $\times$ 10 ⁻³
Total 0.0067							28.0	28.2	18.1	11.9	Total 0.0266

Calculated fast neutron removal length for density of $3.5 \text{ g cm}^{-3} = 10.7 \text{ cm}$ (observed value for same density = 9.5 cm). (Fig. 6.6.5.)
Calculated average thermal neutron capture m.f.p. for density of $3.5 \text{ g cm}^{-3} = 47.9 \text{ cm}$.
See also the footnotes to Table 6.6.3.

Table 6.6.5. Mechanical properties of various concretes

Property	Ordinary	Barytes	Iron shot	Ferrophosphorus
Mixture (wt. %)	Portland cement 8.2 % Sand 28.7 % Gravel 56.4 % Water 6.7 %	Coarse barytes agg. (1 in.) 45.2 Fine barytes agg. (½ in.) 39.2 Portland cement 9.4 Water 6.2	Iron shot (¾ in.-1 in. dia) 50.4 SAE shot ½ in. dia. 22.8 SAE shot ¾ in. dia. 15.2 Portland cement 8.9 Water 2.7	Portland cement 8.04 Coarse f.p. 57 Fine f.p. 30.7 Water 4.24
Density	2.3 g cm ⁻³ (144 lb ft ⁻³)	3.5 g cm ⁻³ (219 lb ft ⁻³)	5.9 g cm ⁻³ (370 lb ft ⁻³)	4.8 g cm ⁻³ (300 lb ft ⁻³)
Crushing strength at 28 days	3500 lb in. ⁻²	3600 lb in. ⁻² (4200 at 112 days)	3663 lb in. ⁻²	3870 lb in. ⁻² (120 days)
Transverse rupture strength	1000 lb in. ⁻²	845 lb in. ⁻²	~970 lb in. ⁻²	—
Thermal conductivity (BThU hr ⁻¹ ft ⁻¹ °F ⁻¹)	1	1*	1.2	—
Typical chemical composition (wt. %)	H 1 O 52.9 Si 33.7 Al 3.4 Fe 1.4 Ca 4.4 Mg 0.2 C 0.1 Na 1.6 K 1.3	H 0.43 O 31.0 Si 0.74 Al 0.57 Fe 8.78 Ca 4.53 Ba 41.93 Mg 0.38 Na 0.13 S 9.94 Mn 0.07	H 0.33 O 5.82 Si 0.91 Al 0.33 Fe 87.5 Ca 3.96 Mg 0.13 Mn 0.35 S 0.05	H 0.47 O 6.66 Si 0.86 Al 0.34 Fe 61.4 Ca 3.61 Mg 0.12 P 26.3

(These data are taken from references 21, 22, 23, and 77.)

* The thermal conductivity is greater for concretes containing a larger percentage of water. A change in water content of ~3% by weight (of the finished concrete) causes a change in thermal conductivity of the order of 0.1-0.2 BThU/hr-ft²-°F, the change being roughly independent of temperature up to 450°F.

e-folding length⁽⁷⁾ observed for fast neutrons was 6.3 cm. The shield was rather expensive to construct: the total volume of 1200 cubic yards cost \$1,460,000, an average of \$1200 per cubic yard. Half of this was made up of the cost of the steel casing, but the aggregate itself accounted for a further \$250 per cubic yard.⁽²⁷⁾

Ferrophosphorus consists principally of a mixture of the phosphides of iron; a typical sample contains 70% by weight of Fe, 24.5% P, 3% Mn, 1% Si, 1% Ti, and has a density of 6.3–6.8 g cm⁻³. It is a suitable aggregate for use with magnesium oxychloride cements,⁽²⁰⁾ and the concrete formed in this way continues to increase in strength over a period of months, and shows no sign of corrosion or internal stresses. Densities of 285–300 lb ft⁻³ are obtainable.

(d) *Other materials.* A large number of other materials have been investigated in the hope that they would improve the shielding properties of concrete, either by increasing the water content, or by reducing capture gamma radiation through the addition of boron. Thus it has been shown that colemanite (2 CaO·3B₂O₃·5H₂O) is probably the most convenient boron-bearing ore and that the addition of limonite (2 Fe₂O₃·2H₂O) has the effect of doubling the water content of typical concretes, provided their temperature is kept below 100°C.

Engineering Considerations

A shielding engineer cannot be expected to have any detailed knowledge of concrete technology, which is nowadays a highly specialized branch of engineering. Nevertheless, it is highly desirable that he should be aware of the rudiments of the subject, so that he can more easily specify designs that can be constructed without difficulty. In what follows we have therefore summarized a few points which have some relevance to shielding work.

Mixes. The ingredients of mixed concrete may be specified either by weight proportion or by volume proportion. The former method is used particularly when close control must be kept on the quality of the finished material; but for the ordinary run of work it is still more usual, at least in Great Britain, to use proportions by volume. The proportions of sand and coarse aggregate are arranged so that the fines just fill the voidage created by the large aggregate. Thus a typical specification may be written 1 : 2 : 4, it being understood that the materials are listed in the conventional order: cement, sand, coarse aggregate. The size of the coarse aggregate may vary from $\frac{3}{4}$ inch for intricate sections, or $\frac{3}{4}$ –1½ inch for ordinary reinforced concrete work, to much larger sizes for mass concrete. The cement ratios vary from 1 : 6 (volume) for high strength concrete (i.e. 1 volume of cement to 6 volumes of sand and aggregate combined) to 1 : 12 for low strength mass concrete. The higher the proportion of cement the greater is the shrinkage of the concrete. In the belief that the single most important factor in the shield composition was the hydrogen density, the earlier pile shields were constructed with a fairly high cement content (1 : 6). However, since current reinforced concrete practice tends towards the use of 1 : 8 mixes (largely because they are cheaper, but also because there is less shrinkage) and since the part played by the hydrogen is now better understood (page 263), it is probable that future shields will be built with considerably “leaner” mixtures.

The water is added to give the prescribed water/cement ratio, which is usually in the region of 0.35 to 0.7 (by weight); the exact value used depends on the required strength and other properties of the concrete, including the necessary workability. Excessive water results in segregation of the constituents.

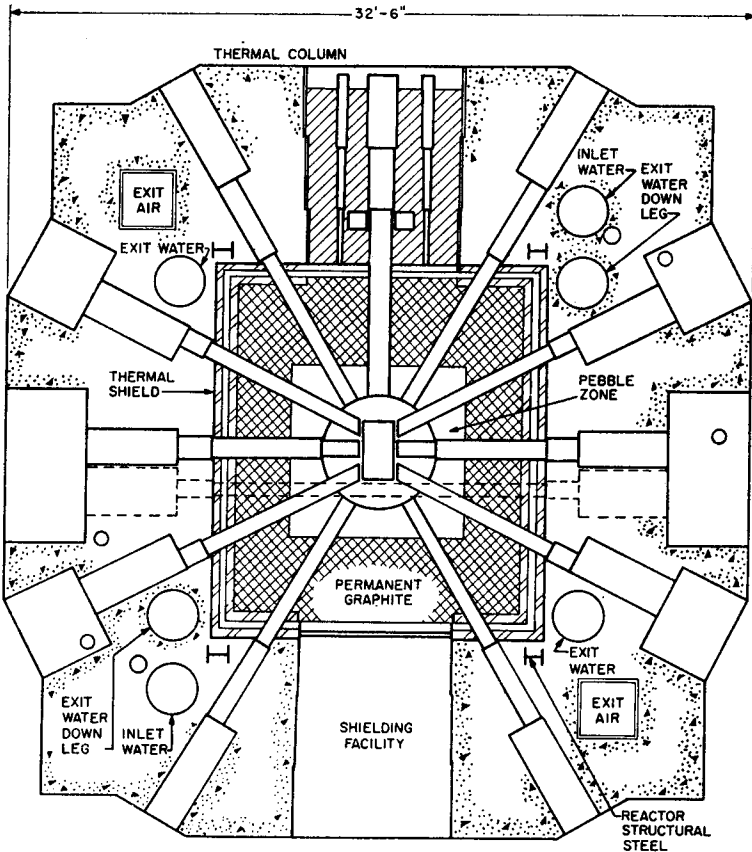


Fig. 6.6.6. Plan view of the U.S. Materials Testing Reactor (MTR).

The graphite reflector consists of two zones, one of which is permanent, and the other a removable assembly of pebbles through which coolant gas is circulated. (From reference 25.)

the fines rising to the surface and giving rise to regions of reduced strength and density. (See also page 273, below.)

Placing. Concrete is usually poured into temporary or permanent frameworks, or "shuttering," which are made of timber or steel. Provision must be made for access to the working volume, to allow the concrete to be placed in "lifts" of 2 to 4½ feet, and to enable the work to be inspected. An interesting alternative is the method of prepacking and subsequent pressure grouting. Its first application to reactors was in the construction of the complicated

shield of the American Materials Testing Reactor (MTR)^(24,25) (Fig. 6.6.6). The aggregate was packed into the nine feet thick shield volume by hand, great care being taken to pack up to and under all horizontal tubes. A cement grout was then flooded into the shield volume through previously placed flooding and sounding tubes, and the tubes were slowly withdrawn upwards as the operation proceeded.

The chief difficulties encountered in placing concrete using normal techniques are in achieving good compaction, particularly where the concrete is heavily reinforced, in avoiding segregation, and in providing adequate construction and thermal expansion joints. Large holes and “honeycombing” are avoided

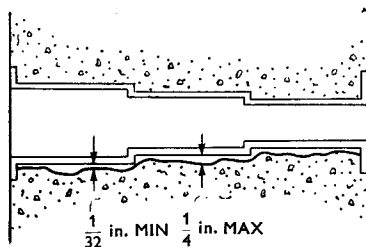


Fig. 6.6.7. Showing type of gap which tends to occur under horizontal tubes through a concrete shield.

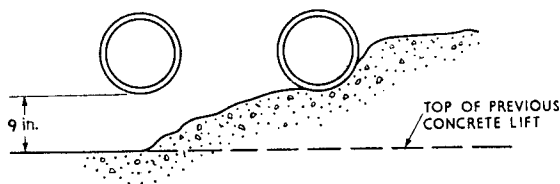


Fig. 6.6.8. Showing preferred termination of lift in relation to horizontal tubes passing through shield.

either by “punning” (i.e. ramming) or by vibrating the concrete. Segregation can be controlled by using a properly-designed mix (e.g. by a proper choice of water/cement ratio) and by avoiding excessive vibration. Shrinkage depends on the aggregate/cement ratio and on the arrangement of the construction joints; some of the associated difficulties can be minimized by terminating a lift just below, rather than just at, a horizontal interface, such as the underside of a horizontal tube. If no special precautions of this kind are taken, a void tends to occur under perhaps one-third of the length of the tube, the gap being about $\frac{1}{4}$ inch across (Figs. 6.6.7 and 6.6.8).

Construction and expansion joints. The vertical interval between construction joints is determined by factors such as the difficulty of placing concrete inside deep shuttering, by the amount of concrete that can conveniently be placed in a working day, and by the rise in temperature that can be tolerated during the setting of the concrete. The horizontal interval between joints is affected by the same factors, and also by the general shape of the shield, and by the

necessity to allow for shrinkage on hardening and for expansion during heating. In either case the aim in choosing the interval is to ensure that no dangerous cracks develop during the life of the shield. What constitutes a “dangerous” crack—from the point of view of radiation leakage—is not easily defined. The

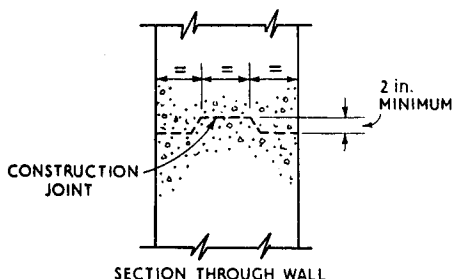


Fig. 6.6.9. Rebated construction joint used in the construction of a reactor shield.

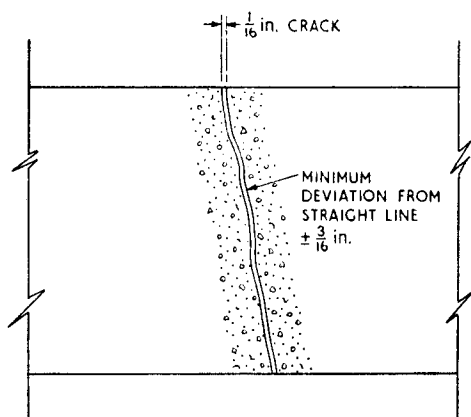


Fig. 6.6.10. Typical crack through mass concrete shield.

policy has been in the past to rebate all construction joints, as shown in Fig. 6.6.9, (so that even if a crack occurs at a joint, there will be no “straight-through path” along which radiation can escape) and to minimize shrinkage cracks by steel reinforcement or by providing expansion joints. However, the application of the duct streaming formulae given in Section 4.12 shows that there is really no need to work to unusually high standards: the shrinkage cracks that can be tolerated in ordinary high-quality concrete work—those which do not affect the structural strength or the aesthetic appearance of the finished work—are not, by and large, such as will cause any hazard from the point of view of radiation leakage, especially as such cracks are hardly ever along a straight line (Fig. 6.6.10).

Control of concrete density. Since the efficiency of a concrete shield depends mainly on its density, a great deal of effort is usually devoted to obtaining concrete of the highest possible density and uniformity. Vibrators are often used to increase the density; but if too much vibration is employed there is a tendency for segregation to occur at the top and bottom of each lift, as illustrated

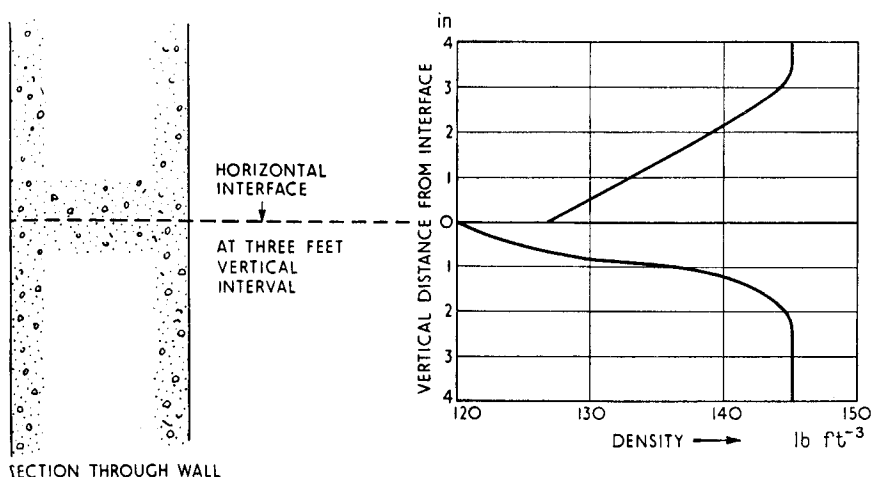


Fig. 6.6.11. Typical density distribution across a lift showing the effect of segregation. (After Dick.)

in Fig. 6.6.11. The fines are drawn towards the top of the lift, and a semi-honeycombed layer is left at the bottom. This segregation can be combated by introducing a layer of mortar $\frac{3}{4}$ inch thick on the top of each lift immediately prior to the placing of the next lift; the mortar then compensates for the fines that may be absent in the bottom layer. Although segregation is known to have occurred at many of the horizontal joints in the shields of existing reactors, no noticeable deterioration in the performance of the shield has been reported. It therefore seems as though specifications have in the past been too tightly drawn, and that some relaxation could be made in the requirement for uniformity of density.

This conclusion is confirmed by two calculations, the results of which are shown in Fig. 6.6.12. In both it was assumed that an infinite plane isotropic emitter was shielded by an infinite plane slab shield, having an attenuation factor about equal to that of a reactor shield ($\sim 10^8$). The curves show the effect of replacing part of the shield with a region of slightly lower density. In Fig. 6.6.12a this region is assumed to be conical in shape, and in Fig. 6.6.12b to be in the form of a plane slab of reduced density—the latter being a reasonably correct representation of the situation near a horizontal construction joint when segregation has occurred. The calculations assumed that the radiation travelled in straight lines, and that it was exponentially attenuated without build-up; consequently they tend to overestimate the effect of the region of

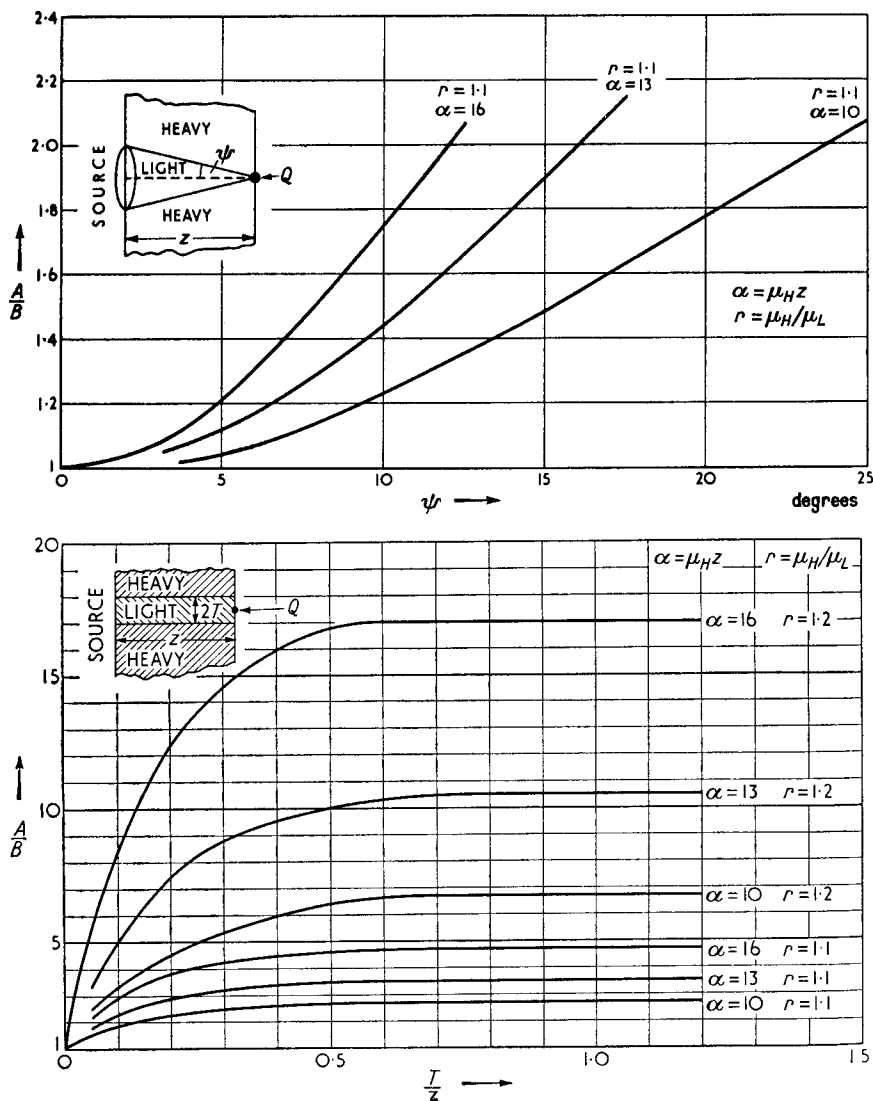


Fig. 6.6.12. Effect of regions of reduced density on radiation transmitted by a shield.

(a) Conical region of low density

(b) Slab region of low density

The intensity which would have been transmitted had the shield been of uniform density is denoted by B . The symbol A denotes the intensity at the point Q when part of the shield is replaced by a material of density lower by a factor r . The linear absorption coefficients in the lighter and denser portions of the shield are denoted by the symbols μ_L and μ_H respectively. (FOSSEY.)

reduced density, which in practice would be somewhat blurred by multiple scattering. A comparison of the density changes observed near an actual construction joint (Fig. 6.6.11) and the predicted increase in dose shows that to all intents and purposes such density changes can be ignored. In short, it appears as though the standards set for good quality civil engineering are already sufficiently stringent for reactor shield construction.

Cost of concrete shields. The cost of a concrete reactor shield depends on a large number of factors, some of which—location and accessibility of site, and the local supply of materials and labour—are outside the control of the designer; but for other factors design is all-important, and attention to detail and ingenuity at the design stage can result in substantial economies in building. Even in a simple reinforced structure the cost of the placed concrete is three or more times the cost of the materials (Table 6.7.1), and in a complicated installation, such as a research reactor provided with many experimental holes, the cost of reinforcement and shuttering may put up the total cost of the shield by a further factor of three. Ideally, therefore, there should be collaboration from the earliest stages between the shield designer and the civil engineer, so that no opportunities are lost of simplifying construction, where this can be done without prejudicing the usefulness of the reactor. To give a specific example: the design of any hole through the shield should, if possible, be such that the liner for the hole does not need to be positioned with very high accuracy. Although it is quite feasible in casting monolithic concrete shields to work to tolerances as close as 0.010 inch for the lining-up of experimental holes, to do so is expensive, because it calls for a rigid steel framework within the shield, or elaborate jiggling, together with a great deal of supervision. A cheaper method, which resulted in considerable savings in the construction of the PLUTO (Harwell) reactor shield, is to leave oversize holes in the concrete, through which the liners are inserted when the concrete is set. After positioning the liners correctly, the surrounding annulus is pressure-grouted with steel-shot concrete. With this technique one can rely on having no shrinkage voids between the concrete and the liner tube.

As far as the costs of materials for concrete are concerned, local variations of price are liable to be important; but superimposed on these variations is a general tendency for shields to become progressively more expensive as their density increases. Indeed, one set of American data,⁽²⁶⁾ reproduced in Fig. 6.6.13, leads to the remarkable conclusion that density is the only significant factor in determining the cost of materials, irrespective of exactly what those materials might be. As long as this price trend continues, the application of the higher density concretes will be restricted to situations where the advantage of having a thin shield offsets the extra cost of material (see Section 6.7). The kind of advantages conferred by a thin, high density shield are simplified design of all gear for the remote handling of in-pile materials and shorter, and therefore cheaper, coffins or flasks for temporary storage of active fuel elements. In addition, with some types of sub-soil it may permit the use of a smaller foundation area, thereby reducing excavation costs. A fourth advantage is that if the reactor is "contained" in a gas-tight building, the saving of a foot in thickness enables the diameter of the building to be reduced by two feet, and the height

by one foot, without reducing the working volume within the building. For reactors with vertical fuel channels there may be a further saving of one foot on the height of the building, because the smaller crane lift needed for the shorter plugs enables the crane to be placed nearer to the top of the reactor. The resulting economies may be considerable. For instance, the cost of the

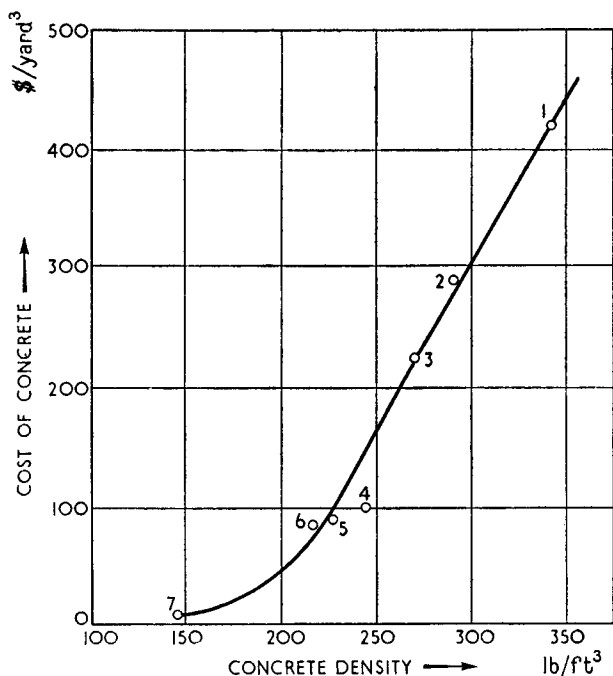


FIG. 6.6.13. Cost of materials (in 1954) for concrete, as a function of density.

The materials were as follows:

1. Steel punchings, dispersed in iron aggregate
2. Ferrophosphorus
3. Steel punchings, plus limonite ore (e.g. Brookhaven)
4. Magnetite (4.95 g cm^{-3})
5. Magnetite (4.6 g cm^{-3})
6. Barytes (4.1 g cm^{-3})
7. Normal (2.7 g cm^{-3})

The densities given in the caption are those of the aggregates. (HENRIE.⁽²⁶⁾)

gas-tight building constructed round the Harwell heavy-water high-flux reactor DIDO (£125,000) would have been increased by roughly £2,000 for each extra two feet of height, and by £4,000 for each additional two feet on the diameter (1955 costs). Finally, a thin shield is an advantage in a physics-research reactor, because it permits higher external beam intensities.

Resistance of concrete to radiation damage. An early indication that radiation damage in concrete is not very serious was given by the behaviour of the Oak Ridge reactor concrete shield, which, though unprotected by a thermal shield,

suffered no apparent deterioration during the course of $5\frac{1}{2}$ years' operation. During this time the integrated dose of radiation at the shield surface⁽¹¹⁾ amounted to 3×10^8 r and 10^{18} neutrons cm^{-2} . This behaviour is confirmed by a careful series of measurements carried out during 1953 to 1956 at Harwell. Forty-eight concrete specimens 2 in. $\times$ 2 in. $\times$ 8 in., suitable for strength determination by transverse rupture, were prepared in a 1 : 3 Portland cement and $\frac{3}{8}$ in. aggregate mix. Some of these were broken as received, some were irradiated in the BEPO reactor, and some were kept as age controls. The blocks were irradiated in a thermal flux of about 10^{12} neutrons $\text{cm}^{-2} \text{sec}^{-1}$, the fast flux being about equal in magnitude; the gamma dose-rate was about 10^6 r hr^{-1} . The total energy deposition was about 0.01 watt cm^{-3} of concrete (density 2.2 g cm^{-3}). If we assume that the total energy deposition is correlated with the radiation damage suffered by the material, then by knowing the energy deposition per cm^3 in any given concrete shield, we may infer its probable radiation life from the data given in the table which follows.

Table 6.6.6. Effects of irradiation on the properties of concrete

- (a) *Blocks as received:* Transverse rupture: 790, 920, 930, 940, and 1000 lb in.⁻².
 (b) *Non-irradiated blocks, oven-tested for five months and subsequently checked for weight-loss and broken:*
 2 blocks at 50°C, 1030 and 1220 lb in.⁻², 2.2% weight loss
 2 blocks at 100°C, 1030 and 1180 lb in.⁻², 3% weight loss.
 (c) *Irradiated blocks, irradiation temperature approximately 50°C.*

Block	Time (months)	Thermal flux $\times 10^{12}$ (n $\text{cm}^{-2} \text{sec}^{-1}$)	Integrated thermal flux n cm^{-2} ($\times 10^{19}$)	Total rate of energy deposition (watts cm^{-3})	Weight loss (per cent)	Rupture stress (lb in. ⁻²)
Q	2	1.1	0.5	0.011	2	1073
R	2	1.1	0.5	0.011	2.1	1076
S	6	1.2	1.6	0.012	2.4	918
T	6	1.2	1.6	0.012	2.6	810
U	12	1.3	3	0.013	2.2	810
V	12	1.3	3	0.013	2.6	940
W	24	1.4	7	0.014	—	734
X	24	1.4	7	0.014	—	627

(Observations by CHISHOLM-BATTEN.)

These results suggest that there was no significant change in strength during the first year of irradiation (integrated thermal neutron flux $\sim 3 \times 10^{19}$ n cm^{-2}), though the two-year blocks show what might be a significant decrease. There is no evidence of the loss by irradiation of significant quantities of water of hydration. We may infer from these figures that, for the majority of the reactors so far built, it is extremely unlikely that there would be any appreciable change in the properties of the shield due to radiation damage during the operational lifetime of the reactor. Radiation damage to concrete is, in fact, a less serious

problem in practice than overstressing due to nuclear heating. In this respect concrete is markedly superior to organic materials.

Nuclear heating of concrete and the necessity for reinforcement. A steel thermal shield 6 to 10 inches in thickness is a feature of many of the reactors which have already been built. It has the extremely important advantage that the major portion of the energy escaping from the reactor is absorbed in a material whose immunity from radiation damage is beyond question and in which no serious problem of thermal stress or heat removal arises. However, it is expensive (Table 6.7.1) and the large capital cost is only slightly offset by the resulting reductions in thickness of the main concrete shield; thus there is a strong financial incentive for determining whether or not the thermal shield is essential. The data given in the previous paragraph lead to the conclusion that for many reactors radiation damage to the concrete is not an important factor. The decision whether or not to provide a thermal shield in such cases therefore turns on the problem of stresses induced by nuclear heating.

Since concrete has such a low thermal conductivity, very appreciable temperature rises (of the order of 1°C per milliwatt of incident energy per cm^2) (Fig. 6.4.2) can take place due to nuclear heating. Such temperature rises cause stresses to be developed, and since concrete has virtually no strength in tension steel reinforcement is normally provided. The amount of reinforcement that is needed depends on the shape of the structure (cylindrical annuli being preferred), on the exact pattern of nuclear heating, and on the restraints imposed on the concrete; but provided the maximum temperature difference in the concrete does not exceed 30° to 50°C both experiment and calculation⁽²⁸⁾ indicate that very little, or "nominal," reinforcement is all that is necessary. The amount of reinforcement rises very rapidly with increasing temperature difference until, for 90°C temperature difference, it is impracticable to include the necessary quantity in the structure. If such large temperature rises are accepted as part of the design, it would then be necessary to reduce the stresses by providing suitable expansion joints (which would effectively divide the structure into a number of smaller units). Whether this solution is preferable to the use of a thermal shield is an economic matter which can only be settled after detailed investigation in a particular case. It should be mentioned that these conclusions regarding the quantity of reinforcement do not take any account of stress-relieving due to "creeping" of the concrete; this phenomenon is known to occur, but little quantitative information is available.

6.7. MINIMUM COST BULK SHIELDS

Now that atomic energy has reached the industrial stage a great deal of thought has been given to the problem of reducing the cost of reactor shields; but although many materials have been studied it appears that the first and most obvious choice—ordinary concrete—is in most cases also the cheapest.

Let us consider the effect of geometry on the cost of a reactor installation which uses a homogeneous type of shield, and in which gamma shielding is the determining factor. Reactor gamma rays are predominantly concentrated in the energy region where Compton scattering is the most important attenuation

process. Thus to a first approximation we can say that, whatever the material, it will be necessary to provide a typical reactor with ~ 670 grams of shielding material per cm^2 of shield surface (corresponding to an attenuation of the order of 10^9). For an idealized spherical reactor, with a core of radius r , and a shield of outer radius R , the weight of the shield in grams is

$$W = \frac{4}{3} \pi \rho (R^3 - r^3)$$

In addition

$$(R - r) = 670/\rho$$

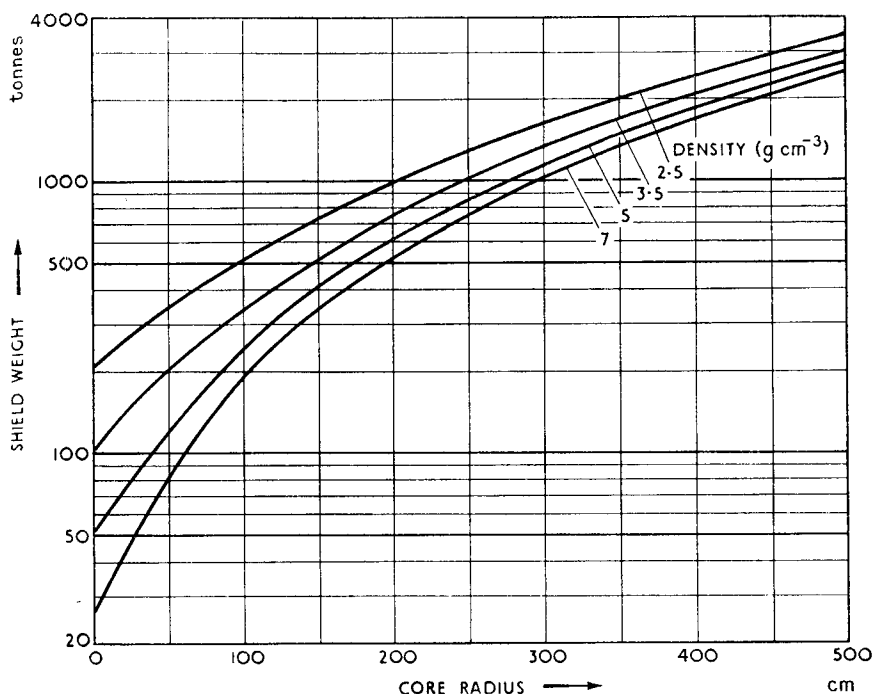


Fig. 6.7.1. Shield weights as a function of density and core radius. The weights were evaluated using equation (6.7.1) and are based on rough gamma shielding requirements only. The shield is assumed to be of uniform composition throughout its volume and to be of an idealized spherical shape.

where $\rho \text{ g cm}^{-3}$ is the density of the shield. Combining these two expressions, we have

$$W = 4\pi \cdot 670 \left\{ r^2 + \frac{670}{\rho} r + \frac{1}{3} \left(\frac{670}{\rho} \right)^2 \right\} \quad (6.7.1)$$

Thus the mass of concrete required is nearly independent of density for large cores and varies as ρ^{-2} for very small cores (see Fig. 6.7.1). However, as we have already seen in Fig. 6.6.13, the cost per unit volume of concrete rises

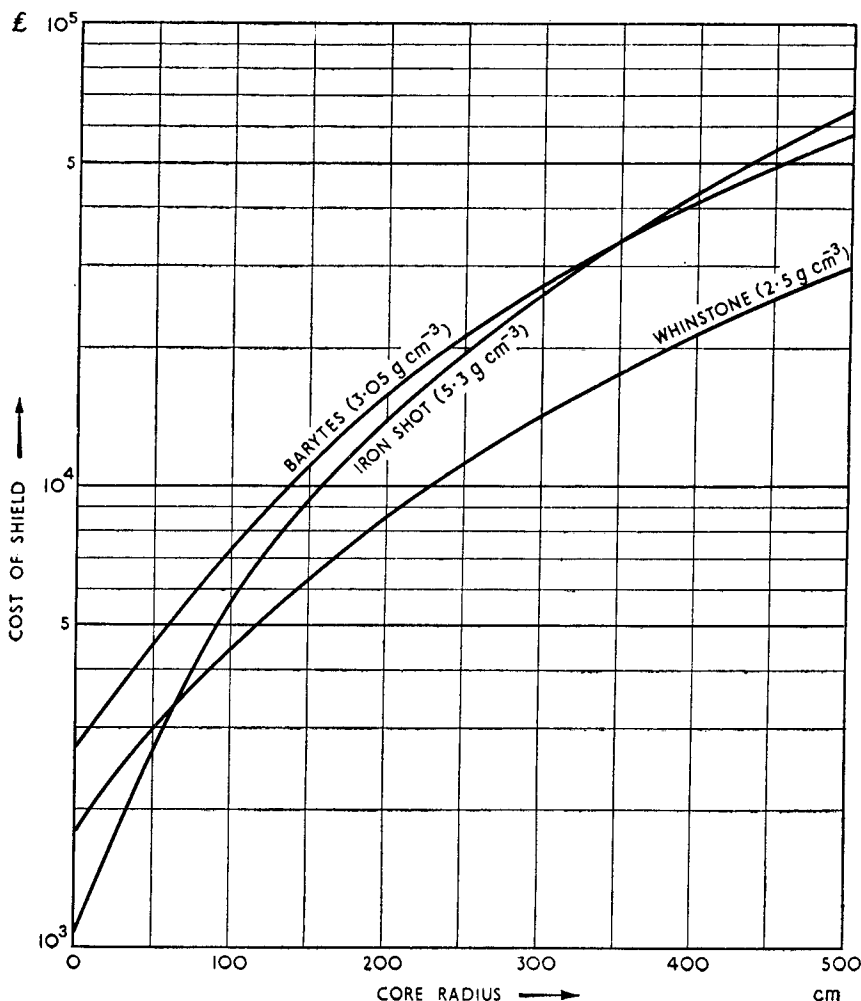


Fig. 6.7.2. Cost of ordinary, barytes, and iron shot concrete for various sizes of idealized spherical reactors. The costs are as given in Table 6.7.2, and include all site charges. It should be noted that the cost for ordinary concrete is subject to considerable variation, depending on the complexity of the shield. The figure of £8.7 per tonne used here is based on the cost of the Windscale reactor shield. The weights on which these costs are based are those shown in the previous figure.

very sharply with increasing density; and this sometimes more than compensates for the reduced mass. The results of a rough comparison of the costs of various kinds of concrete, based on equation (6.7.1) and on the prices given in Table 6.7.2, are shown in Fig. 6.7.2. It will be seen that the natural application of the more expensive high-density concretes is to reactors with very small cores.

The conclusion that ordinary concrete is one of the cheapest shielding materials is further borne out by the data given in Tables 6.7.1 and 6.7.2. In

Table 6.7.1. Cost of shielding materials in Great Britain (1955)
(SIDEBOTHAM and STANDEN⁽²⁹⁾)

Material	Density lb. ft ⁻³	Cost		Remarks
		£ per ton	Shillings per ft ³	
<i>Cements and Concretes</i>				
Portland cement	196	4.75	8.4	
Ciment Fondu		14		40% Al ₂ O ₃
High-alumina cement		120		70% Al ₂ O ₃
Calcined magnesite	219	20	39.1	Commercial dehydrated MgO
Magnesium chloride	98	17	14.9	MgCl ₂ ·6H ₂ O
Anhydrous magnesium chloride	146	84	110	MgCl ₂
Magnesium sulphate	105	17	15.9	MgSO ₄ ·7H ₂ O
Calgon		132		Sodium hexametaphosphate Na(PO ₃) ₆
<i>Aggregates</i>				
Hematite	325	15	43.6	Mainly Fe ₂ O ₃
Limonite	237	4	8.5	Hydrated Fe ₂ O ₃ (free on rail)
Ilmenite	300	7.5	16.1	53% TiO ₂ , ex Malaya (cost, insurance, and freight)
Barytes	280	20	42.8	Mainly BaSO ₄
Borocalcite	140	48	60	44% B ₂ O ₃
Galena	465	75	312	Mainly PbS
Whinstone	200	1.75	3.1	Used in Windscale biological shields
Sharp sand		1.15		
Mild steel punchings		5	21.8	
Blast furnace slag	172	0.8	1.2	37% CaO, 29% SiO ₂ , 22% Al ₂ O ₃ , etc.
Copper refinery slag	234	0.15	0.3	37% FeO, 34% SiO ₂ , 9% ZnO, 11% Al ₂ O ₃ , 7% CaO, etc. (ex works)
<i>Metals</i>				
Cadmium	534	1290	6620	99.9% pure, in stick form
Lead	710	108	685	
Steel plate	488	32	140	
Steel plate (placed)	488	102	446	Based on total cost of PIPPA thermal shield
<i>Hydrogenous Compounds</i>				
Water	62.5	0.014 to 0.021	0.008 to 0.012	Demineralized
Polythene	58	1480	760	Polymerized ethylene
Kerosene	56	21	10.5	Commercial grade paraffin
Asphalt	81	18	13	
Refinery residue oil	58	9.4	4.9	Low grade fuel oil
Ammonia	51	112	—	Pure liquid NH ₃
Lithium hydride	51	44,800	20,400	Technically pure
Titanium hydride	245	12,320	26,950	Technically pure
<i>Boron Compounds</i>				
Boron	145	89,000	115,000	Technically pure
Boron carbide	159	2,240	3,180	
Boral sheet (material only)	160	4,700	6,720	Core of 30% B ₄ C, 70% Al, 0.175 in. thick, clad both sides in Al
Boral sheet (placed)	160	13,440	19,200	As used in DIDO, Harwell
Boron steel	481	32	140	0.15% B
<i>Building Bricks</i>				
Common brick	138	1.95	2.4	Includes 20-mile transport charge
Common brick (placed)	138	8.8	10.8	Scaffold charge included
Concrete brick	150	2.23	3.0	Includes 20-mile transport charge
Concrete brick (placed)	150	9.08	12.2	Scaffold charge included

20 shillings = 1 pound (£).

Table 6.7.2. Relative values (1955) of materials as gamma and fast neutron shields (29)

Material	Density, ρ lb. ft. ⁻³	Cost of shield shillings per ft ³	Gamma attenuation		Fast neutron attenuation		Remarks
			μ (5 MeV) (cm ⁻¹)	Relative value = constant $\times \frac{\mu}{\text{cost}}$	Σ_{rem} (cm ⁻¹)	Relative value* = constant $\times \frac{\Sigma_{\text{rem}}}{\text{cost}}$	
1 : 2 : 4 P.C. concrete, whinstone aggregate	155.0	12.0	0.0715	1.0	0.084	1.0	As used in Windscale biological shields
1 : 2 : 4 P.C. concrete, limonite aggregate	170.4	15.17	0.0817	0.904	0.104	0.99	Bog iron ore, mainly hydrated Fe ₂ O ₃
1 : 2 : 4 P.C. concrete, hematite aggregate	207.4	32.18	0.0999	0.521	0.105	0.47	Iron ore, mainly Fe ₂ O ₃
1 : 2 : 4 P.C. concrete, ilmenite aggregate	196.9	19.97	0.0944	0.793	0.102	0.73	Titanium ore, ex Malaya, > 50% TiO ₂
1 : 2 : 4 P.C. concrete, amblygonite aggregate	146.4	43.11	0.0695	0.271	0.081	0.27	Lithium ore, ex Portuguese E. Africa
1 : 2 : 4 P.C. concrete, galena aggregate	266.4	148.7	0.1627	0.184	0.084	0.08	Lead ore, mainly PbS
1 : 2 : 4 P.C. concrete, barytes aggregate	188.5	30.7	0.0952	0.521	0.088	0.41	Barium ore, mainly BaSO ₄
1 : 2 : 4 P.C. concrete, borocalcite	129.5	34.39	0.0599	0.292	0.088	0.37	Boron ore, approx. 45% B ₂ O ₃
1 : 2 : 4 P.C. concrete, blast furnace slag aggregate	143.0	10.46	0.0665	1.067	0.077	1.06	Mainly calcium, silicon, and al. oxides
1 : 2 : 4 P.C. concrete, copper refinery slag aggregate	169.1	11.64	0.0756	1.090	0.089	1.09	Contains oxides of iron, silicon, aluminium, zinc, and calcium
1 : 2 : 4 P.C. concrete, steel punchings aggregate	276.0	27.14	0.1377	0.852	0.130	0.69	Punchings obtained as scrap from metal perforators
1 : 2 : 4 P.C. concrete, lead aggregate	369.5	312.0	0.2410	0.129	0.094	0.04	Lead in the form of shot
P.C. concrete with 86% by wt. cast iron shot (used for Fig. 6.7.2)	333.0	72.2	0.1655	0.385	0.153	0.30	Max. density obtained by using graded shot
Portland cement, set	124.0	18.4	0.0568	0.518	0.075	0.59	Portland cement, fully hydrated and set

Ciment Fondu, set	112-0	42-8	0-0624	0-245	0-089	0-30	40% Al_2O_3 , hydrated
High-alumina cement set	112-0	368-0	0-0584	0-027	0-111	0-04	"Secar 250" 70% Al_2O_3 hydrated
Plaster of Paris, set	144-0	37-9	0-0688	0-305	0-106	0-40	Hydrated calcium sulphate, $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$
Magnesium oxychloride cement, set	135-0	92-0	0-0631	0-115	0-136	0-21	Hydrated MgCl_2 and MgO
82% by wt. steel punchings in MO cement	344-0	31-2	0-1655	0-890	0-183	0-84	Max. practical density, using these materials
Calcined magnesite	219-0	39-1	0-0991	0-425	0-109	0-40	Commercial dehydrated magnesite, MgO
Common brick	138-0	10-8	0-0665	1-034	0-071	0-95	Cost includes placing with P.C. mortar
Concrete brick	150-0	12-2	0-0686	0-944	0-080	0-94	Cost includes placing with P.C. mortar
Steel plate	488-0	446-0	0-2410	0-091	0-208	0-07	0-25% C, cost includes erection (Windscale thermal shield)
Water	62-5	0-01	0-0303	508-6	0-097	1400	Untreated town's water. Cost does not include containment
60% steel punchings, by vol. in water	320-0	10-27	0-1620	2-643	0-164	2-3	Cost does not include containment
Asphalt	81-2	13-0	0-0376	0-485	0-099	1-1	Oil refinery by-product. Cost does not include containment
Lead	710-0	685-0	0-4860	0-119	0-122	0-026	Material cost only
Cadmium	534-0	6,220	0-035	0-0009	0-141	0-003	Material cost only
Boron	145-0	115,000	0-0398	0-0001	0-141	0-0002	Material cost only
Boron carbide	159-0	3,180	0-0615	0-0032	0-151	0-007	Material cost only (B_4C)
Boral	160-0	19,200	0-0747	0-0006	0-098	0-0007	Sintered B_4C -Al complex. Cost includes installation
Boron steel	481-0	463	0-2410	0-087	0-205	0-064	0-15% B, cost includes erection
Lithium hydride	51-2	20,400	0-0220	0-0002	0-083	0-0006	Material cost only
Titanium hydride	245-0	26,950	0-1255	0-0008	0-187	0-001	Material cost only
Liquid ammonia	38-8	38-8	0-0056	0-024	0-091	0-34	Material cost only
Kerosene	56-0	10-5	0-0274	0-438	0-125	1-7	Material cost only. Comm. grade paraffin
Polythene	58-0	760	0-0282	0-0062	0-124	0-02	Material cost only. Polymerized ethylene (CH_2) _n
Oil refinery residue	58-0	4-9	0-0273	0-935	0-101	3-0	Material cost only. Mainly C, H, and S

* The fast neutron "relative value" is really of significance only for hydrogenous materials.

20 shillings = 1 pound (£).

The abbreviation P.C. denotes Portland cement.

the latter an attempt is made to express in monetary terms the value of a shield for large installations, where geometrical considerations do not specially favour high density materials. The gamma-shielding value is expressed as the ratio of the linear absorption coefficient for 5 MeV photons to the cost of unit volume (the choice of 5 MeV is arbitrary, but reasonable as a test of the value of a reactor shield). The units are normalized so that the "value" of ordinary concrete is unity. The index of value as a fast neutron shield is taken as the ratio of the macroscopic removal cross-section for 8 MeV neutrons to the cost per unit volume, with ordinary concrete again being of unit value. Removal cross-sections were estimated to be 0.9 times the macroscopic total cross-section at 8 MeV for hydrogen, and 0.75 times the macroscopic total cross-section for other materials. It should be remembered that the "neutron values" of shields not containing hydrogen should not be assessed solely on fast neutron data; but unfortunately a truly representative index cannot yet be given since so little is known about the behaviour of intermediate neutrons in such materials.

It must, of course, be stressed that tables such as these can give only a first orientation. What is decisive is the economy of the whole installation, and not simply that of the shield. We have already noted in the previous section particular circumstances in which the use of expensive materials is justified by the engineering advantages which they confer. Nevertheless, for most applications the body of evidence in favour of concrete is impressive.

6.8. METAL-HYDROGEN BULK SHIELDS

Metal-water Shields

It was shown in Fig. 3.3.3 that the total cross-section per unit weight for fast neutrons is considerably greater for light elements than for heavy elements; and as the removal cross-section for fast neutrons (page 180) does not differ greatly from the total cross-section, it follows that the most efficient fast neutron shields (on a weight basis) are those containing large amounts of hydrogen. The most obvious choice is water. Some materials contain more hydrogen per unit volume (Table 6.8.2), but their use tends to be governed by considerations such as flammability, liability to radiation damage, and chemical and thermal stability.

It is also necessary to shield the high-energy gamma radiation emitted by a nuclear reactor, and for this purpose the heavy elements show very marked advantages (Fig. 2.6.4). Consequently, we may expect that the reactor shields having the smallest mass per unit area will be those relying on water or some other hydrogenous material for neutron attenuation and a heavy element (usually lead) for gamma ray attenuation. Moreover, as we shall see later, such a separation of the functions of gamma and neutron attenuation has the additional advantage that it allows the heavy material to be placed to a large extent immediately next to the core, an arrangement which greatly reduces the volume of heavy material that is necessary, and so permits a striking reduction in weight. Although metal-hydrogen shields are probably the only reasonable choice for a mobile reactor installation, they are more expensive than conventional concrete shields, and just as bulky, and therefore find little application

for non-mobile systems. We shall assume in the very approximate treatment which follows that the main aim of the designer is the reduction of shield weight, and that water is to be the hydrogenous material.

The extent to which the separation of the functions of shielding neutrons and gamma rays can be achieved depends very largely on the production of gamma radiation due to neutron capture by the water. The effect of the positioning of the gamma absorbing material on this source of radiation can be studied by considering three groups of neutrons: those which are fast, those which are thermal, and those which have intermediate energies. We shall first consider the relative magnitude of the biological dose-rates contributed by these three energy groups in a water shield containing no heavy gamma-absorbing material. If we assume an infinite plane source of collimated fission neutrons, then the fast flux ϕ_0 varies roughly as e^{-z/λ_0} , where z is the distance into the shield measured from the source, and λ_0 is the relaxation length for the fast group. For positions sufficiently far from the source for a state of equilibrium to have been established between the source neutrons and the "build-up" neutrons of lower energy—which in practice means that z is greater than about 30–40 cm—then the ratio of the thermal to the fast flux may be shown to be

$$(\phi_{th}/\phi_0) \approx [\lambda_0 \Sigma_a (1 - L^2/\lambda_0^2)]^{-1} \exp(L_s^2/\lambda_0^2) \quad (6.8.1)$$

The constants for water have the following values: $L_s^2 \sim 32 \text{ cm}^2$, $L^2 = 7.8 \text{ cm}^2$, $\lambda_0 \approx 10 \text{ cm}$, and Σ_a (the macroscopic thermal absorption cross-section) = 0.020 cm^{-1} ; it will be noticed that these figures satisfy the necessary condition for the flux of lower energy neutrons to be in equilibrium with, and controlled by, the fast neutrons, i.e. that λ_0 should be considerably greater than either L or L_s . Inserting these values in equation (6.8.1) we find that $\phi_{th}/\phi_0 \sim 8$. For water thicknesses ($\sim 180 \text{ cm}$) which give the attenuation necessary in a reactor neutron shield (i.e. 10^8 – 10^9) a large part of the fast neutron dose is contributed by neutrons with energies more than 2 MeV. (Some are "un-collided" neutrons; but, as a comparison of Figs. 6.8.1 and 4.10.1 shows, a considerable proportion of the dose is also contributed by neutrons which have suffered a few small-angle collisions, but which are still fast; intermediate neutrons below 0.1 MeV make a negligible contribution.) For these fast neutrons 1 MPL is given by a flux $\sim 30 \text{ n cm}^{-2} \text{ sec}^{-1}$; while for thermal neutrons $1 \text{ MPL} \equiv 2000 \text{ n cm}^{-2} \text{ sec}^{-1}$. Thus under equilibrium conditions the thermal dose is expected to be $\sim (8 \times 30/2000) = 0.12$ times the fast dose. Measurements⁽⁷⁹⁾ show that at depths $> 100 \text{ cm}$ the thermal flux $\sim 300 \text{ n cm}^{-2} \text{ sec}^{-1}$ when the fast dose at the same point is 1 mrep hr^{-1} . Assuming that the r.b.e. for fast neutrons (page 6) is 10, and that a thermal flux of $2000 \text{ n cm}^{-2} \text{ sec}^{-1}$ is equivalent to 7.5 mrem hr^{-1} , then the ratio of the biological dose-rates is found to be 0.11, in good agreement with the theory. Thus in a water shield the dose contributed by neutrons is very largely due to fast neutrons.

It will be seen from Fig. 4.8.1 that, except for the first 50 cm or so, the neutron flux in water falls off with an e -folding length of about 10 cm. An estimate of the contribution to the biological dose from capture gamma radiation can therefore be made using the formulae of Table 5.2.1, modified to take account

of build-up. The results of such a calculation are given in Fig. 6.8.2, which shows that capture gamma radiation becomes more important than neutrons as a source of biological damage when the water thickness exceeds about 75 cm. Now if we neglect for a moment the attenuation of neutrons by the heavy component of the shield (in a typical case it will actually contribute a factor ~ 100 out of a total required attenuation of 10^8 or 10^9) a shield for a typical

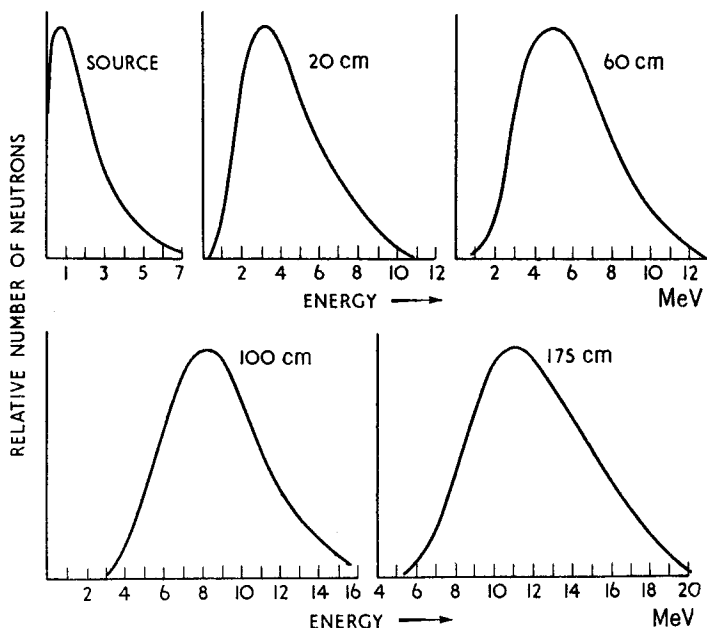


Fig. 6.8.1. Calculated energy distribution of uncollided neutrons from a fission source after penetrating various thicknesses of water.⁽⁸³⁾ (cf. Fig. 4.10.1.)

reactor needs a water thickness of the order of 180–200 cm, in order to cut down the neutron dose-rate to ~ 1 MPL (Fig. 4.8.1). For such thicknesses the capture gamma dose-rate would be of the order of 100 MPL. It would, of course, be possible to complete the shielding by adding a further thickness of water of about one metre (Fig. 2.6.5); but, as light elements are poor absorbers of gamma radiation, it is more economical in weight to use some of the heavy material which has in any case to be provided to shield the gamma radiation emitted by the core and the thermal shield. If there had been no capture radiation from the water, this material would have been positioned as close as possible to the core; but by moving part of it nearer to the outside of the shield it can be made to serve the dual purpose of shielding both the water capture radiation and the core radiation. However, to avoid an unacceptably large increase in weight, the amount of gamma shielding moved towards the outside of the shield must be kept to a minimum.

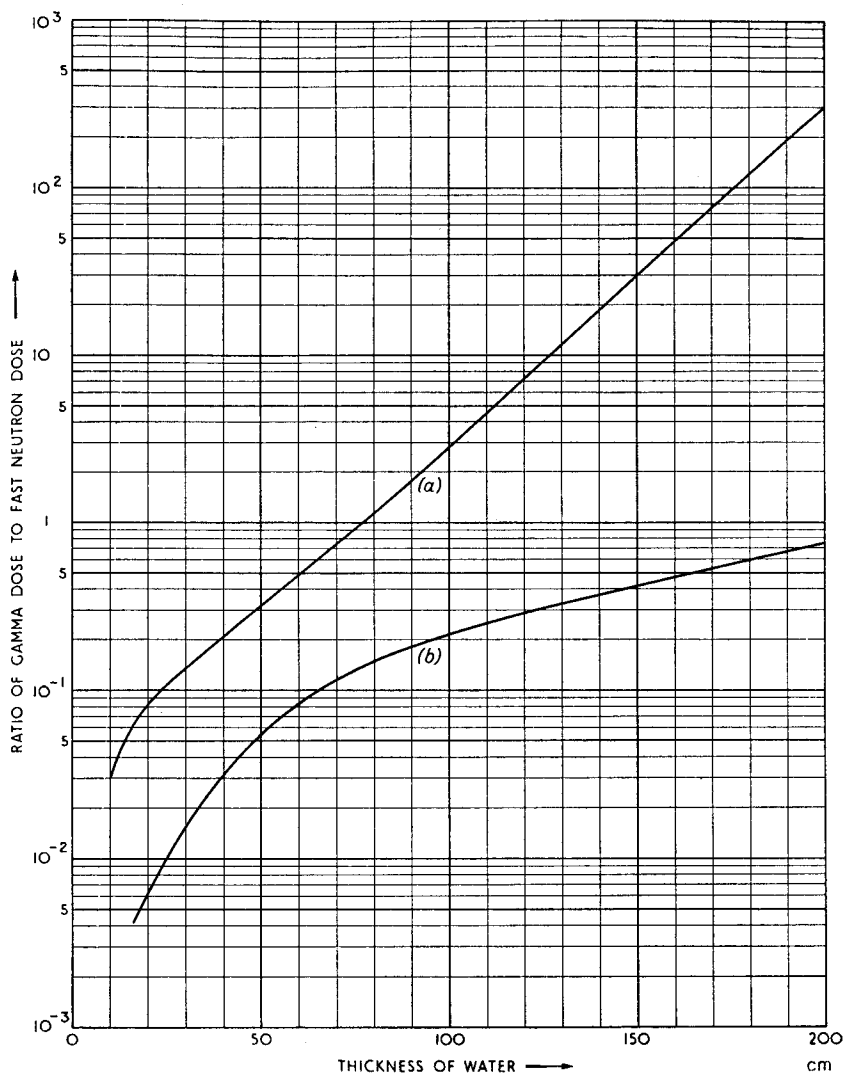


Fig. 6.8.2. Calculated biological dose due to capture gamma rays, expressed as a fraction of the fast neutron biological dose, at the surface of a thick water shield which is shielding a fission neutron source, (a) for untreated water, (b) for water containing sufficient B^{10} for effectively complete replacement of the hydrogen capture radiation by the softer B^{10} radiation.

A first approximation to a suitable layout can be deduced from Fig. 6.8.2. We shall suppose that the shielding of the gamma radiation emitted by the core, reflector, and thermal shield is taken care of by a main gamma shield of suitable thickness, located between the reactor vessel and the water neutron shield; and we shall also make the simplifying assumption that the heavy material which we shall later be moving nearer to the outside of the shield can be regarded as a pure gamma absorber which has no effect on the neutrons, and which therefore does not itself act as a source of capture gamma radiation. It is simplest to work inwards from the outside of the shield. The thickness of water may be taken, for the sake of argument, as sufficient to reduce the fast neutron biological dose rate at the outside of the shield to 0.5 MPL. (This figure and the other radiation levels quoted in this example have no particular sanctity, but they are reasonable values to choose.) From Fig. 6.8.2 it will be seen that the capture gammas from the outer 60 cm of the water shield are not by themselves sufficiently intense to be troublesome, since they contribute no more than half the neutron dose; there is therefore no point in placing any gamma shielding nearer than 60 cm to the outside of the water shield. At this point, however, one can usefully insert a thickness of lead sufficient to attenuate the capture radiation from the inner portions of the water shield until it contributes (say) a further 0.25 MPL, making a total of one MPL. The attenuation required would be in the region of a factor of 100–300 for 2 MeV photons; i.e. about four or five inches of lead.

This arrangement can be improved, and the total weight of the shield reduced, by dividing the lead into a number of plates distributed through the water, with the dimensions of the lead plates and of the water spaces chosen so that as much as possible of the lead is kept close to the core, without at the same time permitting the total gamma dose-rate at the surface to rise above 0.5 MPL. Various arrangements can be evolved very rapidly with the aid of Fig. 6.8.2, and, after applying a correction for the “removal” of the fast neutrons by the lead, the lightest arrangement for a particular reactor can be determined by trial and error.

In principle, it should be possible to obtain a further worthwhile reduction in weight by borating the water, so as to suppress the hydrogen capture gamma radiation. As we have seen, in a water shield with a thickness of (say) 150–200 cm, it is desirable to reduce the capture gamma radiation by a factor of the order of 100. If we were able to dissolve so much boron in the water that virtually all the neutrons were captured by boron instead of hydrogen (giving 0.5 MeV instead of 2 MeV capture gammas), we would obtain curve (b) of Fig. 6.8.2. For the water thicknesses in which we are interested this would reduce the capture gamma dose to a level comparable with the fast neutron dose, thus permitting gamma shielding to be virtually dispensed with in the outer portions of the shield; moreover the softness of the boron capture radiation would facilitate any gamma ray shielding which might be necessary. By using concentrated solutions of certain boron compounds it should be possible to come quite close to the idealized conditions of curve (b) (see Table 6.8.1). However, there would be many practical difficulties, such as the supply of the material, corrosion, and the general undesirability of having very large quantities

of a powerful neutron “poison” in the vicinity of a reactor. Although lithium gives no capture radiation, it is less useful than boron owing to its lower capture cross-section. The reduction of this source of gamma radiation does not necessarily, of course, affect the *total* thickness of gamma shield, which may be determined by the core radiation; but it does allow the heavy material to be still further concentrated towards the inside of the shield.

Table 6.8.1. Suppression of penetrating capture radiation by use of concentrated solutions of boron or lithium compounds

Compound	Solubility in grams per 100 ml water at stated temperature ^(22,30)	$\frac{\text{Captures in boron and lithium}}{\text{Captures in other elements}}$ in the saturated solutions at stated temperatures*
$\text{NH}_4\text{HB}_4\text{O}_7 \cdot 3\text{H}_2\text{O}$	10 (cold water)	30
H_3BO_3	5 (20°C)	17
B_2O_3	2.2 (20°C)	13
LiBO_2	16 (45°C)	73
$\text{K}_2\text{B}_4\text{O}_7 \cdot 5\text{H}_2\text{O}$	26.7 (30°C)	60
NaBF_4	108 (26.5°C)	180
$\text{Na}_2\text{B}_2\text{O}_4$	26 (20°C)	80
$\text{Na}_2\text{B}_4\text{O}_7 \cdot 10\text{H}_2\text{O}$ (borax)	2.7 (20°C)	6
$\text{LiC}_2\text{H}_3\text{O}_2 \cdot 2\text{H}_2\text{O}$ (lithium acetate)	300 (15°C)	20
LiBr	143 (0°C)	8

* The boron is assumed to have its natural isotopic composition.

The simplified approach to the design of a light-weight bulk shield, outlined above, takes no account of the inelastic scattering or capture gamma radiation produced in those portions of the gamma shield which are distributed in the water. It seems probable that the gamma rays from inelastic scattering can be neglected. A crude estimate of their importance can be made by equating the number of inelastic scattering events to the number of source neutrons “removed” by the heavy material. Fig. 4.10.1 gives the fast neutron energy spectrum in water at various distances from a fission neutron source, and in conjunction with the data of Table 3.4.1 enables a very approximate idea of the spectrum of inelastic scattering gamma radiation to be formed. Knowing the source strength and quantum energy distribution, a straightforward calculation will show whether or not the inelastic scattering gamma rays play any important part in determining the shield layout. Normal methods of neutron theory can be used, in conjunction with the data of Table 3.5.1, to estimate the importance of capture gamma radiation emitted by the heavy material. The capture radiation arises very largely from neutrons which have first been thermalized in the water, and subsequently captured by the heavy material; so that an improvement can be made, if necessary, by coating the heavy material with boral.

There are a number of practical difficulties associated with the mechanical design of metal-water shields. The gamma shielding, if made of lead, cannot

be regarded as self-supporting, and normally has to be cast into steel housings; the design of such steel supports is discussed in considerable detail by ROCKWELL.⁽¹⁾ The steel reinforcement is a potential source of very energetic gamma radiation (Table 3.5.1), the suppression of which will tax the ingenuity of the designer. In some situations the lead can be used in the alternative form of lead shot. About 60 per cent of the volume can be filled in this way, but the method has the disadvantage that the shot tends in time to bed down under its own weight, leaving a gap at the top of the shield. To prevent corrosion it is desirable to avoid contact between the lead and any steelwork that is wetted by water. The water which forms the neutron shield must be contained in a vessel strong enough to support its very considerable weight with certainty, since a sudden rupture of the tank would necessitate an immediate shutdown. It is necessary to provide for the escape of the appreciable quantities of gas which are produced by radiation decomposition of the water (see below). Finally, when the engineering of the complete reactor is worked out in detail, it will be found that the conception of a close-fitting shield (such as we have been using here) is little more than a mere abstraction. In practice the provision of (for instance) coolant ducts, and shielded heat exchanger spaces, causes the design to depart very radically from the idealized layout discussed above.

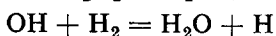
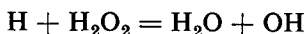
Effects of Radiation on Metal-water Shields

Under the influence of radiation, water is decomposed by the two reactions:⁽³¹⁾



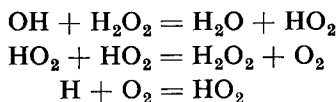
The relative yields of these reactions depend strongly on the ionization density produced by the radiation. Densely ionizing radiations, such as knock-on protons or alpha particles, cause reaction (2) to predominate, while the more diffuse ionization from gamma radiation favours the first reaction, with the formation of the active radicals H and OH. The total yield of water decomposition by the two processes is independent of the ion density. In pure water the overall process initially produces 0.4 molecule of hydrogen per 100 eV of energy absorbed in the water for radiations of low ionization density, about one molecule for mixed reactor radiations, and about 1.8 molecule for densely ionizing radiations. In the absence of dissolved substances the radicals H and OH react with one another to form H_2 , H_2O_2 , and H_2O .

If these were the only reactions taking place water would be decomposed very rapidly when used as the coolant or moderator in a high-flux reactor. However, there also exists a series of back reactions in which the radicals formed in reaction (1) act effectively as catalysts for promoting the reverse of reaction (2), so reforming water. The reactions involved are thought to be the following:



These reactions are facilitated by an increase in the concentration of gaseous hydrogen dissolved in the water. If, however, the water contains too large a

quantity of hydrogen peroxide, a further reaction breaks the chain of recombination:



The radicals formed in process (1) are highly reactive, and will combine with impurities, such as chlorides, which may be dissolved in the water. They are then no longer available for promoting the back reactions, so that the hydrogen formed in reaction (2) appears as bubbles of gas, and the hydrogen peroxide accumulates in the solution and eventually decomposes to water and oxygen. Whether or not gases are evolved when water is irradiated therefore depends on the relative rates of the reactions (1) and (2), (gas formation being more probable the smaller the yield of reaction (1), i.e. the higher the ionization density), on the purity of the water, and on the relative amounts of hydrogen and hydrogen peroxide retained in the water. For these reasons water used as a reactor coolant has to be kept very pure by the use of ion exchangers, and to achieve minimum decomposition an atmosphere of hydrogen is retained above the water. In practice the hydrogen produced by water decomposition will often reach a steady-state concentration; since, if the system is totally enclosed, the hydrogen pressure builds up until the reverse reaction becomes effective. There is often a tendency for hydrogen to be present in excess of the oxygen and hydrogen peroxide, because of the additional evolution of hydrogen and consumption of oxygen in metal corrosion processes.

It is much more difficult in a reactor shield to maintain the clean conditions necessary for the inhibition of water decomposition; and although the problem is less serious than when the water is inside the core, the volume of gas that can be liberated is nevertheless great enough to warrant some consideration at the design stage. This may be shown by the following illustrative calculation. Consider a water shield such that at the inner surface the incident current of fast neutrons is $10^{10} \text{ n cm}^{-2} \text{ sec}^{-1}$, and assume that the back reactions are completely inhibited by impurities in the water and by failure to keep the system under the correct atmospheric conditions. Then, excluding any further amount which would in practice be contributed by thermal neutrons and gamma rays, the energy per cm^2 per second carried into the shield by the fast neutrons is of the order of

$$\begin{aligned}&10^{10} [\text{Kinetic energy (say 4 MeV)} + 2.2 \text{ MeV binding energy}] \\ &= 6 \times 10^{16} \text{ eV cm}^{-2} \text{ sec}^{-1}\end{aligned}$$

If the diameter of the inner surface of the water shield (assumed spherical) is 2 metres, the total number of molecules of hydrogen evolved per second is

$$\sim \frac{4\pi \times 10^4 \times 6 \times 10^{16} \times 1.0}{100}$$

which corresponds to the evolution of 250 litres at S.T.P. per day. The quantity of hydrogen evolved in the shielding water of the American Submarine Intermediate Reactor (with a maximum gamma flux of 10^{11} and a neutron flux of $10^{10} \text{ cm}^{-2} \text{ sec}^{-1}$) was found in practice to be 1 litre per minute, or 50 ft³

per day. It should be noted that the inclusion of boron in solution in the water promotes gas evolution, as some of the energy is then dissipated in the water by the densely ionizing alpha particle from the $B^{10}(n, \alpha)$ reaction⁽³²⁾ (Section 3.6).

In comparison with the effect of radiation on the water, its effect on the metal which forms the other component of the shield is trivial. This metal would in all probability be either steel, or lead supported by steel. The mechanical properties of steels appear to be unaffected⁽³³⁾ by exposure to an integrated slow neutron dose of $\sim 10^{20}$ neutrons cm^{-2} . There are relatively few reactors in which this dose would be exceeded in the shield during the operational lifetime of the reactor.

Other Hydrogenous Materials

If the reactor core is sufficiently small, the weight of the water portion of the shield may considerably exceed the weight of the heavy material. For instance, a spherical reactor with a core 4 feet in diameter, surrounded by a shield consisting of 1.5 feet of lead ($\sim 10^9$ attenuation in gammas, $\sim 10^2$ in fast neutrons) and 5 feet of borated water ($\sim 10^7$ in neutrons), would have a shield weight of 120 tons, made up of 70 tons of water and 50 tons of lead. The great weight of the neutron shield is due to the large volume that is necessary, which in turn is mainly determined by the number of hydrogen atoms per cm^3 of water. There are a number of materials which contain more hydrogen per unit volume than water, and some of which have at the same time lower densities

Table 6.8.2. Concentration of hydrogen in various materials, and rough values for their susceptibility to radiation damage

Material	Formula	Density (g cm^{-3})	Atoms of hydrogen per cm^3 ($\times 10^{23}$)	Rough value for exposure giving 25% change in most sensitive property (rad)
Water	H_2O	1.00	6.7	See pages 290 and 291
Titanium hydride	Variable	3.3 to 3.8	8 to 9	—
Paraffins	$\text{C}_n\text{H}_{2n+2}$	0.87 to 0.91	7.8 to 8.0	10^7 – 10^8
Polythene	$(\text{CH}_2)_n$	0.87 to 0.91	7.8 to 8.0	See Fig 6.8.4
Lithium borohydride	LiBH_4	0.67	7.6	—
Aromatic hydrocarbons	—	0.85 to 1.05	4.7 to 6.8	$\sim 3 \times 10^8$ to 10^{10}
Fuel oil	—	0.89	6.4	$\sim 10^8$
Lithium hydride	LiH	0.82	6.21	Probably large ($\sim 10^{11}$)
Lucite (Perspex)	$(\text{C}_5\text{H}_8\text{O}_2)_n$	1.2	5.8	$\sim 5 \times 10^7$
Ethyl cellulose	$(\text{C}_{10}\text{H}_{18}\text{O}_5)_n$	1.11	5.5	$\sim 5 \times 10^7$
Polystyrene	$(\text{C}_8\text{H}_9)_n$	1.05	5.4	$\sim 10^{10}$
Masonite	—	1.3	4.7	$\sim 10^9$
Wood (ash)	—	0.64	2.5	$\sim 10^9$

From references 22, 34, and 35. The effects of radiation damage on plastics are discussed in detail in references 35 and 36. Note that the threshold doses quoted by different observers are not always based on a uniform method of measurement, and may therefore be not exactly comparable.

(see Table 6.8.2). Such materials might therefore be expected to have applications in the field of propulsion reactors, where weight, and not expense, is the prime consideration. The probable behaviour of such neutron shields can be deduced without difficulty from the published removal cross-sections and from the behaviour of water.

The materials which have been considered as possible replacements for water fall into two main categories: the hydrides (including the boranes) and organic compounds. The hydrides^(37,38) are of interest because they form a possible way of combining in one material both the light and the heavy constituents of a reactor shield, or (in the case of lithium and boron hydrides) of obtaining an efficient neutron shield with a minimum of capture gamma radiation. There are two classes of hydride, those which are ionic in nature, such as LiH, and those which are solid solutions of hydrogen in a heavy metal. Of the simple ionic compounds, lithium hydride has the highest hydrogen concentration—about the same as that of water—and is thermally relatively stable, but it reacts vigorously with the water vapour in air, and is in other ways a difficult material to use.⁽⁷⁶⁾ The radiation resistance of simple ionic solids is frequently high, and the reformation of a compound (e.g. LiH) from its constituent elements may also occur to some extent. Hydrides of the solid solution type can be made to contain appreciably greater quantities of hydrogen per unit volume than water (e.g. titanium hydride, Table 6.8.2) but their instability makes them unsuitable as engineering materials. The boranes tend to be very unstable, though decaborane, $B_{10}H_{14}$, melts without decomposition at 100°C; however, it reacts vigorously with moist air at this temperature. To sum up, it appears as though the reactive nature of all the hydrides prevents them from being seriously considered for most shielding purposes; and added to this, their cost would be prohibitive for all except military projects (Table 6.7.2).

On the other hand, some organic materials which contain relatively large amounts of hydrogen are easily handled and are also fairly cheap (Tables 6.7.1 and 6.7.2). After prolonged exposure to radiation they undergo physical changes^(39,40) due to the combined effect of decomposition into gaseous products and of polymerization. The gas evolution is usually characterized by a "G-value," which is equal to the number of molecules of hydrogen produced for every 100 eV liberated by the absorption of radiation. The G-value for hydrogen evolution ranges from 10^{-2} for the more resistant materials (aromatic compounds, particularly the polyphenyls) to between 1 and 10 for the less resistant materials (long-chain compounds such as the paraffins). In general, the evolution of gas cannot be inhibited, as in the case of water, by maintaining an equilibrium hydrogen pressure over the system.

The useful lifetime of many organic liquids is determined by the very rapid increase in viscosity which occurs after a certain threshold dose (see Fig. 6.8.3). Rough values for the threshold are given for typical materials in Table 6.8.2. The restriction on integrated dose, due to this increase in viscosity, is not a serious one in the context of shielding applications; and any change of properties can be greatly reduced by a slow circulation or by replacement of the liquid. A specific application of organic liquids where they may be preferred to water on the grounds of safety is in the shielding of a sodium cooled thermal reactor.

For organic solids harmful modifications to the mechanical properties occur at integrated doses which are similar to the liquid thresholds already mentioned. The observed changes are discussed in great detail by BOPP and SISMAN.^(35,36) A typical effect is the reduction in the tensile strength and impact strength of plastics (see Fig. 6.8.4). Solid organic materials which have been used in reactor shields include polythene and masonite (compressed wood). The former has relatively low radiation resistance and is also difficult to use if the shield

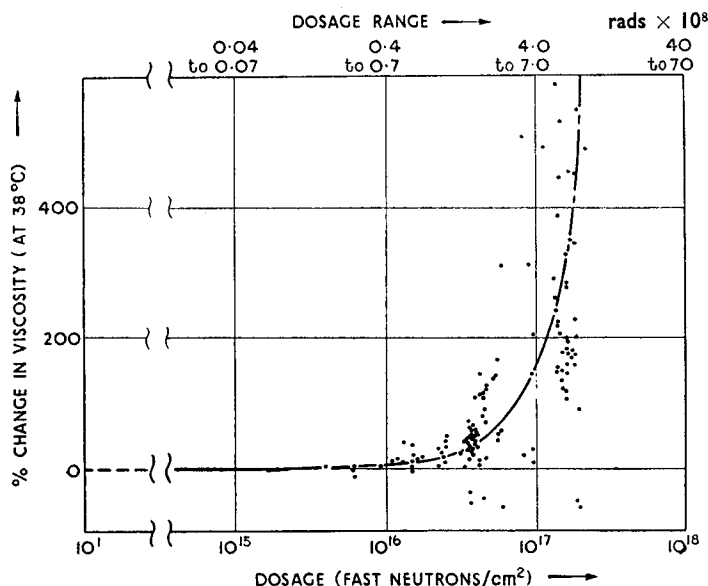


Fig. 6.8.3. Viscosity change in 24 organic liquids plotted as a function of rad-dosage and pile dosage. The pile radiation was composed of 10 thermal neutrons and 5 gamma photons per fast neutron (CALKINS⁽³⁴⁾).

has to undergo large temperature changes between the shut-down and operating conditions,⁽¹⁾ because it has a thermal expansion coefficient 13 times that of iron, and because it softens at about 110°C. It is a satisfactory material for use in the outer regions of a shield. Masonite was used in the Hanford shield, which consisted of alternate layers of masonite and steel. The average density of the shield was 4.27 g cm⁻³, and it contained 83.2% of iron and 0.995% of hydrogen by weight. The fast neutron relaxation length was 5.3 cm.⁽²⁷⁾ In wood alone an *e*-folding length of 8.2 cm has been observed.

For reasons discussed above in connection with water shields, there are advantages in reducing capture radiation by adding boron to hydrogenous materials. A factor of 100 reduction in hydrogen capture radiation is given by including about 8 per cent by weight of natural B₄C in polythene; this is only one-quarter of the amount which can be included without causing manufacturing difficulties. By mixing boron powder and methyl methacrylate

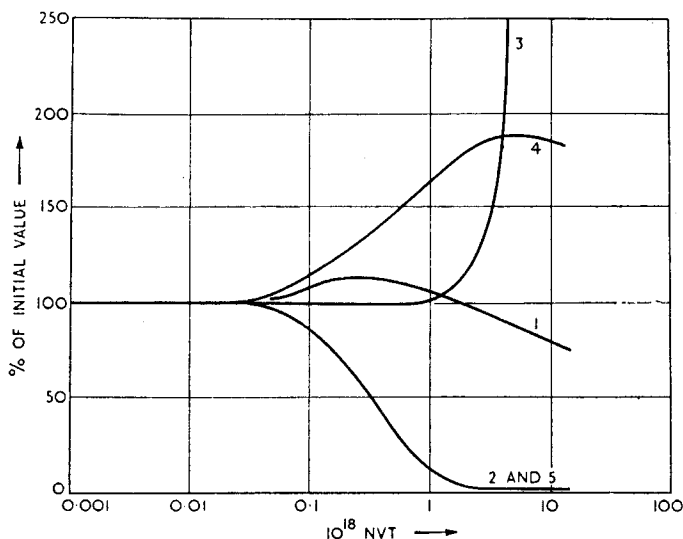


Fig. 6.8.4. Effect of irradiation on the mechanical properties of polythene. The abscissa is the integrated thermal flux per cm^2 . The fast flux (>1 MeV) was 0.04 times the thermal flux, and the epithermal flux was 0.6 times the thermal flux.

A similar effect would be given by a dose of gamma radiation of intensity
 $\sim [(\text{Integrated neutron flux in } \text{cm}^{-2}) \div 10^9] \text{ röntgens.}$

The curves refer to the following properties, whose initial values are shown in brackets:

1. Tensile strength (1400 lb-in.⁻²)
 2. Elongation (250%)
 3. Elastic modulus (3×10^4 lb-in.⁻²)
 4. Shear strength (1400 lb-in.⁻²)
 5. Impact strength (11.2 ft-lb per inch of notch)
- (SISMAN and BOPP.⁽³⁵⁾)

monomer, and polymerizing the mixture, one obtains a material containing up to 1.6 g cm^{-3} of boron, mixed with about one-tenth as much of the plastic binder.⁽⁸⁰⁾ Owing to their low thermal conductivity, cooling of borated plastics must be carefully planned. It should be remembered when making estimates of heating in organic materials that the energy deposition due to fast neutron scattering is relatively more important than in (say) concrete, owing to the small amount of gamma energy released when a neutron is captured in hydrogen.

Partial Shielding

It is convenient at this point to mention the question of partial shielding, since this is a technique which is essentially linked with the use of lightweight shields. The phrase usually refers to the method of saving weight in a propulsion reactor by reducing the thickness of shielding in places to which access cannot be obtained during the operation of the reactor. Thus, in a nuclear-propelled merchant ship, although it may not be possible to avoid providing full shielding on the top and sides of the reactor compartment, only a partial shield is

needed for the bottom. This partial shield should include sufficient gamma ray shielding to permit inspection of the hull without undue difficulty when the ship is dry-docked and when the reactor is fully loaded, but shut down. In addition, sufficient neutron shielding should be provided to prevent the hull itself from becoming unduly radioactive. In practice it will be found that the partial shield needs to be about half the thickness of the complete shield.

It is necessary to ensure that the radiation which emerges from the partial shield during operation of the reactor and which is backscattered from the sea does not cause too high a dose-rate in any of the inhabited compartments of the ship. It has been shown experimentally⁽¹⁾ that a sufficiently accurate estimate of the backscattered gamma radiation can be made simply by taking into account the effects of singly and doubly backscattered photons. Calculations involving no more than double scattering are sufficiently straightforward to be performed on hand-computing machines. It was also found that if the intensity of the gamma radiation backscattered from the sea-water was not excessive, then the neutron dose was also negligible.

For an aircraft reactor it would be essential to employ partial shielding over a large portion of the reactor surface. When flying at high altitude the low density of the air would minimize backscattering, but the obvious design difficulty is to have an acceptably low degree of backscattering when the aircraft is on the ground.

MISCELLANEOUS TOPICS

6.9. ACTIVATION INDUCED BY IRRADIATION IN A NEUTRON FLUX

The various types of activity that can be induced by neutron bombardment are summarized in Section 3.8. When calculating the induced disintegration rate, a distinction must be drawn between activities induced by capture of low energy neutrons, and those caused by fast neutron reactions.

Irradiation by Slow Neutrons

The simplifying assumption is usually made that the neutrons are captured only when they have been thermalized. For irradiation in thermal reactors this assumption is usually satisfactory, though there may be a considerable error for materials with important capture resonances (see Chapter 3) in the epithermal energy region (e.g. indium).

If m grams of an element of "chemical" atomic weight $\bar{A}$ are placed in a thermal neutron flux θ neutrons $\text{cm}^{-2} \text{sec}^{-1}$, the rate of formation of unstable nuclei of mass $(A + 1)$ by neutron capture in an isotope of mass A is

$$\left(\frac{dn}{dt}\right)_{A+1, \text{form}} = \theta f_A(m/\bar{A})N(\sigma_{A, \text{act}} \times 10^{-24}) = F \text{ per second,} \quad (6.9.1)$$

where N is Avogadro's number, f_A is the fraction of nuclei which are isotopes of mass A , and $\sigma_{A, \text{act}}$ barns is the measured isotopic activation cross-section

for the formation by thermal neutron irradiation of the unstable nucleus of mass $(A + 1)$. Since separated isotopes are rarely used, one is usually more interested in the product $(f_A \cdot \sigma_{A, \text{act}})$, rather than in the two quantities separately; this product is referred to as the *activation cross-section per average atom*. In standard tables cross-sections are usually quoted for a neutron energy of 2200 metres per second, the most probable velocity for a Maxwellian spectrum at 20°C, and these values are normally used in calculations of activity (see discussion by HUGHES⁽⁴³⁾ on correct choice of cross-sections). It should be noted that equation (6.9.1) can be applied only so long as the sample absorbs neutrons to a sufficiently small extent to permit the value of θ to be taken as effectively constant throughout the sample. When this is not so, allowance has to be made for the depression of the neutron flux in the sample.

The unstable nuclei obey the usual law of radioactive disintegration (equation 2.10.5). If the number of nuclei of mass $A + 1$ originally present is zero, the number at time t after the commencement of the irradiation is

$$n_{A+1}(t) = \frac{F}{\lambda} (1 - e^{-\lambda t}). \quad (6.9.2)$$

Thus, if the irradiation continues for a time long compared with the reciprocal of the decay constant λ , the amount of the product builds up to an equilibrium value, such that the rate of decay equals the rate of formation. When irradiation ceases, the number of unstable nuclei remaining decreases in proportion to $e^{-\lambda t}$.

In principle, the process of calculating the activity of any material that has been irradiated in a known thermal neutron flux involves no more than a knowledge of its chemical composition, the activation cross-sections for the constituent elements, the mode of decay of the radioactive products, and the application of equation (6.9.2). However, owing to the enormous variation which is found between the thermal activation cross-sections of different elements, a minor constituent of a mixture may easily be responsible for the greater part of the activity. As an example, consider the activities induced in a one-gram sample of a typical manganese-containing stainless steel, irradiated for a year in a flux of thermal neutrons. The steps of the calculation of the activity are summarized in Table 6.9.1.

Immediately after extraction of the sample from the reactor the manganese activity is predominant, as the figures in the last five columns of the table show. However, the activity has a short half-life, and dies away rapidly until the iron or cobalt eventually takes its place as the principal gamma ray emitter. Cobalt is an objectionable impurity which tends to occur in nickel-bearing steels at concentrations up to 0.05%.

In this example the most abundant gamma rays also happen to be the most penetrating, so that the manganese activity is even more important when "seen" through a gamma shield (columns 13 and 14). Examples can also be found in which the most penetrating radiation is not the most abundant. The activity which is the most important then depends on whether the source is to be observed with or without shielding; and, if there is to be a shield, on how thick it is, and on its composition. Before dismissing any component activity

Table 6.9.1. Activities induced in a manganese containing stainless steel by irradiation for one year in a thermal neutron flux of 8.9×10^{12} neutrons $\text{sec}^{-1} \text{cm}^{-2}$

Element	Activation cross-section* per average atom for 2200 m sec ⁻¹ neutrons (barns)	Half-life*	γ-ray energy and abundance† (% of disintegration)	Atomic weight	Activation cross-section per gram of element (cm ² g ⁻¹)	Percentage weight of element in steel	Cross-section per gram of steel (cm ² g ⁻¹)	Saturation activity (disg ⁻¹ sec ⁻¹)	Fraction of saturation activity reached after 1 year's irradiation	Activity after irradiation for 1 year (disg ⁻¹ sec ⁻¹)	Dose-rate at 1 metre from a 1 gram unshielded sample (mr/hr)	Dose-rate at 1 metre from a 1 gram sample, through lead shield of thickness: (a) 1 cm (mr/hr) (b) 5 cm (mr/hr)	
Iron	0.0028	46 d	1.1 MeV 50% 1.3 MeV 50%	55.9	3.0 × 10 ⁻⁵	72.3	2.2 × 10 ⁻⁵	1.93 × 10 ⁸	0.91	1.76 × 10 ⁸	3.2	2.4	0.3
Chromium	0.475	27.8 d	0.32 MeV 3%	52	5.5 × 10 ⁻³	18	9.9 × 10 ⁻⁴	8.74 × 10 ⁹	0.95	8.31 × 10 ⁸	0.13	0.004	—
Nickel	0.030	2.57 hr	0.37 MeV 29% 1.12 MeV 43% 1.49 MeV 29%	58.7	3.1 × 10 ⁻⁴	8	2.46 × 10 ⁻⁵	2.18 × 10 ⁸	1	2.18 × 10 ⁸	3.37	2.5	0.17
Manganese	13.4	2.58 hr	0.822 MeV 100% 1.77 MeV 30% 2.06 MeV 20%	54.9	0.147	1	1.47 × 10 ⁻³	1.3 × 10 ¹⁰	1	1.3 × 10 ¹⁰	338	222	29
Cobalt	37	5.28 yr	1.17 MeV 100% 1.33 MeV 100%	59	0.37	0.026	9.6 × 10 ⁻⁵	8.49 × 10 ⁸	0.124	1.05 × 10 ⁸	3.99	3.1	0.36

* HUGHES, D. J., and HARVEY, J. A.; *Neutron Cross-sections* BNL 325 (1955).

† N.B.S. Circular 499 (U.S. Govt. Printing Office, Washington, U.S.A.), 1950 *et seq.*

as being too small to justify the labour of detailed calculation, a check must always be made to see whether this will still be true after the radiation has been attenuated by a shield.

Suppression of Activation induced by Thermal Neutrons

On pages 254 and 288 we have already mentioned the possibility of reducing capture radiation by adding boron to an irradiated material. The technique may equally well be applied to the reduction of induced activity, since boron gives no radioactive products as the result of slow neutron bombardment. The addition of the boron does not affect the fast flux from which the thermal flux is derived, and consequently, (as we may see from equation 6.8.1), the thermal flux, and hence the degree of activity, varies inversely as the total macroscopic absorption cross-section, just as for capture radiation. Tabulations of the cost of the available boron-containing materials are given in reference 11.

Irradiation by Fast Neutrons

As explained in Chapter 3, energetic neutrons, such as those produced in the fission process, are capable of inducing reactions additional to those observed with thermal neutrons. Most of these reactions have cross-sections which are rapidly varying functions of neutron energy, at least near the reaction threshold energy. Two examples have already been given in Figs. 3.6.2 and 3.6.3; data for other reactions will be found in the standard compilations.^(19,41,42)

Because of this rapid variation with energy, the determination of the activity induced by fast neutrons requires a knowledge of the fast energy spectrum in the reactor core. As explained in Chapter 4, this is difficult to calculate, and it is only in a few special cases that simple approximations can be used. A simple case is a material with a reaction threshold greater than about 4 MeV, undergoing irradiation in a hydrogen-moderated reactor. In these circumstances the build-up of fast neutrons is of reduced importance, and (provided the variation with energy of the reaction cross-section σ_{act} is known) an order-of-magnitude estimate of the activity can be obtained from an estimation of the high energy flux by means of the removal cross-section technique. Consider a point source emitting isotropically one fission neutron per second, separated by a distance X of absorbing medium from a sample whose activity is required, and which contains n atoms of the element under consideration. Then, by using formula (3.9.1) for the fission neutron spectrum, we find that the saturation activity (dis sec⁻¹) induced in the sample after long exposure to high energy neutrons is approximately

$$\int_{\text{threshold energy}}^{\infty} \frac{1}{4\pi X^2} \left[\sqrt{\frac{2}{\pi e}} e^{-E} [\sinh (2E)^{1/2}] n \sigma_{\text{act}}(E) \exp \left(- \int_0^X \Sigma_{\text{rem}}(E, x) dx \right) \right] dE. \quad (6.9.3)$$

The energy variation of Σ_{rem} can be estimated as explained on page 183. For an extended emitter, such as a reactor core, equation (6.9.3) must be integrated over the whole emitting volume; this is most easily done by using the "lumped source" approximation (page 243).

When using equation (6.9.3) a safety factor should be added, since it takes no account of neutrons which are degraded in energy, but which still have an energy above the threshold. This build-up should be fairly small in a water-moderated reactor, but may be important in a graphite assembly. In water itself the flux distributions given on page 190 eliminate the need for further calculation of the spectrum.

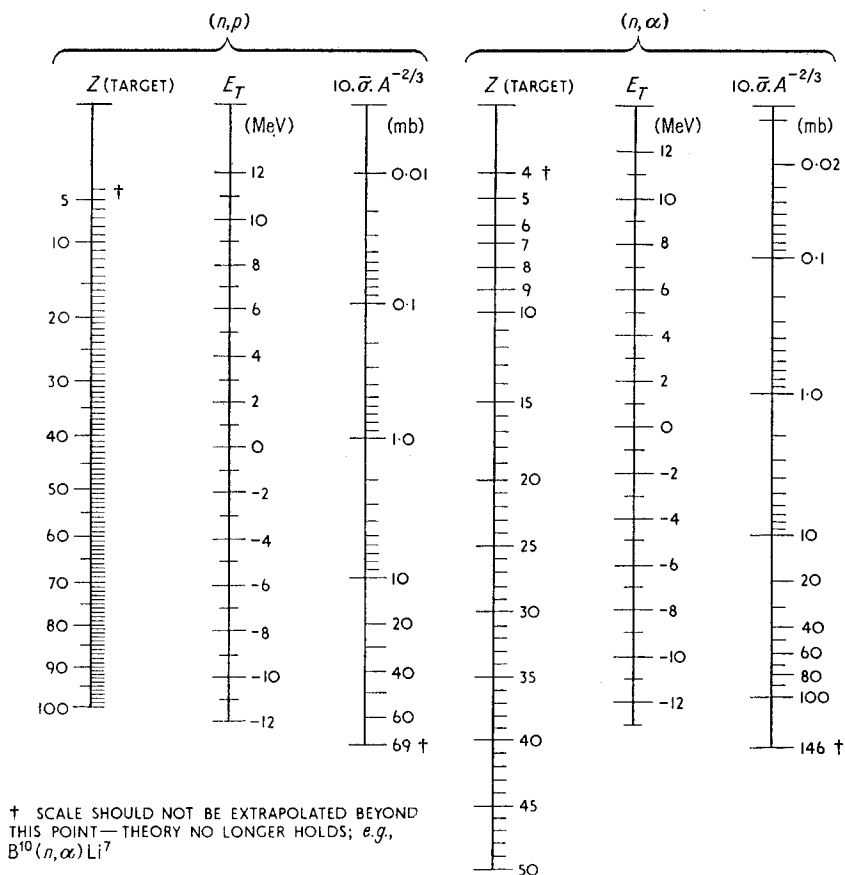


Fig. 6.9.1. Nomogram for estimation of (n, α) and (n, p) cross-sections, averaged over a fission neutron flux (INTHOFF⁽⁴⁴⁾).

At considerable depths inside a shield the semi-empirical equation (4.8.7) can be used. This equation gives a $1/X$ spatial dependence of the fast flux from a point fission source. A more exact theory (reference 4, Chapter 4) shows that the spatial variation should be given by X^{-p} , where $1 < p < 2$; thus equation (4.8.7) gives an overestimate of the fast flux, and can therefore be safely used in activity calculations.

A further simple case is irradiation in an undegraded fission neutron spectrum. Approximate values of (n, α) and (n, p) cross-sections $\bar{\sigma}$, averaged over the fission spectrum, can be found using the nomogram opposite. It is based on a treatment by HUGHES⁽⁴³⁾ and INTHOFF⁽⁴⁴⁾ of the theory of these reactions; an accuracy to within a factor of two for the lighter elements ($Z < 20$) is claimed. The first scale of the nomogram (Fig. 6.9.1) gives the atomic number Z of the target element. The second gives the threshold energy E_T of the reaction. This is related to the " Q " of the reaction (page 137) by the relation

$$E_T = -Q \left(\frac{A+1}{A} \right)$$

where A is the mass number of the target element. Tabulations of Q values are given in references 47 and 48; alternatively they may be calculated from atomic masses (references 45 and 46) with the aid of equation (3.6.1). The third scale gives the value of $10\bar{\sigma}A^{-2/3}$. The nomogram is used in the usual way, and by placing a straightedge across the three scales the value for $\bar{\sigma}$ is obtained. There is some evidence⁽⁴⁹⁾ that the values of $\bar{\sigma}$ given by this method are systematically too high for Z greater than about 20. Some representative (n, p) and (n, α) cross-sections for fission neutrons are given in Table 6.9.2.

Table 6.9.2. Cross-sections of threshold reactions for fission neutrons

Reaction	product half-life	E_T (MeV)	$\bar{\sigma}$ (mb)	$10\bar{\sigma}/A^{\frac{2}{3}}$ (mb)
${}^4\text{Be}^9(n, \alpha)\text{He}^6$	0.89 s	0.3	10	23.2
${}^5\text{B}^{11}(n, \alpha)\text{Li}^8$	0.89 s	7.2	0.085	0.174
${}^8\text{O}^{16}(n, p)\text{N}^{16}$	7.5 s	10.2	0.014	0.023
${}^9\text{F}^{19}(n, p)\text{O}^{19}$	30 s	3.9	0.5	0.7
${}^9\text{F}^{19}(n, \alpha)\text{N}^{16}$	7.5 s	1.5	4.5	6.4
${}^{11}\text{Na}^{23}(n, p)\text{Ne}^{23}$	40 s	3.5	0.7	0.88
${}^{11}\text{Na}^{23}(n, \alpha)\text{F}^{20}$	11.6 s	4.0	0.4	0.5
${}^{13}\text{Al}^{27}(n, p)\text{Mg}^{27}$	10.2 m	2.1	2.8	3.1
${}^{13}\text{Al}^{27}(n, \alpha)\text{Na}^{24}$	14.8 h	3.3	0.6	0.67
${}^{12}\text{Mg}^{24}(n, p)\text{Na}^{24}$	14.8 h	4.9	1.0	1.16
${}^{12}\text{Mg}^{25}(n, p)\text{Na}^{25}$	62 s	4.1	2.0	2.3
${}^{14}\text{Si}^{28}(n, p)\text{Al}^{28}$	2.4 m	4.0	4	4.3
${}^{14}\text{Si}^{29}(n, p)\text{Al}^{29}$	6.7 m	3.3	2.7	2.9
${}^{15}\text{P}^{31}(n, p)\text{Si}^{31}$	170 m	0.7	19	19
${}^{15}\text{P}^{31}(n, \alpha)\text{Al}^{28}$	2.4 m	2.0	1.43	1.44
${}^{16}\text{S}^{32}(n, p)\text{P}^{32}$	14.3 d	1.0	30	30
${}^{16}\text{S}^{34}(n, \alpha)\text{Si}^{31}$	170 m	0.9	3.0	2.9
${}^{17}\text{Cl}^{35}(n, p)\text{S}^{35}$	87 d	-0.7	16	15
${}^{17}\text{Cl}^{35}(n, \alpha)\text{P}^{32}$	14.3 d	-1.0	3.0	2.8
${}^{17}\text{Cl}^{37}(n, p)\text{S}^{37}$	5.0 m	3.5	0.24	0.22
${}^{27}\text{V}^{51}(n, \alpha)\text{Sc}^{48}$	44 h	2.4	0.08	0.058

From measurements by HUGHES, SPATZ, and GOLDSTEIN.⁽⁴³⁾ See also ref. 49.

Although the absolute values of fast neutron activation cross-sections are on the whole very much smaller than for thermal neutrons, the fast neutron reactions nevertheless have considerable practical importance. This is particularly true of the activities induced in water (Sections 3.6 and 6.10). As another example we may quote aluminium, which is very commonly used for active experimental equipment because the activity induced in it by thermal neutrons has the very short half-life of 2.2 minutes. However, if the aluminium is irradiated in the core of a reactor, much longer-lived (though weaker) activities are also induced by the fast neutrons (e.g. Na^{24} , from (n, α) reaction), and these determine how quickly after withdrawal the aluminium can be safely handled.

It should be noted that although thermal activation can be reduced by the addition of boron or lithium (as mentioned above) there is no comparable treatment which can be applied to reduce activation by fast neutrons. The essential difference between the two cases is that for thermal neutrons materials of very large cross-section (several hundred barns) can be used to reduce the thermal flux by a large factor; whereas for fast neutrons there is no very wide spread in the range of available cross-sections, and the most that can be done is to "remove" the fast neutrons efficiently by ensuring that as much hydrogen as possible is present.

*Shielding Required for Gamma Emitting Materials**

The dose-rate D existing at a distance d from an isotropic point source of radiation, in which S disintegrations per second are taking place, which emits photons of energy E MeV in a fraction f of all disintegrations, and which is shielded by a thickness t of absorber is

$$\begin{aligned} D &= (\text{flux of photons per cm}^2 \text{ per sec}) \times \frac{0.3}{40\xi} \text{ r hr}^{-1} \\ &= 0.6 \frac{SfB}{d^2\xi} e^{-t\rho\chi} \text{ mr hr}^{-1}, \end{aligned} \quad (6.9.4)$$

where ξ is the factor for converting flux to dose-rate, given in Fig. 1.5.1, ρ is the density of the absorber, χ is the mass absorption coefficient for narrow-beam geometry (Fig. 2.6.4), and B is the dose-build-up factor for a point isotropic source for photons of energy E (Fig. 2.7.3). When the isotope in question emits more than one photon energy, the quantity D must be determined by summing the effects of the component radiations. The energy E and the fraction f may be found by reference to one of the standard tables of radioactive isotopes (see, for instance, refs. 41 and 42). The thickness t to give any required value of D may be found from equation (6.9.4) by trial and error, or in certain special cases from Figs. 6.9.2 and 6.9.3, and Table 6.9.3. For other simple geometries the flux may be determined with the aid of the formulae given in Chapter 5.

Over a considerable range of gamma-ray energies (0.1 to 2.5 MeV) the value of ξ is approximately equal to $4000/E$ (MeV). By substituting this value in

* Beta emitters and small laboratory sources of neutrons are discussed in Section 6.13. Neutron emitters are also discussed in connection with coolant circuit shielding in Section 6.10.

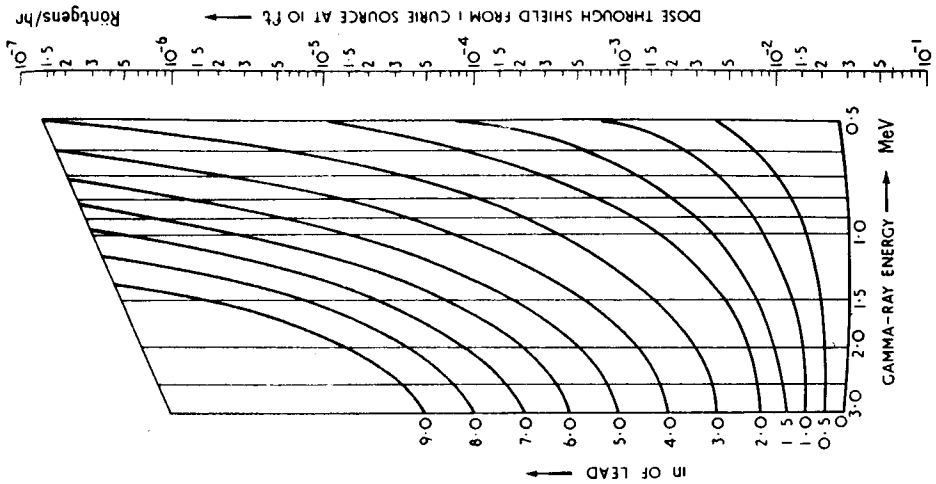
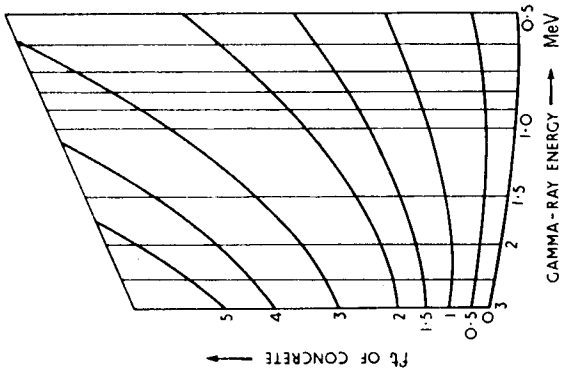


Fig. 6.9.2. Nomogram giving the dose at 10 feet from a one curie gamma emitting point source through lead and concrete shields. Ordinary concrete of density 2.35 g cm⁻³ is assumed in the calculation. It is assumed that the source emits one photon per disintegration.

Select the point on the appropriate network corresponding to the photon energy and thickness of shield. A straight line joining A to this point, if extended until it cuts the dosage axis, will then give the dose rate at 10 feet (3.04 metres) from the source.



A +

equation (6.9.4) we obtain the useful rule of thumb that the unshielded dose-rate at a distance of one metre from a source is

$$\approx 1.5 \times 10^{-8} S f E \text{ mr hr}^{-1} \quad (6.9.5)$$

A quantity colloquially known as the “*k* factor” of a radioactive isotope is defined as the dose in röntgens per hour at a distance of one centimetre from a point source of strength one millicurie.* For an isotope emitting only one photon energy E MeV:

$$\text{“}k\text{ factor”} \approx 5.5 \cdot f E \text{ rhc/mc}$$

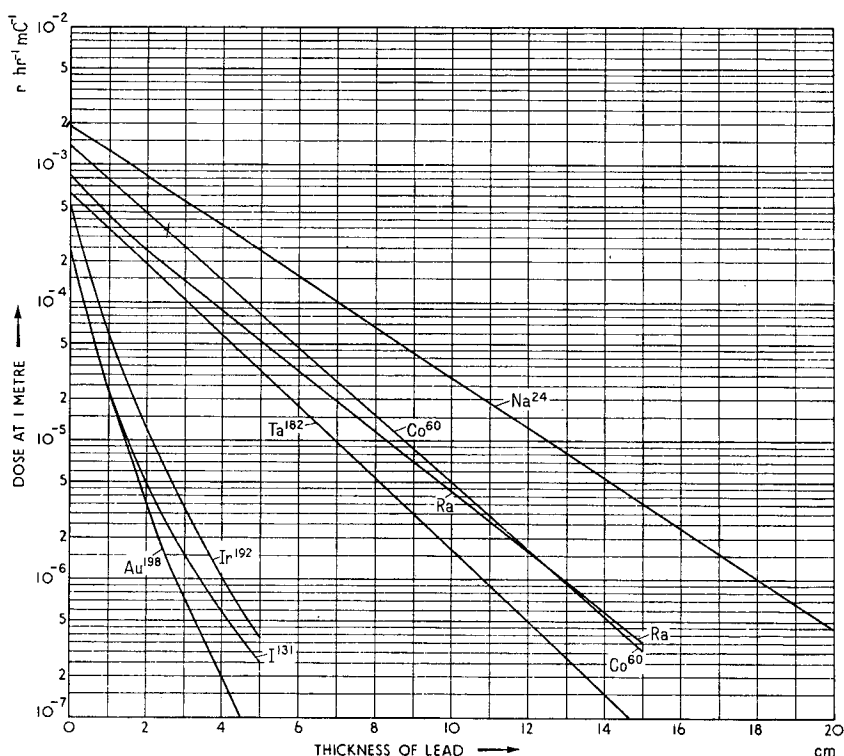


Fig. 6.9.3. Dose in r/hr at 1 metre from millicurie sources of a number of gamma-emitting isotopes, as a function of thickness of lead shielding.
(From measurements by EASTWOOD and WEST.)

* A radioactive source has a strength of one *curie* when 3.700×10^{10} atoms disintegrate per second. Note that the definition refers to numbers of atoms disintegrating, and not to the number of photons emitted. The latter may be greater or less than the number of disintegrations, depending on the decay scheme. For a material such as radium, which is the parent of several generations of daughter products, the source strength conventionally refers to the number of *parent* atoms that are disintegrating. The value of the unit is chosen so that one gram of radium has a source strength of about one curie (actually 0.976 international curies). An alternative unit of source strength is the *rutherford*, equal to a disintegration rate of 10^6 atoms per second.

Table 6.9.3. Table of “*k* factors”

The following table gives the dose-rate in röntgens per hour at 1 cm from a point source of strength 1 millicurie (3.7×10^7 dis/sec). The values are calculated from the data given in references (41) and (50). It has been assumed in the case of positron emitters (denoted by β^+ in the third column) that all beta-rays are absorbed in the source with the emission of two quanta of energy 0.51 MeV. Values in brackets are experimental determinations.

Isotope	Half-life	Disintegration scheme Energy in MeV (per cent abundance)	“ <i>k</i> factor” (<i>r</i> per hour-mc at 1 cm)
Na ²²	2.6 yr	1.28 (100), β^+ (89)	11.6
Na ²⁴	15.0 hr	1.37 (100), 2.76 (100)	18.3 (18.7)
K ⁴²	12.5 hr	1.53 (18)	1.4 (1.52)
Mn ⁵²	5.7 d	0.73 (100), 0.94 (100), 1.45 (100), β^+ (33)	19.7
Mn ⁵⁴	291 d	0.84 (100)	4.7
Fe ⁵⁹	45.1 d	1.10 (57), 1.29 (43)	6.2 (6.8)
Co ⁵⁸	71 d	0.81 (100), 1.62 (0.5), β^+ (14)	5.3
Co ⁶⁰	5.25 yr	1.17 (100), 1.33 (100)	13.0 (12.8)
Zn ⁶⁵	245 d	1.11 (45), β^+ (2)	2.8
Ga ⁷²	14.1 hr	0.63 (21), 0.83 (83), 0.89 (10), 2.20 (30), 2.49 (10), 2.51 (17) and others	>14.2
As ⁷⁶	26.5 hr	0.55 (41), 0.64 (8), 1.20 (9), 1.4 (0.8), 2.05 (1.6)	2.3
Br ⁸²	35.9 hr	Many up to 1.5	14.6 (14.8)
I ¹²⁸	25 min	0.46 (16), 0.54 (1)	0.45
I ¹³⁰	12.5 hr	0.41 (23), 0.53 (100), 0.66 (100), 0.74 (69), 1.15 (29)	12.0
I ¹³¹	8.04 d	0.08 (2), 0.28 (5), 0.36 (80), 0.64 (9), 0.72 (3)	2.2 (2.25)
Cs ¹³⁷	30 yr	0.662 (82)	3.1
Tm ¹⁷⁰	127 d	0.084 (3) and bremsstrahlung from β^- (0.87, 0.97 MeV)	*0.04
Ir ¹⁹²	74.4 d	Many up to 0.6	(5.0)
Au ¹⁹⁸	2.69 d	0.411 (96.4)	2.3 (2.4)
Hg ²⁰³	47 d	0.279 (83)	1.1
†Ra ²²⁶	1590 y	Many	(8.44 ± 0.07)

Experimental measurements by National Physical Laboratory Standards Committee⁽⁵¹⁾ (except radium).

* Figure represents *k* factor for unshielded source from γ and X rays. The *k* factor in practice will probably be greatly increased due to bremsstrahlung.

† Measurement by GHOSH, KASTNER, and WHYTE⁽⁵²⁾ with 0.5 mm platinum filtration. The best value for the number of disintegrations per mg of radium is $3.61 \pm 0.03 \times 10^7$ dis/sec, or 0.976 international millicuries (see ref. 53).

Shielding for Irradiated Uranium Fuel Elements

Methods of calculating the shielding required for mixed gamma-emitting fission products have already been given in Chapter 2, but the necessity for carrying out these calculations can in some cases be avoided by using the values in Fig. 6.9.4, scaled up or down as necessary. The values were determined with the aid of equation (5.3.3) modified to take account of build-up in the lead shield by TAYLOR's method. They are for one day after shutdown, and are correct within a factor of two for decay times from one to ten days. The dose-rate for a finite line of fuel elements of the same source strength per unit length is practically the same as for an infinite line, provided the length of the

line is greater than the thickness of the shield. This follows from the insensitivity of the $i(\mu t, \psi)$ function occurring in equation (5.3.3) to ψ , when $\psi > 30^\circ$ and $\mu t > 10$; 90% of the dose at the surface of the shield is due to the part of the source lying within a "cone of effectiveness" of semi-angle 30° which has its apex at the surface of the shield and its axis directed along the normal

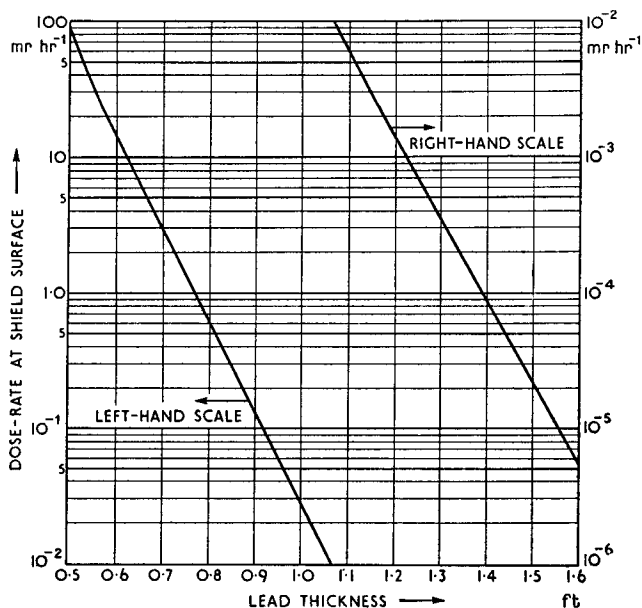


Fig. 6.9.4. Gamma dose-rate at the surface of a cylindrical lead shield containing an infinite line of thin uranium fuel elements which have been irradiated for a long period at an average rating of 100 watts ft⁻¹, and then cooled for one day. Self-absorption of gamma radiation in the fuel elements is neglected.

to the shield surface (Fig. 5.1.3). For the conditions to which Fig. 6.9.4 refers the majority of the dose-rate at the surface is due to the 1.65 MeV La¹⁴⁰ radiation; for instance, when the shield is 0.7 ft (21 cm) thick, this radiation accounts for 60% of the total dose. The 2.4–2.6 MeV group (Fig. 2.10.4) contributes a further 30%.

Specification of Dose-rate at the Surface of a Shielding Flask

The specification of the thickness of a transport or storage flask for a piece of irradiated equipment, or for any other gamma source, is a matter that calls for some judgment. If gamma shielding were inexpensive one would always adopt the obvious solution of making the dose-rate at the surface of the flask everywhere less than one MPL. However, for the intense sources produced by irradiation in a reactor the cost and weight of shielding are so great that this course should not be followed without some good reason. A study should be made of how frequently and how closely it is necessary to approach the flask, since very large savings can be achieved by relying on distance to cut

down the dose-rate and by planning operations so that they take the minimum possible time. However, these techniques should be applied with discretion: in an industrial installation the dose-rate at any accessible portion of the shield should not exceed 10 rads per hour, owing to the difficulty of keeping adequate control of exposure times.

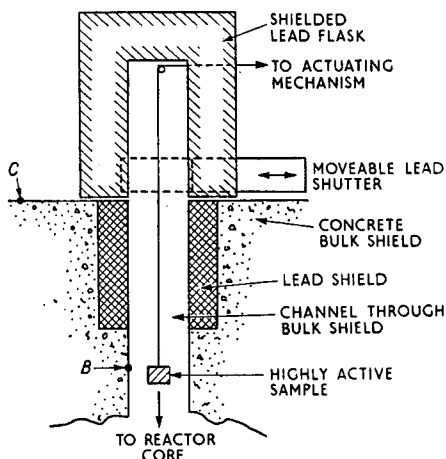


Fig. 6.9.5. Schematic arrangement for extracting a highly active source from a reactor.

An outstanding example of what can be done with the minimum of shielding was the movement of the calandria of the damaged NRX reactor from the reactor hall to the burial ground, in 1953.^(54,55) The calandria was a cylindrical vessel 10.5 ft high and 8.75 ft in diameter, and the gamma dose-rate at the bottom tube-plate was ~ 300 r/hr (i.e. one median lethal dose in 100 minutes). Nevertheless, the whole operation was carried out with scarcely any shielding, and by careful planning no person taking part received a dose of more than 4 r. An example of the use of distance rather than shielding is the wing-tip loading system which was developed in 1952 for conveying radioactive materials by air from Great Britain to South Africa. A third example, in which shielding is rendered unnecessary by limitation of the exposure time, is found in those isotope handling facilities⁽⁵⁶⁾ where highly active cylindrical sources, a few inches long, are driven rapidly along tubes by means of compressed air. Although such an arrangement saves money on shielding, some extra expenditure is necessarily incurred in making the engineering of the system sufficiently reliable.

Extraction of Highly Active Gamma Emitters from a Reactor

The apparatus for extracting highly active gamma emitters from a reactor usually takes the form shown schematically in Fig. 6.9.5. A lead "coffin" or "flask" is offered up to the reactor face, and the sample is withdrawn by some mechanism built into the flask. Ultimately, when the source has been positioned inside the flask, the movable door is shut, and the coffin is then safe to handle. However, before this stage is reached, there is an intermediate situation which

needs special attention. As the source approaches the surface of the reactor shield, the operator may be exposed to a high dose-rate from the gamma rays of the source "shining" through the comparatively small thickness of concrete *BC*. Here again, it is most undesirable that the dose-rate at the operating position (or at the face of the shield, if it is easily accessible), should exceed 10 r/hr. Where necessary, extra shielding should be provided to keep the dose-rate below this level. The extra shielding could be in the form of a flange on the flask, or a lead or steel insertion in the concrete, as indicated by the shaded region in the figure. The latter policy has been adopted for the large experimental hole shown as item 17 in Fig. 6.2.3.

Gamma Dose-rate Outside a Large Tank of Radioactive Material

A frequently occurring problem is the calculation of the dose-rate existing immediately outside a large volume of gamma-emitting material. This material might be, for instance, the low-activity fission product solutions which are the waste products of a chemical separation plant, and which are normally stored in large effluent tanks. The dose-rate could, of course, be calculated by use of formulae such as those given in Chapter 5; but there is also an elegant approximate method based on considerations of energy-balance. The argument is most straightforward when applied to dilute aqueous solutions.

First consider the dose-rate at a point inside a tank whose dimensions are very much larger than the mean free path of the gamma radiation in the liquid. Provided the specific activity is uniform throughout the liquid, and if the point is sufficiently far from the edge of the tank, the energy absorbed per gram of liquid per second must equal the energy emitted per gram per second. If now a small quantity of tissue is placed in the tank, this equilibrium will not be disturbed, and since tissue and water have similar gamma-absorbing properties, the energy absorbed per gram of tissue will also be very close to the energy given out per gram of solution. Thus if there are g photons each of energy E_0 MeV emitted per second per gram of solution, energy is deposited in the tissue at a rate gE_0 MeV sec⁻¹ g⁻¹. Now consider what happens if we divide the tank in two by a vertical plane through the point at which the tissue is situated, and remove one half of the tank to infinity. Then from symmetry considerations the dose received by the tissue will be halved, if we neglect backscattering of the gamma radiations from the half of the tank which has been removed (the calculation gives an overestimate because of this approximation). Since 1 röntgen corresponds to the deposition of 93.1 ergs, or 5.8×10^7 MeV, per gram of water or tissue (page 15), the gamma dose-rate immediately outside a large effluent tank containing this concentration of gamma-emitting material is

$$3.1 \times 10^{-5} gE_0 \text{ röntgens per hour}$$

This applies to a tank which subtends a solid angle of 2π at the point of interest. If we assume that the point of interest is moved to some distance from the tank, so that the tank (which is now assumed to be finite but still large compared with the mean free path of the radiation in the solution) subtends a solid angle Ω , then the dose-rate would be given approximately by $\Omega/2\pi$ times

the dose at the surface. Note that in calculating the dose outside the tank we have considered only gamma radiation, since the tank wall will almost always be opaque to beta radiation. However, in calculating the gamma activity of the solution, account should be taken of the small amount of bremsstrahlung produced by the stopping of beta particles by water (see Sections 2.11 and 6.13). An allowance should also be made for the gamma attenuation introduced by the tank wall. For non-aqueous solutions the expression for the dose-rate should be modified by making use of the Bragg-Gray relation (page 91), so as to allow for the difference in the stopping powers (for Compton and other electrons) of the gamma-emitting medium and the small element of tissue.

6.10. ACTIVITY INDUCED IN COOLING CIRCUITS

An important special case of activation is the activity induced in the reactor coolant. The intensity of the emitted radiation determines the amount of shielding which has to be provided for the heat exchangers during the operation of the reactor, while the half-lives of the induced activities determine how soon after shut-down it is possible to carry out maintenance work. The usual and most economical arrangement is to place the heat exchangers in a shielded compartment which itself forms part of the main (or "primary") reactor shield, and which is separated from the reactor core by a "secondary" shield thick enough (a) to reduce neutron activation of the heat exchanger structure to a negligible level and (b) to permit entry into the heat exchanger space soon after the reactor has been shut down. An example of the use of such an arrangement in a shore installation is shown in Fig. 6.1.1. The design problem of ensuring access to heat exchangers after shutdown is colloquially described as the "accessibility problem."

A number of mechanisms contribute to the activity of the coolant stream: (a) direct activation of the coolant (and its impurities) by exposure to the neutron flux; (b) solution or erosion of active material from the walls of the coolant channels; (c) recoil of fission products from the surface of fuel elements which are contaminated with uranium; (d) recoil of radioactive nuclei from the walls of the coolant channels, due to the momentum acquired in the process of neutron capture; and for circulating fuel reactors we must add (e) the activity of the fuel itself. Which of these sources of activity is the most important depends very much on the nature of the coolant. For instance (c) is negligible for sodium-cooled reactors, but has a bearing on the detection of burst fuel elements in carbon-dioxide cooled reactors of the Calder Hall type; (d) can probably always be neglected, but (b) is the factor which determines the residual activity of a water circuit. The degree of induced activity can vary within very wide limits; thus the Calder Hall heat exchangers are unshielded, whereas the shielding required for the heat exchangers of a high-flux water-cooled or sodium-cooled reactor is comparable in thickness with the main reactor shield.

In what follows we shall derive simple expressions for the intensity of the important kinds of induced activity. We shall assume that the coolant is circulated in a closed system in a simple series arrangement without bypasses, and that the time taken to complete one cycle is T . The coolant is assumed for

simplicity to be in a constant neutron flux for a time τ per cycle, and for the remaining time $(T - \tau)$ to be in a flux-free region.

Activities Directly Induced by Thermal Neutron Bombardment

Assume that for the time τ per cycle the coolant is in a constant thermal neutron flux θ . Let Σ_{act} be the macroscopic activation cross-section of a given nuclide per cm^3 of coolant. Then after one traversal of the core the activity of the coolant, due to this nuclide, is, by equation (6.9.2),

$$A_1 = \theta \Sigma_{\text{act}} (1 - e^{-\lambda\tau}) = \theta \Sigma_{\text{act}} \cdot \alpha \text{ dis sec}^{-1} \text{ cm}^{-3},$$

where λ is the decay constant, equal to $(1.44 \times \text{half-life})^{-1}$, and α is an abbreviation for the term in brackets. During the next complete cycle (core exit to core exit) this activity decays by a factor $e^{-\lambda T}$, which we will call β , and a further A_1 disintegrations per second are induced by the second traversal of the core. The total activity is now

$$A_2 = \alpha\theta \Sigma_{\text{act}} \beta + \alpha\theta \Sigma_{\text{act}}.$$

Thus after r cycles the activity at the core exit is

$$\begin{aligned} A_r &= \alpha\theta \Sigma_{\text{act}} \{\beta^{r-1} + \cdots + \beta + 1\} \\ &= \alpha\theta \Sigma_{\text{act}} \frac{(1 - \beta^r)}{(1 - \beta)}. \end{aligned}$$

The half-lives of a large number of the activities which are important in practice are short compared with the time for which the reactor operates. The activity then reaches the saturation value

$$A_{\text{sat}} = \theta \Sigma_{\text{act}} \frac{(1 - e^{-\lambda\tau})}{(1 - e^{-\lambda T})} \quad (6.10.1)$$

For sodium or potassium (but not water or Li^7) the time T which the coolant spends in making one complete circuit is much less than $1/\lambda$. When this is so, equation (6.10.1) may be simplified to

$$A_{\text{sat}} = \theta \Sigma_{\text{act}} \frac{\tau}{T} \text{ dis cm}^{-3} \text{ sec}^{-1}; \quad (6.10.2)$$

that is, the saturation activity is just that which would be given by the time-average of the flux experienced by the coolant. The same rule would then apply to coolant circuits with bypasses for cooling the thermal shield and the reflector. The following Table lists coolants which require gamma shielding as a result of neutron irradiation. The shielding for lithium is necessitated by the extremely energetic beta particle which is emitted; when stopped this gives rise to bremsstrahlung, the intensity of which can be calculated from the data given in Chapter 2. The intensity can be greatly reduced by absorbing the beta particles in a material of low atomic number; in this case the best material is clearly the coolant itself.

Table 6.10.1. *Gamma Activities Induced in Pure Coolants by Neutron Irradiation*

Coolant	Element and % wt. of element in coolant	Target nucleus and % abundance in element	Half-life of induced activity	Isotopic activation cross-section per atom of target nuclide	γ -radiation per disintegration	Activation cross-section per gram of coolant ($\text{cm}^2 \text{g}^{-1}$)
Li	Li (100%)	Li ⁷ (92.5)	0.85 sec	0.033 b†	13.4 MeV β (100%) + bremsstrahlung	2.6×10^{-3}
Na	Na (100%)	Na ²³ (100)	15.0 hr	0.56 b†	2.78 MeV (100%) 1.38 MeV (100%)	1.47×10^{-2}
K	K (100%)	K ⁴¹ (6.9)	12.4 hr	1.0 b†	1.5 MeV (20%)	1.0×10^{-3}
H ₂ O	O (88.9%)	O ¹⁶ (99.76) O ¹⁷ (0.039)	7.35 sec 4.14 sec	16 μb^* 7 μb^*	6.2 MeV (85%) 0.9 MeV neutron (100%) 1.6 MeV (70%)	5.32×10^{-7} 8.56×10^{-11} 1.3×10^{-8}
Air	O (23.2%) A (1.3%)	O ¹⁶ O ¹⁷ O ¹⁸ A ⁴⁰ (99.6)	As above 109 min	As above 0.53 b†	As above 1.37 MeV (100%)	1.39×10^{-7} 2.24×10^{-11} 3.4×10^{-9} 1.0×10^{-4}
CO ₂	O (72.6%) A (0.001%)‡	O ¹⁶ O ¹⁷ O ¹⁸ A ⁴⁰	As above As above	As above As above	As above As above	4.35×10^{-7} 7.01×10^{-11} 1.0×10^{-8} 8 $\times 10^{-8}$

† Cross-section for 2200 m sec⁻¹ neutrons.

* Fast neutron reactions—value given is averaged over a fission spectrum. (See Section 3.6.)

‡ Typical argon content from air impurity in CO₂.

Neutron Activation of Impurities in the Coolant Stream

In practice coolants are, for one reason or another, impure, and the impurities become active and contribute to the radiation intensity in the heat-exchanger space after shutdown. We can conveniently illustrate the problem by reference to water, which has the merit for this purpose that the impurity activities are much greater in relation to the intrinsic coolant activity than with some other coolants, such as sodium.

An examination of Table 6.10.1 shows that the water activities themselves die away very rapidly after shutdown, and are no embarrassment as far as maintenance work on the heat exchangers is concerned. However, owing to the effect of the reactor radiations on the water, active radicals are present during operation which cause the gradual corrosion of the materials forming the pipes and pressure vessel. The corrosion products—the so-called “crud”—are radioactive, or may become so owing to their subsequent exposure to the neutron flux, so that active atoms will be carried round with the coolant stream until they decay radioactively, or are deposited in some place where the water velocity is low, or are taken out by purifiers, leaks, or by routine replacement of the water. The result is that the activity builds up to an equilibrium level at which the amount of fresh radioactive material just balances the losses. A mathematical formulation which enables this level to be predicted, provided certain engineering and metallurgical data are available, is given by ROCKWELL.⁽¹⁾ In a particular case—actually the Idaho prototype of the Nautilus reactor—the values given in the following table were found for the equilibrium intensities of the main primary coolant activities, when running at full power and full water flow.

Table 6.10.2. *Equilibrium Coolant Activities Observed in the Idaho Reactor*

Radioactive isotope	Half-life	Saturation activity of primary water at full power, full flow (μc per ml (except N^{17}))	Source of activity
N^{16}	7.3 sec	100	O^{16} in water
N^{17}	4.1 sec	$800 \text{ n cm}^{-3} \text{ sec}^{-1}$	O^{17} in water
K^{38}	7.7 min	5×10^{-2}	?
A^{41}	1.8 hr	4×10^{-2}	Air in water
F^{18}	1.9 hr	4×10^{-2}	?
Mn^{56}	2.6 hr	5×10^{-3}	Steel
Na^{24}	15 hr	$\sim 1 \times 10^{-3}$	Na in water
Co^{60}	5.3 yr	2×10^{-5}	Steel
Fe^{59}	45 days	1×10^{-5}	Steel
Ta^{182}	111 days	6×10^{-6}	Steel

(From ROCKWELL and COHEN.⁽⁵⁷⁾)

Activities Induced by Fast Neutron Bombardment

The activities induced in water (which is probably the case of greatest practical importance) are discussed in Section 3.6, and the methods of estimating

the degree of activity are summarized in Section 6.9. The shielding required by the N^{16} gamma activity can be determined from the data of Sections 2.6 and 2.7. The other important activity is the N^{17} delayed neutron emitter. Figure 6.10.1 shows the attenuation of N^{17} neutrons by water. The relaxation length for these fast (0.9 MeV) neutrons is 4.3 cm in water and polythene and

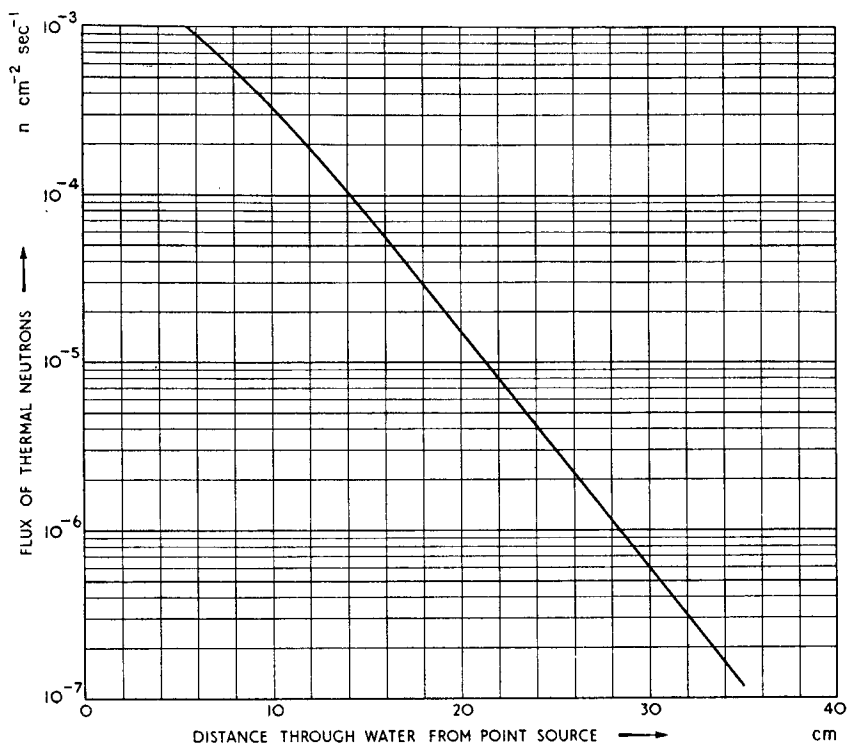


Fig. 6.10.1. Flux of thermal neutrons as a function of distance through a water shield from a point source emitting one N^{17} neutron (0.9 MeV) per second. (SHURE and ROYS.⁽⁵⁸⁾)

14 cm in concrete. The effective removal cross-section of iron is 1.1 barns per atom.⁽¹⁾

Shielding for Circulating Fuel Reactors

The high activity and long half-lives of the fission products in the fuel-cum-coolant stream of a circulating fuel reactor make the problem of accessibility more difficult to solve than in the cases discussed above. The shielding calculations required are straightforward except in two respects. First, when the reactor is in operation the fuel will be circulating rapidly, so that short-lived fission products will not be confined to the reactor core. Little is known about the radiations they emit, except that they are unusually energetic. The design difficulty caused by lack of knowledge of their quantum energies can be

partly avoided if the gamma shield is made of lead, since the relatively slight variation in the absorbing power of lead for gamma energies around 3.5 MeV means that the calculated attenuation is less sensitive than usual to uncertainties in the quantum distribution. The total decay energy of these fission products is known tolerably well (see Section 2.10), and a rough value—which will in

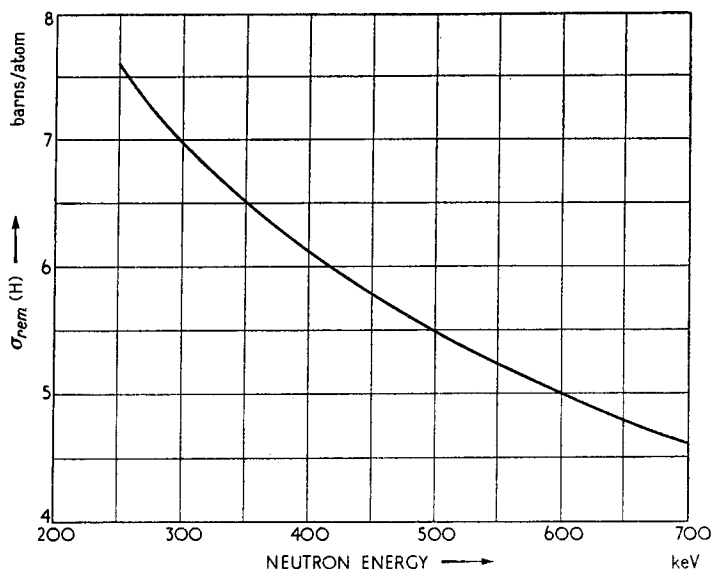


Fig. 6.10.2. Removal cross-section of hydrogen for neutrons in the energy range which includes the delayed neutron emitters (see Table 3.9.2). (After BLIZARD⁽⁵⁹⁾.)

fact be an upper limit—for the intensity behind a lead shield may be found by assuming that the whole of the energy is liberated in the form of 3.5 MeV quanta.

The second point concerns the delayed neutrons which are emitted by the fuel. These neutrons have energies which are below the energy at which inelastic scattering in (for instance) steel is effective. The only efficient method of causing such neutrons to lose energy is through collisions with hydrogen nuclei. The effective removal cross-section of hydrogen in this energy range is shown in Fig. 6.10.2. Methods of calculating the amount of hydrogenous shielding required are discussed in Chapter 4.

6.11. BEAM TRAPS

Most experiments in nuclear physics which use a reactor as a neutron source are performed outside the reactor. A hole, extending to the reflector or to the core, is opened, and a collimated beam of neutrons and gamma radiation is allowed to escape. The experimental apparatus, which is placed in the beam, usually intercepts no more than a few per cent of the radiation, and the main

beam travels on across the reactor hall. The dose-rate in such an external beam a few inches in diameter, taken from the centre of a reactor with a peak flux of 10^{14} n cm⁻² sec⁻¹, is likely to be of the order of 10^5 maximum permissible levels of radiation; it is therefore essential to absorb the beam in a "beam trap" placed as close to the experimental apparatus as possible. The beam trap should be as small as possible, because there is never any room to spare on the experimental face of a reactor, and it should also scatter back towards the experimental apparatus as few neutrons as possible.

The *gamma radiation* which escapes along with the neutrons has a complex energy distribution resulting from the degradation of fission and capture gamma rays by repeated Compton collisions. Typical spectra are shown in Fig. 6.2.1. Provided the energy flowing per unit time in the beam is known (it can be roughly estimated from the data already given), and provided the quantum energy distribution is assumed to be similar to that shown in Fig. 6.2.1, the amount of gamma shielding necessary in the direction of the beam can be found by using the narrow-beam absorption coefficients (Fig. 2.6.4) without a build-up factor correction. Less shielding is required in the sideways direction. A reasonably accurate estimate could be found by calculating the contribution from singly and doubly scattered photons; but the labour involved would rarely be justified, and more often one has recourse either to experimental measurements carried out with a similar arrangement, or to some extremely crude method of calculation designed to overestimate the transmitted dose.

The *neutron energy spectrum* in the beam is also complex, and the flux of medium and high energy neutrons is particularly difficult to calculate. Because of their effectiveness in causing biological damage it is desirable to pay particular attention to them, and wherever possible to measure the fast neutron dosage in the beam experimentally.

The flux contours shown in Fig. 6.11.1 *et seq.* are helpful in designing water or paraffin-wax beam traps. The measurements used a beam of cross-sectional area 1.76 in.² from the centre of a Hanford reactor. The gamma-ray intensity in the beam was 45 r hr⁻¹, and the neutron beam strength over the whole area was 8.4×10^7 n sec⁻¹. The beam entered (a) a water-tank, and (b) an assembly of paraffin-wax blocks (with a density when stacked of about 1.0–1.1) through a canal about 12 inches in depth, to minimize back-scattering. The measurements were carried out with a neutron detector which was sensitive to low energy neutrons, and fast neutrons were not studied directly. Unborated paraffin-wax gave results which were similar to those for water, except that the neutron pattern was slightly modified, due to the larger number of hydrogen atoms per cm³ in the wax. The addition of boron made little difference to the gamma-flux curves, showing that the neutron capture radiation generated in the beam-trap was unimportant in comparison with the gamma rays scattered from the beam. Placing a block of lead in the beam had a much greater effect on the gamma rays than on the neutrons. The performance of a proposed design of beam-trap of roughly similar construction, but for different beam sizes, can be predicted by simply scaling these results. It should be remembered, however, that the scaling factor for gamma rays must include the beam area as well as the intensity, because an intensity in röntgens per hour is equivalent to a

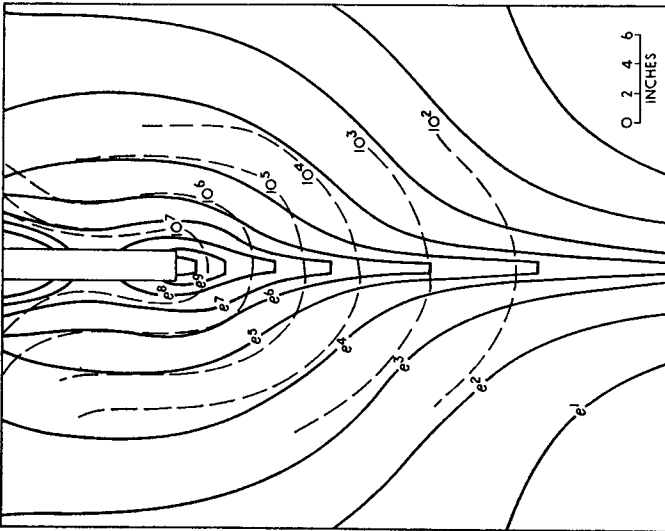


Fig. 6.11.1. Flux contours in water beam-trap (see text). In this diagram and the two which follow, the thermal neutron contours in arbitrary units are shown by dashed lines. Solid lines are contours of gamma-ray intensity in mr hr^{-1} by integral powers of e . (STINSON⁽⁶⁰⁾)

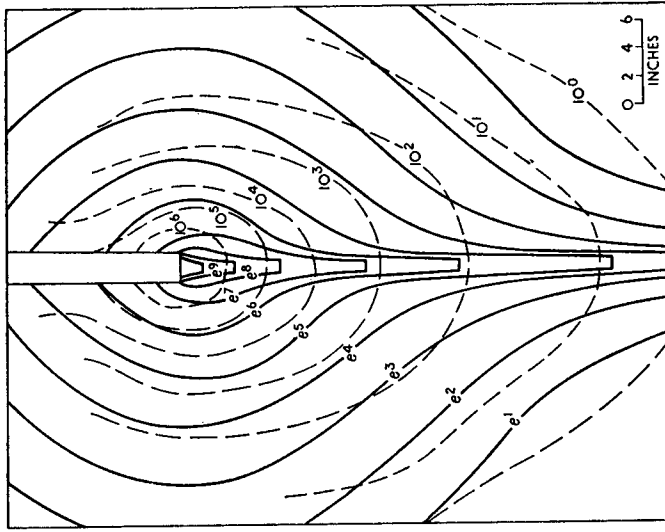


Fig. 6.11.2. Flux contours in a paraffin-borax mixture (H/B atomic ratio = 14)

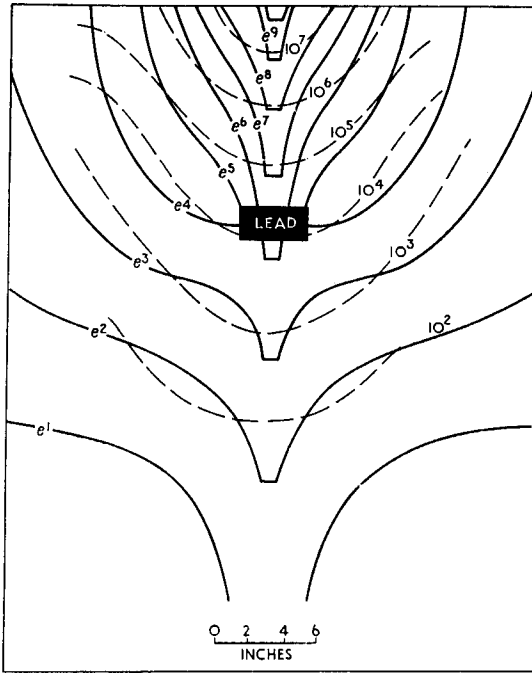


Fig. 6.11.3. Flux contours in water with $2 \times 4 \times 6$ inch lead brick in the beam. The gamma rays are strongly attenuated, but the effect on the neutrons is small.

certain flux of photons per second *per unit area*. Some additional data may be found in reference 32, Chapter 4.

The design⁽⁶¹⁾ of the trap shown in Fig. 6.11.4 is based on the results given in Table 6.11.1. It is intended for use with a beam of diameter 2 inches or less, in the cramped conditions usually found on the experimental face of

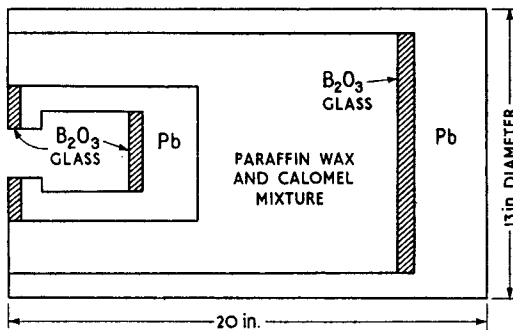


Fig. 6.11.4. Beam trap developed to minimize back-scattering of beams of mixed gamma radiation and neutrons. (NICHOLSON,⁽⁶¹⁾)

a research reactor. The neutron beam impinges on a B_2O_3 glass plate, and some of the scattered neutrons are stopped by the iris diaphragm. The body of the trap is filled with a mixture of mercurous chloride ("calomel") and paraffin-wax; the mixture is prepared by stirring 100 lb. wt. of calomel into a cubic foot of the melted wax.

Efficiency of Beam Traps in Suppressing Backscattered Neutrons

Beam traps for experimental holes should be designed to back scatter the smallest possible fraction of the incident radiation, so as to minimize the unwanted "background" counts recorded by the experimental equipment. In practice the backscattered neutrons tend to be more important than the back-scattered gamma radiation, since more experiments are concerned with neutrons, and the detectors can, if necessary, be made insensitive to gamma radiation; in any case, the albedos for gamma radiation are relatively small (Table 2.8.2).

NICHOLSON⁽⁶¹⁾ has compared the effectiveness of various materials in back-scattering neutrons from one of the BEPO experimental holes (Table 6.11.1). He used a BF_3 counter (mainly sensitive to the lower energy neutrons), located so that neutrons were observed after they had been backscattered through 150° . A slab of polythene 2 cm in thickness (which scatters effectively all the neutrons in the beam) was used as a standard of comparison.

Table 6.11.1. Relative Count-rates Observed in a BF_3 Counter when Neutrons from a BEPO Beam are Backscattered by Various Materials
(NICHOLSON⁽⁶¹⁾)

Material	Count rate	Cadmium ratio of scattered neutrons
Polythene (2 cm)	100	59
B_2O_3 glass (2 cm)	1.4	4.5
B_4C compressed powder, density 1.85 g/cc (1.7 cm)	2.1	5.5
Boric acid powder faced with 0.5 mm iron sheet	3.5	6.2

See also page 192.

6.12. SHIELDING WINDOWS

The necessity for remote handling of highly radioactive gamma-emitting materials inside "hot cells" has focussed attention on methods of allowing the operator to see what is happening without exposing himself to dangerous quantities of radiation. The traditional method has been to view the work through a labyrinth in the shield wall by means of a periscope (Section 2.8), while more recently use has been made of television techniques. Direct viewing *via* a shielding window possesses several advantages over both of these methods.⁽⁶²⁾ Due to the effect of refraction (Fig. 6.12.1) the operator has a very wide field of view, especially when the window is made of lead glass or any other material of unusually high refractive index; he does not have to keep his attention focussed on an eyepiece, and may keep watch on an installation

at the same time as he performs other duties; and a much better perception of depth is obtained. The main disadvantage is that the cost of a shielding window is usually greater than that of a periscope. The window is also liable to be obscured by radiation-induced discolouration, although recent developments described below have largely overcome this difficulty.

A shielding window should give a wide angle of view; it should be free from inhomogeneities and of good transparency; and it should be of high

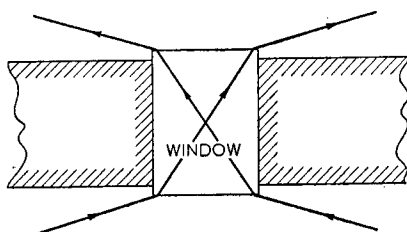


Fig. 6.12.1. Illustrating the increase in the field of view due to the effect of refraction.

density. This last consideration is important, because if the density is considerably less than that of the surrounding wall, the wall would have to be thickened in the vicinity of the window, and the cost would then be increased.

Liquid windows are cheap, easily renewable, and relatively immune from radiation-induced discolouration; the obvious drawback is that the shield may be suddenly lost, if the glass container were to break. To reduce this danger to a minimum the tank should be formed of laminations of tempered glass.

The cheapest of all liquids, water, has a low density, and, being composed of elements of low atomic number, also has a large build-up factor (§ 2.7). Consequently, water shields have to be much thicker than shields of brick or concrete, and this makes them difficult to use as windows in the side walls of a "hot cell." Water finds its principal application as a shielding medium in a rather different situation, where size is unimportant, and where it is convenient to immerse the work in the liquid and to carry out the operations from above by direct viewing. Examples of this kind of application are (i) the fuel discharge channels in the Oak Ridge and Windscale reactors (Fig. 6.1.2a), where the spent fuel is loaded into trolleys which operate under about 12 feet of water, and (ii) the "swimming pool" type of reactor, which needs a water shield 16 feet in depth when the small highly enriched core is operating at a power level of 100 kW (Fig. 6.12.2). Provided the water is kept clean, it has been found that there is no difficulty in carrying out quite intricate maintenance operations on the cores of these reactors. During operation of the reactor, it is necessary to prevent cooling water from being carried rapidly to the surface by convection currents, and so causing a gamma radiation hazard due to the short-lived water activities (Table 6.10.1).

The available range of dense transparent liquids includes acetylene tetrabromide and solutions of zinc bromide, zinc chloride, barium and mercuric

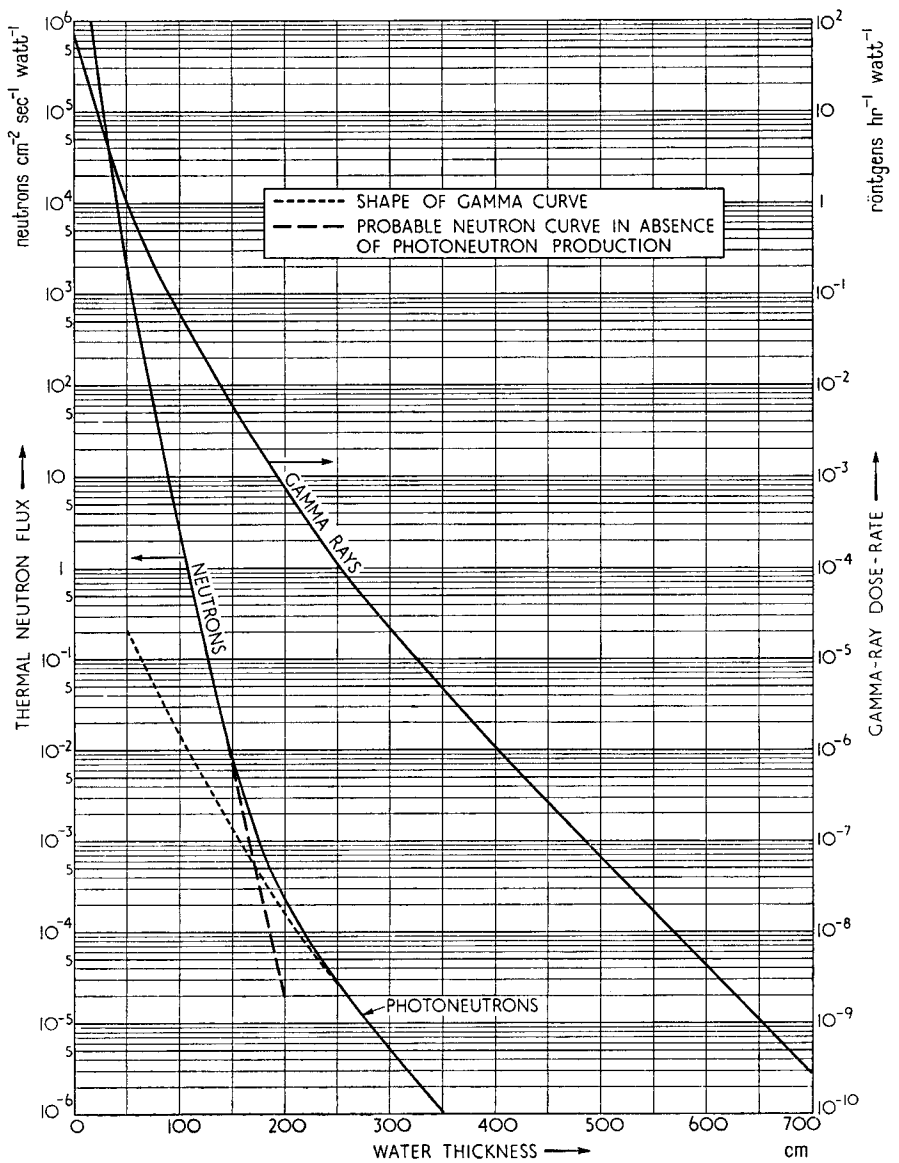


Fig. 6.12.2. Attenuation of radiations in the water shield of the Bulk Shielding Reactor.⁽²⁵⁾

The thermal flux at water thicknesses greater than 200 cm is probably mainly due to photoneutrons produced from the deuterium contained in the water. The distance is measured from the edge of the reactor core.

bromides, and cadmium borotungstate, all of which have densities in the range 2 g cm^{-3} to 3 g cm^{-3} . Acetylene tetrabromide has the highest density obtainable (2.96 g cm^{-3}), but is unfortunately highly toxic. The other liquids suffer from chemical instability or susceptibility to radiation-induced colouration, with the exception of zinc bromide.

Concentrated zinc bromide solution has a density comparable with that of concrete (2.5 g cm^{-3}), good transparency, and good resistance to the effects of radiation.⁽⁶³⁾ The cost (in 1956) of about £1.5 per litre is about one quarter that of non-browning lime glass of comparable density. On exposure to radiation the pure solution becomes coloured, owing to the conversion of bromide ion to free bromine; a haziness is also produced by the oxidation of ferrous iron impurities (which are always present in commercial zinc bromide) to a brown ferric precipitate. The discolouration can be inhibited by the addition of 30 grams of hydroxylamine hydrochloride ($\text{H}_2\text{NOH}\cdot\text{HCl}$) per 100 litres of zinc bromide solution. The additive ensures that the bromine remains in the form of bromide ion, and is itself decomposed to water and nitrous oxide. For windows 3 ft in thickness this treatment is effective at integrated gamma doses of up to 10^7 r ; larger additions can give protection for longer periods. (Note that as the liquid is constantly in circulation due to slight convection currents it is the dose averaged throughout the liquid which determines the rate of discolouration.) Gassing, with the evolution of hydrogen and nitrous oxide, sets in at a dose-rate of about $4 \times 10^4 \text{ r/hr}$, but this need not cause any difficulty in operation if the liquid is stirred, so that the liquid nearest to the source of radiations is continuously renewed. Some care must be taken in sealing the tank, to avoid corrosion by the liquid. The use of stainless steel structural members, with "Teflon" (polytetrafluorethylene) or "Tygon" gaskets is recommended.⁽⁶⁴⁾ A zinc bromide window seven feet in thickness is used in the 60 inch cyclotron at Argonne, and has shown little sign of deterioration from exposure to radiation. A typical installation is shown in Fig. 6.12.3.

Solid windows. The use of a solid glass window avoids the danger of complete loss of the shield in the event of an accident, but introduces additional mechanical complications. In practice the glass shield must be built up from layers of plate glass (as shown in Fig. 6.12.4) or from thick lead glass bricks. For maximum ease of viewing, multiple reflections should be eliminated by filling the interstices between the layers of glass with a liquid of the same refractive index as the glass. It is then necessary to build the window inside a liquid-tight tank, and this adds considerably to the cost.

Normal flint or lime glasses, which are mixtures of sodium, calcium, or potassium silicates, have rather low densities around 2.5 g/cm^3 . Much higher densities can be obtained by including heavy elements, and by using compounds other than silicates. For instance, a lead glass containing 60% by weight of PbO , 21.3% of WO_3 , and 16.1% of P_2O_5 has a density of 6.58 g/cm^3 , and a high refractive index (1.99). In addition, this glass shows good resistance to radiation discolouration.⁽⁶⁵⁾ Unlike most glasses, which turn brown, this goes neutral grey under irradiation; the colour fades rapidly after irradiation ceases. Since lead glass is attacked by certain chemicals, it is sometimes advantageous to protect a window with some transparent inert material, such as a plastic.

A lead glass of density 6.26 g cm^{-3} and containing 4.3% by weight of boron has been developed for thermal neutron shielding,⁽⁶⁶⁾ but its stability under irradiation is poor.

Discolouration under the influence of radiation is a more serious problem with solid than with liquid windows, because the average dose cannot be kept down by circulation or renewal of the shielding medium. The mechanism of

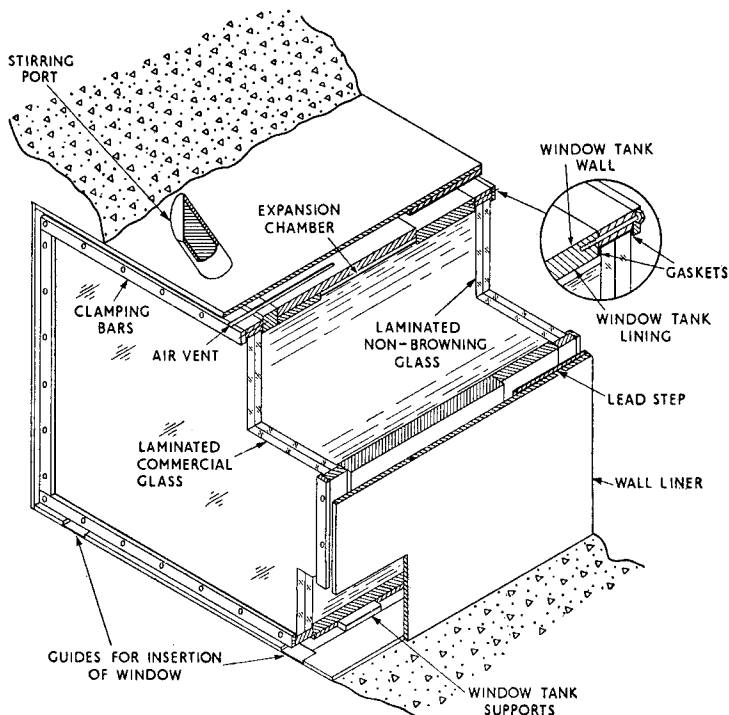


Fig. 6.12.3. Mechanical design of a typical liquid window.⁽⁶²⁾

the discolouration process has therefore been given a considerable amount of attention,⁽⁶⁷⁻⁷²⁾ and is by now fairly well understood.^(74,75) Glasses, like crystals, possess local imperfections which can act as electron traps, to which electrons liberated by ionizing radiations can migrate, and so form "colour centres" which absorb radiation in the visible part of the spectrum. Gamma-induced discolouration tends to show a saturation behaviour at high integrated doses. This is due to an equilibrium which is eventually set up between the rate of production of colour centres and the rate at which they are destroyed by thermal agitation and by a radiation-induced fading process. Thermal agitation tends to cause the colour to fade after irradiation ceases, though in most cases the process is very slow at room temperature. The fading can be greatly accelerated both by heating and by irradiation with visible light. A somewhat different behaviour is observed under neutron bombardment which, by displacing atoms,

creates fresh colour centres. After an initial levelling-off similar to that observed with gamma rays, the discolouration continues to increase slowly but steadily with integrated dose. The discolouration due to neutron-induced damage is much more permanent, and cannot be removed by heating.

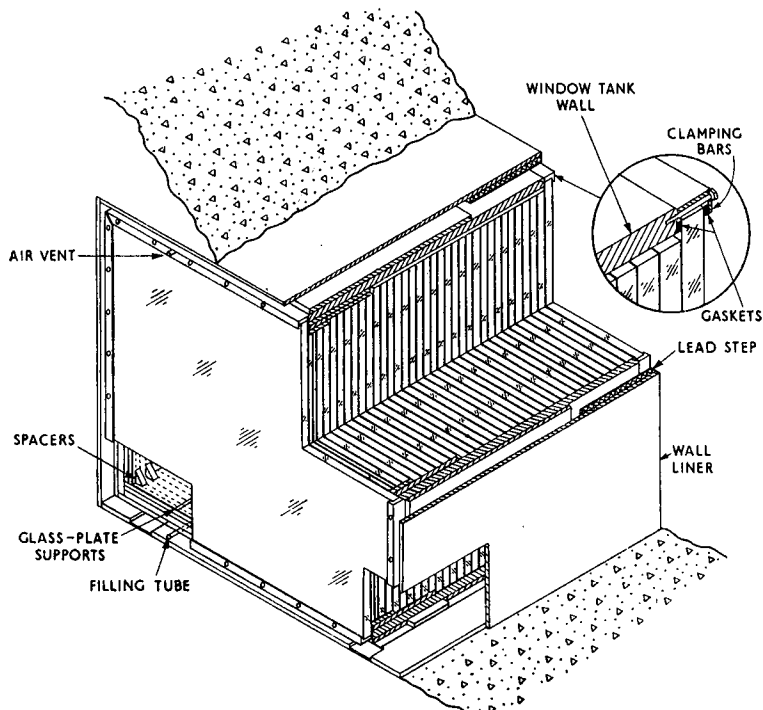


Fig. 6.12.4. Mechanical design of a window composed of laminations of high-density glass.⁽⁶²⁾

Gamma-induced discolouration may be inhibited^(67,73) by including in the glass the oxide of an element which exists in more than one valence state; a plausible explanation of this effect is that radiation then changes the valence state of the added material, instead of forming colour centres. Very few oxides are themselves sufficiently free from colouration to be used for this purpose; both the oxides of cerium, however, have their main absorption bands in the ultra-violet, and do not impart more than a yellow tinge to the glass. Cerium-stabilized plate glasses have been developed which are practically colourless, and which show no detectable change in transmission after exposure to 10^8 röntgens of hard gamma rays.⁽⁷²⁾ Thus the problem of radiation-induced discolouration can be said to have been very largely overcome. It should be noted that although cerium addition improves lead silicate glasses, it gives no improvement with lead borate compositions. In addition, the quantitative behaviour of lead glasses under irradiation is markedly influenced by the temperature and time of heating during manufacture.⁽⁷²⁾

Neutron shielding windows. The non-hydrogenous windows discussed in this section can be made suitable for use with neutrons by adding a section of a transparent organic material, such as perspex, lucite, or plexiglass. Such materials have relatively low resistance to radiation (Table 6.8.2), and should therefore be placed as far towards the outside of the shield as the intensity of capture gamma radiation which they emit will allow. Such composite neutron-gamma windows are suitable for viewing, for example, the heat-exchanger space of a water-moderated reactor.

Table 6.12.1. *Materials for Shielding Windows*

Type of glass or solution	Density (g cm ⁻³)	Refractive index	Optical transmittance for sodium D line*§ (per cent)	Colour	Radiation stability index† (röntgens)	Fading index‡	Cost§ (\$ per pound)
Commercial lime	2.52	1.52	94	Green	2.6×10^2	2-5	0.50
Water-white lime	2.52	1.52	99	Colourless	2.6×10^2	2-5	1.25
Non-browning lime	2.68	1.53	97	Light yellow	1.6×10^6	1-3	2.25
Corning 8362	3.27	1.59	98	Yellow	0.8×10^6	1-3	—
X-ray lead	4.88	1.76	92	Deep yellow	2.0×10^2	2-5	2.50
Dense lead	6.20	1.98	95	Yellow-orange	1.5×10^2	10-50	4.00
ZnBr ₂ solution	2.52	1.56	99	Colourless	—	—	0.60 (Also see text)

* The *optical transmittance* is measured through a sample 1 inch in thickness.

† The *stability index* is defined as the dose required to produce a change in optical density of 1% for white light in a half-value thickness for gamma rays. The optical density ρ is defined as $\rho = -\log_{10}(T)$, where T is the transmittance.

The half-value thickness = $0.693/\mu$, where μ is the linear absorption coefficient (Section 2.2).

The exposure rates were 100 and 700 r/min.

‡ *Fading index*: the ratio by which the exposure must be increased in order that the change in optical density is 0.01, two weeks after the termination of the exposure.

§ *Transmittance and price* are for laminated sections. Costs were estimated in September 1952. (From FERGUSON,⁽⁶²⁾)

6.13. SHIELDING REQUIRED FOR RADIOACTIVE MATERIALS DURING TRANSPORT BY NORMAL PUBLIC SERVICES

During the last twenty years there has been such a rapid increase in the number of shipments of radioactive materials by normal commercial transport services, that it has become necessary to adopt a code of practice regarding the shielding of such shipments. It has been found that the protection of persons handling the packages is not the factor which normally determines the

thickness of shielding; a more stringent requirement is that undeveloped photographic plates carried in the same vehicle should not be fogged by the gamma rays emitted by the radioactive material. The risk of fogging depends on how near to the source a package containing photographic materials happens to lie, how long the two remain in close contact, and how sensitive the photographic emulsion is to the gamma radiation. The risk could obviously be made vanishingly small by increasing the thickness of shielding, but this would increase the cost of transporting the radioactive material. In practice, therefore, a compromise has to be reached, and the acceptable level of radiation chosen so that it carries with it very little risk of accidental fogging of photographic materials, but does not unreasonably increase the cost of transporting radioactive sources.

American Regulations

A committee set up by the American National Research Council in 1947 found that if the fastest kinds of commercially available X-ray film were exposed to a total dose from hard gamma radiation of 11.5 mr the resulting fogging would be barely detectable by a trained observer. The American Interstate Commerce Commission (I.C.C.) regulations for the shipment of radioactive materials were therefore framed to keep exposure of photographic materials during transport below this figure. The American regulations have been adopted in their entirety by Canada.

During transport, relatively few parcels remain in contact with each other for much longer than a day without being disturbed. The I.C.C. regulations therefore exempt from any special restrictions packages for which the gamma radiation level at the surface is never more than 10 mr per 24 hours, and which fulfil certain additional requirements. To be exempt (or in "Group 3") the package must emit no significant amount of alpha or beta radiation or neutrons, and must be mechanically strong enough to prevent any leakage during normal transportation. The external dimensions of the packet must be greater than 4 inches, so that it cannot easily be carried close to the person in a pocket.

The I.C.C. regulations stipulate that packages having a surface gamma dose higher than 10 mr per 24 hours (classified as "Group 1" packages) must be kept at least 15 feet from packages containing undeveloped film, and the shielding provided must be sufficient to reduce the 24 hour dose at 15 feet below the photographic equivalent of 11.5 mr of hard gamma radiation. The permitted dose at 1 metre then works out at 10 "radiation units," if the "radiation unit" is defined as the number of mr hour⁻¹ at one metre, multiplied by a factor K (given in Fig. 6.13.1), which takes into account the variation of the sensitivity of photographic emulsion to gamma radiation of different energies. The factor K is found to be nearly independent of photon energy, for energies greater than about 0.2 MeV; but there is a sharp increase in photographic sensitivity at low energies due to photoelectric absorption in silver and bromine. However, the soft radiations are so easily shielded that for no energy does the factor K necessitate the addition of more than about 1 mm of lead. To ensure the safety of persons handling Group 1 packages, the I.C.C. rules limit the dose at the surface of any package to 0.2 r/hour.

Neutron-emitting sources (e.g. radium-beryllium or polonium-beryllium) are classified separately in America as "Group 2" packages. They must be shielded in such a way that the total biological dose does not exceed 10 mrem hr⁻¹ at one metre. Neutron sources always emit some gamma radiation (see Section 3.10), and the effect of this secondary radiation on photographic plates is usually much larger than the effect of the neutrons themselves.

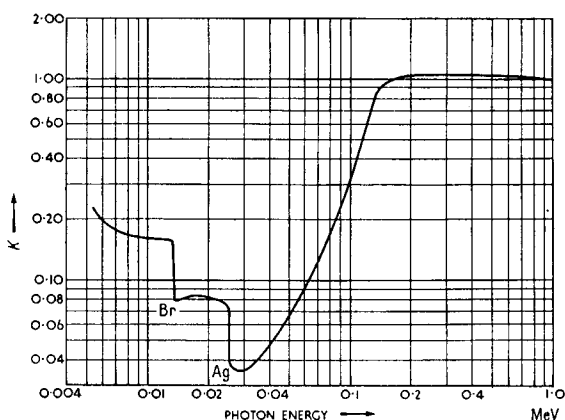


Fig. 6.13.1. Relative sensitivity of photographic plates to gamma radiation of different quantum energies. The factor K is proportional to the number of röntgens required to give a standard degree of blackening (from EVANS⁽⁵⁹⁾).

In addition to specifying maximum radiation levels, the I.C.C. regulations make certain other stipulations intended to decrease the risk in case of an accident. Radium, plutonium, and strontium, three materials which are amongst the most dangerous when ingested (Table 1.6.1), must be packed in an inner metal container of stainless steel, malleable iron, or brass, of not more than 3 inches in diameter or 8 inches in length, with a minimum wall thickness of $\frac{1}{8}$ inch, and having a screw top. Liquid radioactive materials (except those packed in a metal tube) must be surrounded by absorbent material which will completely absorb the liquid in case of breakage, and this absorbent material must all be situated inside the gamma ray shielding.

British Regulations

At present (1957) there are no published regulations covering the transport of radioactive materials in Great Britain, but the standards which have been adopted as an interim measure are as follows:

Rail transport. Packages are divided into two classes:

- Class 1. Radiation level at any point on the surface not greater than 10 mr per 24 hours.
 - Class 2. Radiation level at any point on the surface greater than 10 mr per 24 hours, but not exceeding 300 mr per 24 hours.
- Class 1 packages are not segregated from other freight carried in passenger

trains; Class 2 packages have to be stored at least 4 feet from any other goods.

Sea transport. Long-lived isotopes must be packed so that the radiation at any point on the surface of the container is not greater than 200 mr per hour, and the radiation one yard from the package is less than 10 mr per hour.

Air transport. The conditions recommended by the International Air Transport Association are basically the same as the American I.C.C. regulations summarized above.

Postal transport. Approved types of package are accepted provided the radiation intensity at any point on the surface of the package is not greater than 10 mr per 24 hours.

*Shielding of Beta Emitters During Transport**

The transport regulations stipulate that the thickness of the packaging must be sufficient to attenuate all the beta rays produced in the source. This is extremely simple to arrange, and the required thickness may be found from Fig. 2.11.1. However, beta emission is invariably accompanied by the production of a certain amount of gamma radiation, and this limits the amount of pure beta emitter that can be shipped in one package. As explained in Section 2.11, this gamma radiation is due to two causes. The stopping of beta rays in the shield gives rise to a small amount of penetrating radiation, known as external bremsstrahlung, which can be minimized by absorbing the beta rays in a material of low atomic number. In addition, there is a small amount of inner bremsstrahlung associated with the beta-ray process itself. The order of magnitude of the effect may be appreciated from the following example: it was found experimentally, using an aqueous solution containing 1 mc of P^{32} , that the unshielded gamma-ray dose at one metre was 2.7×10^{-3} mr/hr.⁽⁵³⁾ The dose-rate was reduced to 40% by 0.03 inch of lead shielding and the transmitted radiation was found to have an effective energy of about 0.8 MeV. By comparing this dose-rate with the maximum permitted dose-rate of 10 mr per 24 hours at a distance of 2 inches (which corresponds to the minimum packet size of 4 inches) we conclude that aqueous solutions of P^{32} need to be segregated from photographic materials, or given some gamma shielding, when their strength exceeds about 0.5 mc. The bremsstrahlung production increases with increasing beta energy, so that sources of very energetic beta rays, such as Y^{90} , would need rather more shielding than sources of P^{32} of the same strength.

In addition to bremsstrahlung, there is the further quite distinct process of gamma production associated with the annihilation of positrons (Section 2.4).

Shielding for Radioactive Neutron Sources

The intensities of the neutron sources available in the laboratory are such that comparatively thin shields (usually of water or paraffin-wax) are sufficient. Such thin shields are not small compared with the slowing down length of the fast neutrons, and therefore the "build-up" of the intermediate energy neutrons must be allowed for. BLIZARD⁽⁵⁹⁾ states that the following semi-empirical formula gives an estimate of the fast neutron dose which is adequate for quick

* Shielding of gamma emitters is discussed in Chapter 2 and Section 6.9.

shielding calculations: The dose-rate D , in "maximum permissible levels," at a distance r cm from a source of strength S neutrons per second, is

$$D = \frac{BS \exp [-\Sigma_{\text{rem}} t]}{4\pi r^2 q}, \quad (6.13.1)$$

where t cm is the thickness of the neutron shield, q is the flux of neutrons of the source energy which gives 1 MPL (see Table 1.5.2), and Σ_{rem} is the macroscopic removal cross-section for the given source and shield; it may be obtained from Table 6.13.1, or may be calculated on the assumption that it is equal to 0.9 times the macroscopic total hydrogen cross-section of the shield, for neutrons of the effective source energy. BLIZARD recommends the use of a value of 5 for the build-up factor B for Po-Be or Po-B sources with water or paraffin shields more than 20 cm in thickness.

Table 6.13.1. *Constants for Calculating Shielding for Common Artificial Neutron Sources (equation 6.13.1)*

Source	Effective energy (MeV)†	Yield (n sec ⁻¹ curie ⁻¹)	Macroscopic removal cross-sections (cm ⁻¹)	
			H ₂ O	Paraffin-wax
Ra-Be	5.4	†		
Ra-B	2.9	†		
Po-Be*	4.5	2.3 to 2.6 × 10 ⁶	0.139	0.161
Po-B	2.3	6.1 to 6.9 × 10 ⁶	0.161	0.192
Sb-Be	0.025	†		

* Sources differ in energy spectra depending on method of construction.

† See also Section 3.10.

(From BLIZARD.⁽⁶⁹⁾)

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APPENDIX I

DATA USEFUL IN SHIELDING CALCULATIONS

1. CONVERSION FACTORS

(a) Energy

To obtain Multiply by	MeV	BThU	Joules	kW-hr	g-cal	ergs
MeV	1	1.517×10^{-16}	1.602×10^{-13}	4.44×10^{-20}	3.83×10^{-14}	1.602×10^{-6}
BThU	6.59×10^{15}	1	1.055×10^8	2.93×10^{-4}	252	1.055×10^{10}
Joules	6.25×10^{12}	9.48×10^{-4}	1	2.78×10^{-7}	0.239	10^7
kW-hr	2.25×10^{19}	3.41×10^8	3.6×10^6	1	8.60×10^5	3.60×10^{13}
gram-calorie (15°C)	2.61×10^{13}	3.97×10^{-8}	4.18	1.163×10^{-6}	1	4.18×10^7
ergs	6.24×10^5	9.49×10^{-11}	10^{-7}	2.78×10^{-14}	2.39×10^{-8}	1
gram	5.61×10^{26}	8.50×10^{10}	8.99×10^{13}	2.49×10^7	2.15×10^{13}	8.99×10^{20}

1 watt = 6.24×10^{12} MeV sec⁻¹

1 atomic mass unit = 1.660×10^{-24} gram = 931 MeV

There are approximately 3.1×10^{16} fissions per second per megawatt of reactor heat.

(b) Energy flux and current

Multiply	MeV cm ⁻² sec ⁻¹	MeV cm ⁻² sec ⁻¹	kW ft ⁻²	watts cm ⁻²
by	1.485×10^{-10}	1.409×10^{-13}	3.42×10^8	3.18×10^8
to obtain	watts ft ⁻²	BThU. ft ⁻² sec ⁻¹	BThU. ft ⁻² hr ⁻¹	BThU. ft ⁻² hr ⁻¹

(c) Energy deposition

Multiply	MeV cm ⁻³ sec ⁻¹	MeV cm ⁻³ sec ⁻¹	kW litre ⁻¹	ergs cm ⁻³ sec ⁻¹	fissions cm ⁻³ sec ⁻¹
by	4.54×10^{-9}	4.30×10^{-12}	9.7×10^4	2.83×10^{-3}	3.23×10^{-11}
to obtain	watts ft ⁻³	BThU. ft ⁻³ sec ⁻¹	BThU. ft ⁻³ hr ⁻¹	watts ft ⁻³	watts cm ⁻³

(d) Mass, length, volume, pressure

Multiply	pounds	long tons (2240 lb)	inches	in. ²	in. ³	ft	ft ³
by	453.6	1.016×10^6	2.54	6.452	16.387	30.48	0.0929
to obtain	grams	grams	cm	cm ²	cm ³	cm	(metres) ³

Multiply	ft ³	British imperial gallons	U.S. gallons	lb in. ⁻²	lb ft ⁻³
by	28.32	4.546	3.785	70.306	0.01602
to obtain	litres	litres	litres	g cm ⁻²	g cm ⁻³

(e) Time

$$1 \text{ year} = 3.156 \times 10^7 \text{ seconds} = 5.25 \times 10^5 \text{ min}$$

$$1 \text{ week} = 6.048 \times 10^5 \text{ seconds} = 1.01 \times 10^4 \text{ min}$$

$$1 \text{ day} = 8.64 \times 10^4 \text{ seconds} = 1.44 \times 10^3 \text{ min}$$

2. DEFINITIONS AND FUNDAMENTAL CONSTANTS

1 curie = 3.700×10^{10} disintegrations per second

1 rutherford = 10^6 disintegrations per second

Avogadro's number = 6.023×10^{23} atoms per gram atom

Charge on electron = 4.80×10^{-10} e.s.u. = 1.602×10^{-19} coulombs

Neutron wavelength $\lambda (=2\pi\lambda)$ at energy E eV = $2.86 \times 10^{-9} E^{-1/2}$ cm

Neutron velocity at energy E eV = $1.383 \times 10^6 E^{1/2}$ cm sec⁻¹

Compton wavelength = 2.426×10^{-10} cm

Gamma-ray wavelength at photon energy E eV

$$= \frac{1.239 \times 10^{-4}}{E} \text{ cm}$$

1 röntgen = 83.8 erg g^{-1} in air = 93 erg g^{-1} in water

Mean energy to produce one ion pair in air = 32.5 eV

1 röntgen hr⁻¹ = 9.3×10^{-14} amps cm⁻² in air at S.T.P.

Response of low-energy neutron detector to whole of
incident neutron spectrum

Cadmium ratio = $\frac{\text{Response of detector when surrounded by thin}}{\text{(usually } \sim 1 \text{ mm) cadmium}}$

3. USEFUL RULES FOR GAMMA SOURCES [$0.5 < E_\gamma < 3 \text{ MeV}$].

Power radiated from C curies emitting E MeV per disintegration = $5.94 \times 10^{-3} EC$ watts

Dose-rate due to point source of C curies emitting one photon of energy E MeV per disintegration = $6 CE \text{ r hr}^{-1}$ at one foot and $0.55 CE \text{ r hr}^{-1}$ at one metre

Dose-rate due to a flux F photons cm⁻² sec⁻¹ of energy E MeV = $1.9 \times 10^{-6} FE \text{ r hr}^{-1}$

Dose-rate due to a flux Q watts cm⁻² of gamma radiation = $1.2 \times 10^7 Q \text{ r hr}^{-1}$

APPENDIX II

THERMAL NEUTRON CROSS-SECTIONS

THIS table has been compiled from HUGHES and HARVEY, *Neutron Cross-sections*, U.S.A.E.C. Report BNL-325(1955), and the *Handbook of Chemistry and Physics*, Chemical Rubber Publishing Co., 35th Edition (1953).

Absorption cross-sections are quoted for monokinetic neutrons of 2200 m sec^{-1} velocity. In the majority of cases the cross-section has a " $1/v$ " dependence (see Section 3.5). For cross-sections marked "not $1/v$ " the correct reaction rate in a thermal flux will be obtained by multiplying the 2200 m sec^{-1} cross-section by the factor accompanying this marking in the table (e.g. by 1.3 in the case of cadmium).

APPENDIX II

Atomic number	Symbol	Atomic weight	Density (g cm ⁻³)	Thermal absorption cross-section (barns)	Average scattering cross-section (barns)	Thermal absorption cross-section per cm ² (cm ⁻¹)	Average scattering cross-section per cm ² (cm ⁻¹)	Thermal absorption cross-section per gram (cm ² g ⁻¹)	Average scattering cross-section per gram (cm ² g ⁻¹)
1	H	1.008	0.08988 × 10 ⁻³ 0°C	0.330 ± 0.003	38 ± 4 (gas)	0.018 × 10 ⁻³	2.0 × 10 ⁻³	0.197	23
2	He	4.003	0.17847 × 10 ⁻³ 0°C	71.0 ± 1	0.8 ± 2	3.29	2 × 10 ⁻⁵	6.1	0.1
3	Li	6.940	0.534 20°C	0.010 ± 0.001	1.4 ± 0.3	1.2	0.06	0.67 × 10 ⁻³	0.1
4	Be	9.013	1.8 20°C	755 ± 4	7 ± 1		0.8 × 10 ⁻³	41.9	0.47
5	B	10.82	*	0.0032 ± 0.0002	4 ± 1			0.00016	0.2
6	C	12.01	*	1.88 ± 0.05	4.8 ± 0.2			0.081	0.24
7	N	14.008	0°C	0.330 ± 0.003	10 ± 1	0.101 × 10 ⁻³	0.054 × 10 ⁻³	0.081	0.43
8	O	16.00	0°C	<0.0002	4.2 ± 0.3	<0.1 × 10 ⁻⁶	0.23 × 10 ⁻³	<0.000075	0.16
9	F	19.00	0°C	<0.010	3.9 ± 0.2	<0.5 × 10 ⁻⁶	0.21 × 10 ⁻³	<0.0003	0.12
10	Ne	20.183	0.90035 × 10 ⁻³ 0°C	<28	2.4 ± 0.3	<0.07 × 10 ⁻³	0.064 × 10 ⁻³	<0.08	0.072
11	Na	22.997	0.971 20°C	0.50 ± 0.01	4.0 ± 0.5	0.0127	0.10	0.0131	0.10
12	Mg	24.32	1.74 20°C	0.063 ± 0.004	3.6 ± 0.4	0.0027	0.16	0.0016	0.089
13	Al	26.98	2.699 20°C	0.230 ± 0.005	1.4 ± 0.1	0.0138	0.084	0.0051	0.031
14	Si	28.09	2.42 20°C	0.13 ± 0.003	1.7 ± 0.3	0.0067	0.09	0.0028	0.036
15	P	30.973	*	0.19 ± 0.003	5 ± 1			0.0037	0.10
16	S	32.066	0°C	0.49 ± 0.02	1.1 ± 0.2			0.0092	0.021
17	Cl	35.457	3.214 × 10 ⁻³ 0°C	31.6 ± 1.0	16 ± 3	1.73 × 10 ⁻³	0.87 × 10 ⁻³	0.54	0.27
18	A	39.944	1.7837 × 10 ⁻³ 0°C	0.62 ± 0.04	1.5 ± 0.5	0.017	0.04	0.0093	0.02
19	K	39.100	0.87 20°C	1.97 ± 0.06	1.5 ± 0.3	0.0264	0.020	0.0303	0.023
20	Ca	40.08	1.55 20°C	0.43 ± 0.02	9 ± 2	0.0100	0.21	0.0065	0.13
21	Sc	44.96	3.02 10°C	24 ± 1	2 ± 2	0.97	0.08	0.32	0.03
22	Ti	47.90	4.53 20°C	5.6 ± 0.4	4 ± 1	0.30	0.23	0.070	0.05
23	V	50.95	5.16 20°C	5.1 ± 0.2	5 ± 1	0.36	0.35	0.060	0.059
24	Cr	51.99	7.1 20°C	2.9 ± 0.1	3.0 ± 0.5	0.24	0.25	0.034	0.035
25	Mn	54.94	7.27 20°C	13.2 ± 0.4	2.3 ± 0.3	1.02	0.18	0.141	0.025
26	Fe	55.85	7.8 20°C	2.53 ± 0.06	11 ± 1.0	0.214	0.9	0.0272	0.12
27	Co	58.94	8.9 20°C	3.70 ± 1.5	17.5 ± 1.0	3.4	0.58	0.38	0.071
28	Ni	58.69	8.9 20°C	4.6 ± 0.2	17.5 ± 1.0	4.2	1.6	0.047	0.18
29	Cu	63.54	8.94 20°C	3.69 ± 0.12	7.2 ± 0.7	0.31	0.61	0.035	0.068
30	Zn	65.38	7.14 20°C	1.06 ± 0.05	3.6 ± 0.4	0.070	0.24	0.0098	0.033
31	Ga	69.72	5.91 20°C	2.77 ± 0.12	4 ± 1	0.141	0.21	0.0238	0.036
32	Ge	72.60	5.31 20°C	2.35 ± 0.20	3 ± 1	0.104	0.13	0.0195	0.025
33	As	74.91	5.73 15°C	4.1 ± 0.2	6 ± 1	0.19	0.28	0.033	0.048
34	Se	78.96	4.8 20°C	11.8 ± 0.4	11 ± 2	0.43	0.40	0.090	0.084
35	Br	79.916	3.12 20°C	6.6 ± 0.5	6 ± 1	0.155	0.14	0.050	0.045
36	Kr	83.8	0°C	28 ± 5	7.2 ± 0.7	0.75 × 10 ⁻³	0.19 × 10 ⁻³	0.20	0.052
37	Rb	85.48	1.53 20°C	0.70 ± 0.07	12 ± 2	0.0076	0.13	0.0049	0.085
38	Sr	87.63	2.53 20°C	1.16 ± 0.06	10 ± 1	0.020	0.17	0.0080	0.069
39	Y	88.92	4.80*	1.38 ± 0.14	3 ± 2			0.0094	0.02
40	Zr	91.22	6.4 20°C	0.18 ± 0.02	8 ± 1	0.0076	0.34	0.0012	0.053
41	Nb (Cb)	92.91	8.4 20°C	1.1 ± 0.1	5 ± 1	0.060	0.27	0.0071	0.032

42	Mg	95-95	10-2	2-5 ± 0-2	7 ± 1	0-16	0-44	0-016	0-044
43	Tc	99	100 ± 25	100 ± 25				0-6	0-015
44	Ru	101-7	2-46 ± 0-12	2-46 ± 0-12	6 ± 1	0-18	0-43	0-6	0-035
45	Rh	12-5	150 ± 7	150 ± 7	5 ± 1	0-11	0-36	0-88	0-029
46	Pd	106-7	8-0 ± 1-5	8-0 ± 1-5	3-6 ± 0-6	0-55	0-25	0-045	0-020
47	Ag	107-88	62 ± 2	62 ± 2	6 ± 1	3-62	0-35	0-345	0-033
48	Cd	112-41	2550 ± 100	2550 ± 100	7 ± 1	118	0-33	13-6	0-037
49	In	114-76	(not 1/ <i>n</i> , 1-3)	(not 1/ <i>n</i> , 1-3)	2-2 ± 0-5	7-3	0-08	1-00	0-011
50	Sn	118-70	190 ± 10	190 ± 10	4 ± 1			0-0030	0-020
51	Sb	121-76	0-60 ± 0-10	0-60 ± 0-10	4-3 ± 0-5	0-182	0-142	0-0272	0-0213
52	Te	127-61	5-5 ± 1	5-5 ± 1	5 ± 1	0-133	0-15	0-0212	0-024
53	I	126-91	6-7 ± 0-6	6-7 ± 0-6	3-6 ± 0-5	0-157	0-084	0-032	0-017
54	Xe	131-3	5-851 × 10 ⁻³	35 ± 5	4-3 ± 0-4	0-94 × 10 ⁻³	0-11 × 10 ⁻³	0-16	0-020
55	Ba	132-91	760 mm	29-0 ± 1-5	20 ± 5	0-246	0-17	0-131	0-091
56	Cs	137-36	20°C	1-17 ± 0-10	8 ± 1	0-048	0-12	0-0051	0-035
57	La	138-92	20°C	8-9 ± 0-3	15 ± 5	0-238	0-4	0-0386	0-065
58	Ce	140-13	20°C	0-70 ± 0-08	9 ± 6	0-021	0-3	0-0030	0-04
59	Pr	140-92	20°C	11-2 ± 0-6	25 ± 5	1-33	0-31	0-048	0-104
60	Nd	144-27		46 ± 2				0-192	
61	Pm	145							
62	Sm	150-43						22	
63	Eu	152-0		5500 ± 200	8 ± 1	170		18	0-033
64	Gd	156-9		(not 1/ <i>n</i> , 1-5) 4600 ± 400 (not 1/ <i>n</i> , 0-95) 46,000 ± 2000 (not 1/ <i>n</i> , 0-85)				176	
65	Tb	159-2		1100 ± 150	100 ± 20			0-17	0-37
66	Dy	162-46		64 ± 3	15 ± 4	2-8	0-26	4-1	0-054
67	Ho	164-94		166 ± 16				0-23	
68	Er	167-2		118 ± 6				0-42	
69	Tm	169-4		36 ± 4	12 ± 5			0-125	0-04
70	Yb	173-04		108 ± 5				0-37	
71	Lu	174-99		105 ± 5				0-353	0-27
72	Hf	178-6		21-3 ± 1-0	8 ± 2	4-70	0-36	0-071	0-017
73	Ta	180-88		19-2 ± 1-0	5 ± 1	1-18	0-27	0-071	0-016
74	W	183-92	20°C	84 ± 4	14 ± 4	1-21	0-32	0-063	0-045
75	Re	186-31	20°C	14-7 ± 0-7	11 ± 1	5-6	1-0	0-27	0-045
76	Os	190-2	22-42	430 ± 20		1-05	0-79	0-047	0-035
77	Ir	193-1	17°C	8-1 ± 0-4	10 ± 1	0-53	0-66	1-34	0-031
78	Pt	195-23	20°C	98 ± 1	9-3 ± 1-0	5-78	0-55	0-025	0-028
79	Au	197-2	17-5°C	380 ± 20	20 ± 5	15-5	0-8	0-300	0-06
80	Hg	200-61	20°C	(not 1/ <i>n</i> , 0-95)				1-14	
81	Tl	204-39		3-3 ± 0-5	14 ± 2	0-11	0-50	0-0097	0-041
82	Pb	207-21	20°C	0-17 ± 0-01	11 ± 1	0-056	0-36	0-00049	0-032
83	Bi	209-00	20°C	0-032 ± 0-003	9 ± 1	0-0090	0-25	0-000092	0-026
84	Po	210							
85	At	210							
86	Rn	222	0°C						
87	Fr	223	760 mm						
88	Ra	226-05		510 ± 40				1-35	
89	Ac	227		7-0 ± 0-4				0-018	
90	Th	232-12	20°C		0-20				
91	Pa	231		7-68 ± 0-07					
92	U	238-07	20°C	(not 1/ <i>n</i> , 0-99)	0-36			0-0193	

* These elements occur in allotropic forms of different densities

APPENDIX III

SELECTED ACTIVATION DATA FOR THERMAL NEUTRONS

Element	Target nucleus	Half-life of induced activity	γ -rays emitted per disintegration	Activation cross-section per average atom (barns)	Activation cross-section per gram of element ($\text{cm}^2 \text{g}^{-1}$)
Na	Na^{23}	15.0 hr	2.78 MeV (100%); 1.38 MeV (100%)	0.56	0.015
Al	Al^{27}	2.27 min	1.78 MeV (100%)	0.21	0.0047
A	A^{40}	109 min	1.37 MeV (100%)	0.53	0.0079
K	K^{39}	1.3×10^8 yr	1.46 MeV (100%)	2.8	0.043
	K^{41}	12.4 hr	1.51 MeV (20%)	0.069	0.0011
Cr	Cr^{50}	27.8 d	0.32 MeV ($\sim 3\%$)	0.48	0.0055
Mn	Mn^{55}	2.58 hr	0.822 MeV (100%); 2.06 MeV (20%); 1.77 MeV (30%)	13.4	0.148
Fe	Fe^{58}	46 d	1.1 MeV (50%); 1.3 MeV (50%)	2.8×10^{-3}	3.0×10^{-5}
Co	Co^{59}	5.28 yr	1.17 MeV (100%); 1.33 MeV (100%)	36.0	0.37
Ni	Ni^{64}	2.57 hr	0.37 MeV (14%); 1.12 MeV (29%); 1.49 MeV (14%)	3×10^{-2}	3.1×10^{-4}
Cu	Cu^{63}	12.8 hr	1.34 MeV (0.5%); β^+ (19%) (See page 145)	2.7	0.025
Zn	Zn^{64}	250 d	1.12 MeV (45%)	0.24	0.0023
Zr	Zr^{94}	65 d	0.721 MeV (100%)	0.017	1.1×10^{-4}
	Zr^{96}	17 hr	0.75 MeV (100%)	5.6×10^{-3}	3.8×10^{-5}
Mo	Mo^{98}	67 hr	0.741 MeV (17%); 0.78 MeV (3%)	3.1×10^{-2}	1.9×10^{-4}
Sn	Sn^{112}	112 d	0.393 MeV (100%)	1.2×10^{-2}	6.0×10^{-5}
	Sn^{116}	14.5 d	0.162 MeV (100%); 0.159 MeV (100%)	9×10^{-4}	4.5×10^{-6}
	Sn^{118}	250 d	0.065 MeV (100%); 0.024 MeV (100%)	2.4×10^{-3}	1.2×10^{-5}
Au	Au^{197}	2.7 d	0.411 MeV (100%)	96	0.294
Hg	Hg^{202}	47 d	0.279 (100%)	1.15	0.0034
Pb	Pb^{208}	3.2 hr	No γ -rays	3.1×10^{-4}	9×10^{-7}

Notes:

1. All cross-sections quoted are for 2200 m sec^{-1} neutrons.
2. Data from D. J. HUGHES and J. A. HARVEY, *Neutron Cross-sections*, BNL325 (1955) and U.S.A.E.C. *Reactor Handbook*, Vol. I, 1955.
3. The activity of irradiated lead is mainly determined by its impurities.

NAME INDEX

(Only authors mentioned by name in the text are included in this index)

- | | |
|---|--|
| <p> AEBERSOLD P. C. 15
 ADAIR R. K. 115
 ALBERT R. D. 73
 ALLEN S. J. M. 36
 ANDERSON H. L. 149
 ANDREWS D. G. 227
 ANSLOW G. A. 15
 ARONSON R. 44, 190-1

 BALDWIN G. C. 155
 BARSCALL H. H. 115, 120, 129
 BEATTIE J. W. 93
 BELL R. E. 264
 BENDELL D. 265
 BERGER M. J. 54, 59
 BERNSTEIN S. 71
 BETHE H. A. 11, 23, 76, 83, 87, 133
 BIRAM M. B. 7
 BILLINGS J. J. 25
 BLATT J. M. 107
 BLEULER E. 73
 BLIZARD E. P. 314, 327, 328
 BLOCH F. 79
 BOPP C. D. 294-5
 BORST L. B. 66
 BUECHNER W. W. 76-7
 BURT B. P. 74-5

 CALKINS V. P. 294
 CAVE L. 44
 CERTAINE J. 191
 CHAPMAN G. T. 180, 181
 CHISHOLM-BATTEN A. W. 277
 CLARK O. 193
 CLIFFORD C. E. 199, 202
 COHEN P. 312
 COLGATE S. A. 33
 CORNER J. 44, 60
 CORYELL C. D. 63, 72

 DAVISSON C. M. 33
 DAY R. B. 128
 DICK D. R. R. 273
 DOGGETT J. 54, 60
 DUMOND J. W. M. 23

 EASTWOOD W. S. 304
 EGGLER C. 133 </p> | <p> ELDER F. R. 155
 EMDE F. 212
 EVANS R. D. 33, 326

 FANO U. 43, 44, 49, 50
 FEATHER N. 72, 146
 FELD B. T. 107, 140, 155
 FERGUSON K. R. 324
 FERMI E. 73, 192, 193
 FESHBACH H. 120, 184
 FIELD E. L. 227, 228
 FLAMMERSFELD A. 72
 FLOWERS B. H. 94, 95
 FOSSEY E. B. 274
 FOWLER W. A. 95
 FRY T. M. 233

 GAMBLE R. L. 61, 62
 GARTH R. C. 133
 GHOSH A. 305
 GLENDENIN L. E. 72
 GOLDSTEIN H. 23, 42-4
 GOLDSTEIN N. 301
 GOODMAN C. 122
 GREISEN K. 84-6
 GROSHEV L. V. 136

 HARVEY J. A. 298, 333, 336
 HALL H. 22
 HALLIDAY D. B. 200, 203-6, 248
 HALMSHAW R. 40
 HALPERN O. 154, 193
 HANSON A. O. 139
 HARRISON J. R. 203-4, 253, 265
 HAUSER W. 120
 HAYWARD E. 59
 HEITLER W. 21-23, 76, 78, 83, 87
 HENRIE J. O. 276
 HINTON C. 234
 HOFFMAN J. G. 193
 HORTON C. C. 200-206
 HOWLETT J. 66-68
 HUBBELL J. H. 59
 HUGHES D. J. 130, 132, 133, 297, 298, 301, 333, 336
 HULLINGS W. Q. 256
 HULME H. R. 22
 HURWITZ H. 50, 187 </p> |
|---|--|

- ILIFFE C. E. 235, 246
 INTHOFF W. 300, 301

 JAHNKE E. 212
 JOSEPHS S. A. 68
 JUSTUS K. M. 75, 76

 KASTNER J. 305
 KENDALL J. 233
 KENNEDY R. J. 90
 KERST D. W. 154, 155
 KIEHN R. M. 122
 KINSEY B. B. 134
 KIRKBRIDE J. 61, 62
 KIRN F. S. 90
 KITZES A. S. 256
 KLEMA E. D. 139
 KNAPP R. 40
 KNIPP J. K. 79
 KOUTS H. J. 210

 LAKEY J. A. 53, 203, 204
 LATYSHEV G. D. 36
 LAUGHLIN J. S. 93
 LAWSON J. D. 82, 85, 86, 88
 LAX M. 95
 LEVIN J. S. 130, 133
 LIEDTKE R. A. 136
 LISTON R. H. A. 44, 60
 LIVINGSTON M. S. 11, 193
 LOVE T. 237
 LUENEBOURG R. 193
 LÜSCHER E. 139

 MAIENSCHIN F. 237
 MARTIN H. 140
 McELHINNEY J. 95
 MCKINNEY V. L. 256
 MILLAR C. H. 264
 MILLER D. W. 115
 MILLER W. 80
 MILLS A. M. 265
 MITCHELL J. S. 7
 MITTELMAN P. S. 136
 MONTALBETTI R. 154, 155
 MOON P. B. 31
 MORGAN K. Z. 10
 MOTEFF J. 70, 71
 MOTZ J. W. 237
 MURRAY G. S. 263

 NATHANS R. 154
 NICHOLSON K. 317, 318

 PARKER H. M. 2
 PASECHNIK M. V. 128
 PAVLISH A. E. 259, 261
 PEEBLES G. H. 44, 54
 PELLE R. W. 61
 PERRIN F. 23
 PLACZEK C. 192, 212, 215
 POOLE M. L. 106
 PRICE B. T. 253, 265
 PRICE G. A. 154, 155
 PUTMAN J. L. 73

 RENNIE C. A. 68
 ROBSON J. M. 264
 ROCKWELL T. 256, 290, 312
 ROLLINS W. 1
 ROSSI B. 84-86
 ROYS P. A. 313
 RUSS S. 1

 SAUTER F. 22
 SCHAMBERGER R. D. 205, 207, 208
 SCHIFF L. I. 85
 SCHONLAND B. F. J. 72
 SEGRÈ E. 150
 SELIGER H. H. 75, 76
 SHORE F. J. 205
 SHURE K. 313
 SIDEBOTHAM E. W. 282, 283
 SIEWERS D. C. 95
 SIMON A. 199, 202
 SISMAN O. 294, 295
 SKILLINGS D. J. 263
 SMITH J. H. 220, 221
 SMITH N. M. 210
 SNYDER M. J. 261
 SNYDER W. S. 7
 SPATZ W. D. B. 301
 SPENCER L. V. 43, 44, 50
 SPICER B. M. 95
 STANDEN T. 282, 283
 STEPHENSON R. 10
 STINSON W. P. 316
 STOBBE M. 22
 STORM M. L. 220, 221
 STORRS C. L. 180, 181
 STORY J. S. 68
 SUGARMAN N. 63

 TAIT J. H. 7
 TAYLOR J. J. 50, 53, 212, 243-5, 305
 TRUMP J. G. 74

 UHLENBECK G. E. 79
 UNTERMYER S. 64, 65

- | | |
|---------------------------------------|----------------------|
| VAN DE GRAAFF R. J. 74 | WHYTE G. N. 305 |
| VICTOREEN T. A. 15, 22, 36 | WICK G. 44 |
| | WIEGAND C. 150 |
| WALT M. 120, 128, 129 | WIGNER E. P. 64 |
| WATT B. E. 146 | WILKINS J. F. 42, 43 |
| WAY K. 64 | WU C-S. 73, 76 |
| WEILLS J. T. 64, 65 | WYARD S. J. 78, 79 |
| WEISSKOPF V. F. 107, 114, 129, 184 | WYND J. C. 259 |
| WENDE C. J. W. 214 | |
| WEST R. 304 | YAFFE L. 75, 76 |
| WHITE G. R. 4, 22, 28, 29, 35, 36, 50 | |
| WHITEHOUSE W. J. 73 | ZUNTI W. 73 |

SUBJECT INDEX

A

Absorbed dose, unit 5
 Absorption coefficient, linear 20
 mass 20
 gamma-rays, values 34-39
 absorption cross-sections, thermal 333-335
 absorption edge 21
 absorption, resonance, in U^{238} 238
 "true" 28
 accessibility problem 309
 acetylene tetrabromide 321
 activation, by fast neutrons 296, 299-302
 by slow neutrons 296-7, 336
 cross-section 145, 297
 of coolants 309-314
 of impurities 297
 suppression of 299
 age, neutron 172, 174
 theory 158, 171, 174, 186
 aggregate 258, 261
 barytes 266
 boron-bearing 269
 density, effect on cost 275
 iron-bearing 266
 air, activation of 311
 energy for ion pair 332
 gamma absorption coefficients 29, 38, 39
 permissible levels of contamination 12
 aircraft reactor shields 296
 air-walled ionization chamber 91
 albedo, neutrons 192-197, 296, 318
 alpha particles, biological effects 5, 6
 from neutron absorption 137
 neutrons from bombardment by 149, 150
 range in air 11
 alumina cement 260, 261
 aluminium, build-up factors 49, 51
 fast neutron removal cross-section 180
 gamma cross-sections (values) 27, 31, 34, 35, 38, 39
 slow neutron cross-section 118
 angular current 159
 angular density 158
 angular flux 159
 angular momentum 119, 142
 annihilation of positron 23, 327
 annular ducts, neutron leakage 202
 antimony-beryllium source 151
 shielding 328
 argon, activation of 311
 atomic mass unit 331
 attenuation of gammas (*see* gamma radiation)

attenuation of neutrons (*see* neutrons)

Auger electron 22

Avogadro's number 332

B

Backscattering (*see* scattering)
 barium bromide, for windows 309
 barium-Portland cement 260
 barn 20, 104
 barytes concrete (*see* concrete)
 beam traps 314-318
 BEPO reactor 243, 253, 265, 266
 beryllium, gamma absorption coefficients 34, 35
 in (α, n) source 149, 150
 in (γ, n) source 151
 neutron binding energy 109, 153
 neutron diffusion properties 168
 neutron slowing-down 175
 removal cross-section 180
 beta decay 73, 142, 143
 emitters, transport 327
 particles (*see* electrons)
 betatrons, gamma radiation from 87, 89
 binding energies, of neutrons 107, 109, 110, 112
 biological doses, relative, in water shield 285
 biological effects of radiation 1-6
 biological half-life 13
 biological shield (*see* bulk shield)
 bismuth, removal cross-section 180
 Boltzmann's constant 105
 borai 251, 256
 boranes, as neutron shields 293
 borated graphite 233, 257
 borated plastics 294, 295
 borated thermal shields 232, 233, 252, 256
 borated water, gas evolution in 292
 neutron energy distribution 106
 reduction of capture gammas 287, 288
 boron, cross-section (thermal) 137, 138
 for suppressing activation 299
 in concrete 269
 (n, α) reaction 118, 247, 255
 neutrons from α -bombardment 149, 150
 removal cross-section 180
 use in shielding 102, 138
 boron compounds, in reactors 237, 310
 neutron backscattering by 318
 boron steels 255
 boroxal 256

- bound atom cross-section 118
 - Bragg-Gray relation 91, 309
 - branching ratio 112
 - Breit-Wigner formula 113, 131
 - bremsstrahlung, and neutron production 152, 155
 - energy distribution 77, 82-90
 - external 79, 327
 - inner 79, 81, 327
 - intensity 77, 78, 82-90
 - British thermal unit 245, 331
 - Brookhaven reactor 266
 - build-up factors for gamma radiation 41-56, 88
 - as sum of exponentials 50
 - energy reflection 60
 - finite shields 54
 - for heterogeneous media 53, 243, 244
 - for oblique incidence 54-56
 - values 45-55
 - build-up of neutrons 174, 176, 186-190
 - bulk shield 231, 258-296
 - metal-hydrogen 284-296
 - minimum cost 278-284
 - Bulk Shielding reactor, gamma spectrum 237
 - shield performance 320
- C
- Cadmium, borotungstate, as shielding window 321
 - capture gamma radiation 134
 - difference method 114, 131
 - neutrons 100
 - ratio 188, 332
 - total cross-section (values) 114
 - used in thermal shield 232, 236, 257
 - Calder Hall reactor 309
 - calcium, magic nucleus 110
 - calculations, two-group 236
 - calorimetric method for gamma intensity 92
 - capture gamma radiation (*see* gamma radiation)
 - capture scattering (*see* scattering)
 - capture, resonance, gammas from 236
 - carbon dioxide, activation of 311
 - carbon, gamma absorption coefficients 34, 35, 38, 39
 - neutron binding energy 109
 - neutron diffusion properties 168
 - neutron slowing-down properties 175
 - removal cross-section 180
 - cascade theory 88
 - cataract of eye 5, 7
 - cement, barium-Portland 260
 - heat of hydration 261
 - high alumina 260, 261
 - cement, magnesium oxychloride 259, 260
 - Portland 258-261
 - temperature rise during setting 261
 - water ratio 270
 - centre-of-mass co-ordinates 119
 - channelling effect factor 210
 - characteristic radiation 78
 - chemical compatibility 231
 - chest X-ray, dose from 8
 - chlorides, dissolved, and decomposition of water 291
 - chlorine, removal cross-section 180
 - circulating fuel reactors (*see* reactors)
 - cloudy crystal ball (*see* nucleus, optical model)
 - cobalt-60, dose through lead shield 304
 - cobalt, in steel, activation 297, 298
 - coherent scattering (*see* scattering)
 - colemanite 269
 - collision, elastic, neutron energy-loss 102, 169-171
 - colour centres 322
 - compatibility, chemical 231
 - compound nucleus 104, 111-116, 130, 141
 - Compton electrons, range of 91
 - Compton scattering (*see* scattering)
 - Compton wavelength 24, 332
 - concrete, barytes 37, 249, 265-268, 276, 280-283
 - boron-containing 269
 - capture gamma radiation in 246
 - cost of 259, 275, 276, 278-284
 - cracks in 272
 - creep 278
 - density, control of 273-275
 - engineering considerations 269-278
 - ferrophosphorus 268, 276
 - high density 276
 - iron aggregate 266, 268
 - cost 276, 280-283
 - joints in 271, 272, 278
 - magnetite aggregate 37, 266, 276
 - mixing 269
 - nuclear heating 278
 - placing 270
 - Portland 261, 262, 268
 - cost 276, 280-283
 - gamma attenuation 37, 263, 264, 303
 - hydrogen content 259, 261, 263
 - neutron attenuation 185, 188, 189, 264
 - neutron diffusion properties 168
 - nuclear properties 262
 - slowing-down length 175
 - temperature rise in 249
 - temperature rise on setting 261
 - prepacking method of construction 270
 - radiation damage 276, 277

- concrete, reinforcement 278
 - segregation 270, 273
 - shields 258–284
 - shrinkage 269, 270
 - tolerances in design 275
 - cone of effectiveness 306
 - conversion coefficient, internal 143
 - conversion electrons 143
 - conversion factors (energy, length, mass, pressure, time, volume) 244, 331, 332
 - coolants, activity induced in 309–314
 - layout of circuit 232
 - co-ordinates, centre-of-mass (or C.G.) 160
 - laboratory 160, 169
 - copper, decay scheme 145
 - gamma absorption 31, 33
 - removal cross-section 180
 - cosine distribution, neutrons 192
 - cosmic rays, dose from 8
 - cost of shielding 275, 280–284
 - Coulomb forces 108, 109
 - cross-section 19
 - absorption (thermal) 333–335
 - activation 145, 297, 302, 336
 - bound atom 118
 - Compton 25–27
 - energy absorption 27, 28
 - for formation of compound nucleus 104, 115
 - free atom 117
 - macroscopic 20, 104
 - microscopic 20, 104
 - per average atom 145, 297
 - photoelectric 21
 - potential scattering 115
 - Rayleigh 31
 - removal 176–184, 328
 - Thomson (nuclear) 31
 - total 20, 115
 - true absorption 28
 - “crud” 312
 - crystal, neutron diffraction by 118
 - curie 304, 332
 - current, angular 159
 - due to various source configurations 218–228
 - neutron 158
 - use in heating calculations 240
 - cylindrical ducts, bent 203
 - leakage from 197–201
 - cylindrical sources, current and flux from 227
 - decomposition, radiation (*see* radiation)
 - decrement, logarithmic 171
 - delayed neutrons (*see* neutrons)
 - shielding for 314
 - Delbruck scattering 32
 - delta rays 4
 - density, neutron 158, 159
 - slowing-down 172
 - density of shield, effect on weight and cost 275–280
 - detectors, gamma, sensitivity 95
 - deuterium, in (γ , n) source 151
 - neutron binding energy 153
 - deuterons, range in air 11
 - DIDO reactor 241, 256, 266, 276
 - diffraction of neutrons by crystal 118
 - diffraction scattering (*see* scattering)
 - diffusion constant 159, 161, 168
 - equation 159
 - length 159, 161, 162, 168
 - theory 158, 159, 162–169
 - discolouration, radiation-induced 321–324
 - disc sources, current and flux from 218
 - distance, as shield 306–307
 - dose, absorbed, units 5
 - build-up factor (gammas) 41
 - median lethal 8
 - outside gamma-emitting tank 308
 - dose-rate, from various causes 8
 - permissible 12
 - dosimeter, Hurst-Ritchie 179
 - double Compton scattering 32
 - Dounreay reactor 232, 233, 257
 - duct, in neutron shields (annular, bent, cylindrical, helical, stepped) 197–210
 - duct penetration, gamma rays 57
- E
- e^{-x} 217
 - effectiveness, cone of 306
 - Einstein relation 109
 - elastic scattering (*see* scattering)
 - angular dependence 129
 - electron, Auger 22
 - backscattering of 74–76
 - bound, scattering by 30
 - bremsstrahlung from stopping of 76–90
 - charge on 332
 - Compton cross-section of 25, 26
 - energy absorption cross-section 27
 - energy distribution from β -decay 73
 - energy loss by ionization 82
 - energy loss by radiation 82
 - internal conversion 143
 - K-capture 81, 144
 - orbital electron capture 81, 144
- D
- de Broglie wave 104
 - decaborane 293
 - decay constant 67, 297, 310

- electron, permissible flux of 10
 - positive (*see* positron)
 - quasi-exponential attenuation 74
 - radius of 21
 - range-energy relation 71, 72
 - relative biological efficiency 6
 - rest energy of 21
 - scattering cross-section 27, 28
 - electron-volt 17
 - emergency exposure 12
 - end-point energy 73, 75
 - energy absorption cross-section 28, 38, 39
 - energy-balance calculation 308
 - energy, binding 107–112
 - energy build-up factor (gammas) 41
 - energy conversion factors 244, 331
 - energy-loss of neutron per collision 102
 - energy-mass equivalence 109
 - "energy unit" 15
 - engineering of concrete shields 269–278
 - epicadmium neutrons 100
 - epithermal neutrons 100
 - heating by 252
 - erosion of active material 309
 - erythema, skin, dose to produce 8
 - escape of gamma radiation from reactors 235–245
 - escape of neutrons from surface 192
 - eV 17
 - excitation energy 108, 113, 120
 - excited state 101, 111, 116
 - expansion joints, in concrete 271, 272, 278
 - exponential attenuation 19, 212
 - of gamma rays 33, 36
 - of neutrons 102
 - exponential functions 217
 - exponential integral functions 212–215
 - external bremsstrahlung 79
 - extrapolation length 163
 - eye, cataract of 5, 7
- F
- Fading of induced coloration 322–324
 - fast neutrons (*see* neutrons)
 - Feather rule 72
 - Fermi equations for neutron escape 192, 193
 - Fermi spectrum 73, 74
 - ferrophosphorus (*see also* concrete) 268, 269, 276
 - Fick's law 160
 - fission, energy release 147, 331
 - fission, gamma rays from (*see* gamma radiation)
 - fission products, biological effect 2
 - gamma rays (*see* gamma radiation)
 - neutrons from (*see* neutrons)
 - fission products, recoil of 309
 - total energy from 64, 65
 - fission yield 61–63
 - flask, shielding, permissible surface dose 306–307
 - fluorescent radiation 22, 28
 - fluorescent yield 22, 29
 - fluorine, (α, n) reaction 150
 - fluorine, removal cross-section 180
 - flux, angular 159
 - definition 20
 - from various types of source 218–228
 - most probable 106
 - of gammas, use in heating calculations 239
 - of neutrons 105, 158, 159, 172
 - of neutrons, permissible 10
 - of photons, conversion to dose-rate 4, 9, 302
 - uncollided 20
 - free-atom cross-section 117
 - fuel elements, shielding for 305, 306
 - fuel oil, as neutron shield 292
- G
- G value, for gas evolution 293
 - (γ, n) reaction 90, 110, 150
 - gamma emitters, transport 306, 307, 324–327
 - gamma-radiation, absorption by common materials (values) 37
 - air-scattering 56
 - albedo 56, 60, 209, 243
 - attenuation (basic principles) 32–56
 - attenuation, broad beams 40, 302
 - attenuation, by concrete 37, 263, 264, 265, 266, 303
 - build-up factors (*see* build-up)
 - characteristic 78
 - definition 17
 - detectors, sensitivity of 95
 - discolouration due to 321, 322
 - escape from reactors 235–245
 - following beta-decay 143
 - from betatrons 87, 89
 - from BSR, attenuation in water 320
 - from capture in thermal shield 245–250
 - from fission 61, 62, 68, 69, 70, 258
 - from fission products 62, 64, 66–70, 237, 314
 - from neutron capture (radiative capture) 99, 102, 111, 112, 130, 133, 135, 136, 236, 285
 - in water shield 285–289
 - suppression 288
 - from linear electron accelerators 87
 - from reactors 61–71, 235–245

gamma-radiation, from resonance capture 236
 from stopping of electrons (*see* bremsstrahlung)
 from synchrotrons 87
 heating by 235–245, 250
 high-energy, neutron production by 154
 intensity of high-energy radiation 90–95
 intensity, useful rules 332
 ionization from 94
 meson production by 32
 photographic effect 326
 scattering (*see* scattering)
 shielding required 302, 303
 wavelength 332
 gamma width (*see* width)
 gas evolution from shields 232, 290–293
 gas evolution, G-value 293
 Gauss probability integral 106
 genetic effects of radiation 8
 geometrical formulae 218–229
 glass, for shielding windows 321–324
 gold-198, dose through lead shield 304
 “good geometry” 32
 gram-rad 5
 gram-röntgen 15
 ground state 108
 graphite (*see also* carbon)
 borated 233, 257
 graphite-moderated reactors 188, 244, 254

H

Half-life, biological 13
 radioactive 13
 half-value thickness 41, 74
 Hanford reactor shield performance 294
 heat exchanger layout 309
 heat flux, conversion factors 244, 331
 heat from fission product decay 65
 heat release in shields 231, 235–251
 concrete 278
 heavy water, neutron diffusion properties 168
 neutron slowing-down properties 175
 removal cross-section 180
 helical ducts, neutron attenuation 205
 helium, magic nucleus 110
 helium-3, (*n*, *p*) cross-section 118
 heterogeneous, fast neutron shields 182, 183
 media, build-up factors (*see* build-up)
 reactors, gamma sources 238
 high-energy tail 107
 holes through shields 197–210
 stepped 207
 homogeneous reactor (*see* reactor)
 Hurst-Ritchie fast neutron dosimeter 179
 HVT 41, 74

hydration, cement, heat of 261
 hydrides 292, 293
 hydrocarbons 292
 removal cross-sections 180
 hydrogen, capture gamma radiation 247
 content of concrete shields 259, 261, 263
 evolution, G-value 293
 gamma absorption coefficients 34–39
 neutron cross-section of 117
 pair production 36
 photoelectric absorption in 36
 scattering of neutrons by 119, 170, 174, 176
 hydrogen-metal bulk shields 284–296
 hydrogenous shields (*see also* water, organic materials, paraffin, polythene)
 as fast neutron attenuators 106, 179, 181–191, 284–296, 313, 320

I

Impurities, activation of 297, 312
 incoherent scattering 27, 30
 independent particle model 111
 indium resonance neutrons in water shield 191
 inelastic scattering (*see* scattering)
 neutrons, tables 123–125
 ingestion of radioactive material 2, 12
 inner bremsstrahlung 79, 81, 327
 “instrument tolerance” 231
 integral absorbed dose 5
 integrals occurring in kinetic theory 105
 intensity of high-energy gamma radiation 90–95
 intermediate neutrons 100
 internal conversion 143
 inverse square attenuation 41, 212, 302, 306
 iodine 13
 gamma absorption 38, 39
 iodine-131, dose through lead shield 304
 ion pair, formation energy in air 332
 ionization, chamber, air-walled 91
 density, and biological damage 5, 6, 13
 density, and radiation damage 290
 electron energy-loss by 82
 from gammas, depth distribution 94
 measure of biological damage 3
 iridium-192, dose through lead shield 304
 iron, aggregates 266
 boron-loaded 252–255
 capture gamma 133
 coherent scattering by 30
 concrete 266
 fast neutron attenuation 180, 181
 gamma absorption coefficients 34, 35, 37–40
 gamma build-up factors 45–55, 60

iron, in thermal shields 250, 253
 inelastic scattering by 121, 122
 neutron diffusion properties 168
 nuclear heating in 250
 pyrites 266
 isomeric transition 145

J

Joule 245, 331

K

K electron capture 81, 144
 k factors, table of 305
 kinetic theory, integrals 105
 Klein-Nishina equation 25, 27, 43

L

Laboratory system (*see* co-ordinate)
 Laplacian operator 160
 lanthanum radiation 69, 306
 lead, attenuation of gamma radiation 27,
 31–35, 37–39, 303, 304
 capture gamma radiation 133
 coherent scattering by 30
 gamma build-up factors 45–55, 60
 inelastic scattering by 121
 magic nucleus 110
 neutron diffusion properties 168
 nomogram for gamma attenuation 303
 removal cross-section 180
 shot, use in shields 290
 use in thermal shields 252, 256, 257
 use in water shields 290
 length (*see under* diffusion length, etc.)
 length, conversion factors 332
 lethal dose, median 8
 lethargy 171
 leukaemia 8
 lifetime for nuclear process 112
 lightweight shields 284–290
 limonite 266, 269
 line sources, current and flux from 225
 linear absorption coefficient 20
 for air (values) 29
 linear electron accelerators, gamma pro-
 duction 76, 80, 87
 lipid solvents 13
 liquid windows 321
 lithium, activation of, bremsstrahlung 311
 lithium hydrides 292, 293
 lithium, (n, α) reaction 118, 247
 lithium, removal cross-section 180
 lithium, suppressing capture radiation 289
 use in neutron shielding 102, 138, 289

logarithmic decrement 171
 Los Alamos reactors 237
 lucite 324
 lumped source approximation 243

M

Macroscopic cross-section 20, 104
 magic nucleus (*see* nucleus)
 magnesium oxychloride cement (*see* cement
 and concrete)
 magnetite aggregate 266, 276
 manganese, beta decay scheme 143
 in steel, activation of 297
 masonite, as a neutron shield 292, 294
 mass absorption coefficient, γ -rays, values 29
 for air 34–35
 mass, conversion factors 332
 mass-energy absorption coefficient, γ -rays,
 values 38–39
 mass, energy equivalence 331
 maximum permissible flux, neutrons 10
 photons 9
 maximum permissible level (MPL) of
 radiation 7–15, 230, 306, 307
 Maxwellian energy distribution 100, 105,
 106, 114, 131
 mean free path, capture 161
 mean free path, definition 20
 transport 161
 Mega-electron-volt, definition 17
 meson production 32
 metal-water shields 182, 284–292
 methyl methacrylate, borated 294–5
 mercuric bromide, for windows 309
 MeV 17, 245, 331
 microscopic cross-section 20, 104
 millicurie 304
 moderation 101
 molybdenum, gamma absorption coefficients
 34, 35, 38, 39
 moment method, gamma rays 43
 neutrons 189–191
 momentum, angular 119, 142
 Monte Carlo method 59, 60
 MTR reactor 266, 270, 271

N

N unit 15
 n unit 15
 (n, α) reaction 101, 118, 139, 300, 304
 (n, γ) reaction 101, 130–136
 ($n, 2n$) reaction 101, 123, 138, 141
 (n, p) reaction 118, 139, 300, 304
 natural radioactivity, dose-rate from 8
 Nautilus 312
 neptunium, energy from decay of 65

- neutrino 73, 142, 144
 neutrons, albedo 192–197, 296, 318
 angular flux and current 159
 attenuation, in Portland concrete 264
 in thick shields 158
 in water 284
 beam traps 315
 binding energies 107, 109, 110, 112
 biological effect 6
 build-up (*see also* build-up) 186–189
 cadmium 100
 cataract, due to 7
 contribution to dose in water shield 285
 current 158
 delayed, from fission 63, 148
 delayed, from fission, shielding 314
 density 158, 159
 diffraction 101, 105, 118
 diffusion (*see* diffusion)
 discolouration due to 321, 322
 ducts, effect on attenuation 197–210
 epicadmium 100
 epithermal 100, 252
 escape from surface 192
 fast 100–103
 activation by 296, 299–302
 attenuation 102, 175–188, 284
 attenuation by water 106, 190, 191, 284–292, 313, 320
 capture 132
 heterogeneous shields 182, 183, 190, 191
 threshold reactions 301
 fission 100, 146, 147
 attenuation by water 178–179, 190, 191
 energy distribution 146
 (n, α) and (n, p) cross-sections 300, 304
 number (ν) 147
 removal cross-sections 180
 uncollided, in water-shield 286
 flux 105, 158, 172
 from (α, n) reactions 149, 150
 gamma rays after capture (*see* gamma radiation)
 heating by 235–250
 intermediate 100
 mass 103
 N^{17} (1 MeV), attenuation by water 313
 permissible flux 10
 photoneutrons 32, 90, 151–155, 320
 pile 100
 pile (from BSR), attenuation in water 320
 potential energy in nucleus 108
 resonance 100, 101
 in water shield 191
 scattering (*see* scattering)
 slow 100, 101
 activation by 296–297, 299, 310, 336
 neutrons, slowing-down 169–175, 186–189
 sources, transport and shielding 326–328
 spin 104
 streaming down ducts 197–210
 streaming through metal 209
 temperature (*see* temperature, nuclear)
 thermal 100, 102, 103, 105
 cross-sections 333–335
 ultra-fast 100, 101
 velocity 332
 wavelength 101, 104, 120, 332
 windows, shielding 324
 nickel, removal cross-section 180
 nitrogen, gamma absorption coefficients 38, 39
 N^{16} , N^{17} activities 140
 N^{17} neutrons, attenuation 313
 (n, p) cross-section 118
 NRX reactor 264, 307
 nuclear forces 103, 107
 nuclear lifetimes 112
 nuclear radius 103, 107
 nuclear reaction, (α, n) 149, 150
 fast neutron-induced 301
 (γ, n) 110, 150
 isomeric transition 145
 (n, α) 101, 118, 137, 139, 300, 301
 (n, γ) 101, 130–136, 236–238, 245, 246
 (n, p) 118, 137, 139, 300, 301
 ($n, 2n$) 101, 110, 141
 neutron producing 100, 110, 145, 149, 150
 threshold (also (n, α), (n, p)) 139, 301
 nuclear resonance 131
 nuclear resonance scattering 30, 32
 nuclear temperature 121, 126, 141, 187
 nuclear Thomson scattering 30, 31
 nucleon 104, 111
 nucleus, compound 104, 111–116, 130, 141
 de-excitation of 112, 120
 energy levels 111, 112, 121
 excited state 111, 116
 independent particle model 111
 magic 103, 110–112, 133
 optical model 111, 115
 stability 142
 statistical model 121
 strong interaction model 111–120
 transparency to neutrons 115

 O
 Oil, as neutron shield 292
 orbital electron capture 81, 144
 organic liquids, viscosity and radiation damage 293, 294
 organic materials, as shields 231, 292, 293
 used in Na-cooled reactors 293

- outer bremsstrahlung 79, 81, 327
- oxygen, gamma absorption coefficients 34, 35, 38, 39
 - magic nucleus 110
 - (*n, p*) cross-sections 140
 - removal cross-section 180
- P
- Pair production 18, 22–24, 36, 40
- paraffin, as neutron shield 292–293
 - removal cross-section 180
 - use in beam traps 315
- partial shielding 230, 232, 234, 295, 296
- partial width 112, 113
- pebble shields 210
- penetrability factor 113, 138
- permissible level of radiation (*see* radiation)
 - lifetime dose 8
- perspex 3, 292, 324
- pessimistic calculations 236
- phosphorus-32, shielding for 327
- photoelectric effect 18, 21, 22, 36, 40
 - fluorescent radiation following 22
- photographic plates, sensitivity to radiation 325–326
- photon (*see also* gamma radiation and scattering) 17
 - momentum and energy 24
- photoneutron (*see* neutron)
- pile (*see* reactor)
- pile neutrons 100
- Planck's constant 17, 104, 112
- plane sources, current and flux from 218
- plastics, as shielding windows 324
- plastics, borated 294, 295
- plastics, effect of radiation on 294, 295
- plexiglass 3, 324
- PLUTO reactor 275
- plutonium, fission neutrons from 147
 - ingestion hazard 13, 14
 - use in neutron sources 149
 - transport 326
- polonium, use in neutron sources 149, 326, 328
- polynomial method 43
- polyphenyls, radiation resistance 293
- polystyrene, radiation resistance 293
- polythene, as neutron shield 292, 294
 - attenuation of 1 MeV neutrons 313
 - borated 294
 - neutron backscattering 318
 - radiation effects 295
- Portland concrete (*see* concrete and cement)
- positron, annihilation 23, 327
 - biological efficiency 6
 - emission in radioactive decay 144
 - positron, from pair production 22
- potassium, activation of 311
- potassium-40, in body 8
- potential barrier 113, 120
- potential energy, neutron in nucleus 108
- potential scattering (*see* scattering)
- potential well 108, 111
- prepacking method for concrete 270
- pressure, conversion factors 332
- pressurized water reactor (PWR) (*see* reactor)
- primary shield 309
- propulsion reactor (*see* reactor)
- proton, biological effect 5, 6
 - following neutron absorption 137
 - mass 103
 - range in air 11
- Q
- “Q” of reaction 137
- quantum (*see also* gamma radiation and photon) 17
- R
- rad 5
- radiation, biological effects 1–15
 - damage, in concrete 276, 277
 - damage, in steel 292
 - decomposition, gas evolution 232
 - effect on viscosity of organics 293–294
 - effect on water shield 290, 291
- radiation energy-loss 82
- radiation, fluorescent 22
- radiation-induced discolouration 321–324
- radiation length 82
- radiation, maximum permissible level (MPL) 7–15, 230, 306, 307
- radiative capture (*see* gamma radiation)
- radioactivity, decay constant 67, 310
 - following neutron bombardment 141
 - 296–314
- radium, disintegrations per gram 304, 305
 - dose through lead shield 304
 - hazard from 1, 13, 326
 - naturally occurring 8
 - use in neutron sources 101, 149, 326
 - transport 326
- radius, electron 21
 - nuclear 107
- range-energy relation, deuteron 11
 - electron 72
 - proton 11
- Rayleigh scattering (*see* scattering)
- reactor, BEPO 243, 253, 266
 - Brookhaven 266
 - Bulk Shielding (BSR) 237

- reactor, Calder Hall 197, 309
 - circulating fuel 147, 313
 - coolants, activation of 309-314
 - DIDO 241, 256, 266, 276
 - Dounreay 232, 233, 257
 - extraction of gamma-emitters from 307, 308
 - graphite-moderated 188, 244, 254
 - Hanford 294
 - heterogeneous, gamma sources 238
 - homogeneous 71, 147, 313
 - Idaho submarine prototype 312
 - leakage of gamma radiation 235-245
 - Los Alamos fast 237
 - Los Alamos water-boiler 237
 - MTR 266, 270, 271
 - Nautilus 312
 - NRX 264, 307
 - Oak Ridge 276, 319
 - PLUTO 275
 - pressurized water 232, 244, 257, 309
 - propulsion 181, 230, 284, 293, 295, 296
 - research 230, 241
 - shielding, general 230-318
 - ship 230, 295, 296
 - sodium-cooled 209, 231, 293, 309
 - Submarine Intermediate 291
 - swimming-pool 237, 319
 - water-cooled 140, 244
 - Windscale 230, 234, 235, 280, 319
 - recoil of atoms 309
 - biological effect 6
 - relative biological efficiency (rbe) 5, 6
 - relaxation length 20
 - rem 6
 - removal cross-sections 176-184, 328
 - remote handling devices 231
 - rep 15
 - research reactor, shielding 230
 - resonance capture 236, 238
 - resonance, in total cross-section 99, 131
 - resonance neutrons (*see* neutrons)
 - resonance scattering (*see* scattering)
 - rest energy, electron 21
 - röntgen 1, 4, 332
 - equivalent photon flux 4
 - limitations 4, 90-95
 - rutherford 304, 332
- S
- Scattering of electrons 74, 76
 - scattering of gamma radiation 24-32, 40-60
 - backscattering 56, 60, 209, 243
 - by bound electrons 30
 - by thick plane slab 58, 60
 - by thin scatterer 58
 - scattering of gamma radiation, coherent 27, 30
 - Compton 18, 19, 24-30, 36, 42, 56
 - degradation of energy 42
 - Delbruck 32
 - double Compton 32
 - from sea 296
 - incoherent 30
 - multiple, 40-60
 - nuclear resonance 30, 32
 - nuclear Thomson 30, 31
 - Rayleigh 30, 31, 33, 35, 56
 - Thomson 26
 - scattering of neutrons, albedo 192-197, 296 318
 - angle of 160, 171
 - backscattering by beam traps 318
 - by hydrogen 119, 170
 - capture elastic 116, 120
 - diffraction 101, 105, 120, 169
 - elastic 101, 169
 - elastic, angular dependence 129
 - elastic, heating by 247
 - inelastic 101, 102, 116, 119, 121, 123-125, 127, 138, 169, 176, 187, 252, 289, 314
 - isotropy of 119
 - potential 115, 117, 119, 120
 - resonance 118
 - shadow 101, 120
 - shape elastic 120
 - scattering of positrons 75
 - secant integral functions 212-214, 216
 - secondary shield 309
 - shadow scattering 101, 120
 - shields and shielding (*indexed under other headings*)
 - ship reactor 181, 230, 284, 295, 296
 - shoe-fitting fluoroscopes 8
 - shrinkage of concrete 269, 271
 - slab sources, current and flux from 218
 - slowing-down density 172
 - slowing-down length 175
 - slowing-down of neutrons 169-175, 186-189
 - sodium, activation of 311
 - sodium, capture radiation from 112, 134, 136
 - sodium (Na^{24}), dose through lead shield 304
 - sodium-cooled reactors (*see* reactors)
 - organic shields 293
 - solid windows 321
 - solution of active material 309
 - solvents, lipoid, hazard 13
 - sources, current and flux from 218-227
 - sources of gammas (*see* gamma radiation)
 - sources of neutrons (*see* neutrons)
 - spallation reaction 102
 - spectrum of gammas, from fission 61
 - from reactors 237

- spectrum of neutrons, from (α, n) reactions 150
 from fission 146
 spherical geometry, neutron diffusion in 168
 spherical sources, current and flux 228
 spin 104
 statistical model of nucleus 121
 statistical weight factor 104, 114
 steel, activation of impurities 297
 steel, boron 252, 254, 255
 steel, gamma absorption 37, 40
 steel, radiation damage 292
 steel, stainless, induced activities 298
 steel, use in thermal shield 252, 253, 278
 stepped holes through shields 207
 sticking probability 114
 stopping power, for electrons 91
 streaming of neutrons, down ducts 197-210
 through metal joists 209
 strong interaction nuclear model 111-115, 119, 120
 strontium (radio), hazard 13
 transport 326
 sulphur, (n, p) cross-section 139
 surface, escape of neutrons from 192
 permissible level of radiation at 230, 306, 307
 swimming-pool reactors 237, 319
 synchrotrons, gamma radiation from 87
- T
- Tank, gamma dose outside 308
 tantalum-182, dose through lead shield 304
 Taylor's approximation 50, 53, 212, 243-245, 305
 television 318
 X-ray dose from 8
 temperature distribution in shields 248
 temperature, nuclear 121, 126, 141, 187
 ten-folding lengths, gamma radiation 37
 thermal column 107
 thermal neutrons (*see* neutrons)
 source strength 103
 thermal shield 232, 251-258
 capture gamma radiation 245
 for PWR 232, 257
 necessity for 278
 temperature in 250
 Windscale reactor 235
 Thomson scattering formula 26
 threshold reactions (*see* nuclear reactions)
 thulium-170, k factor 305
 time, conversion factors 332
 tin, gamma build-up factors 49, 52, 54, 60
 titanium, total cross-section 131
 titanium hydride 292-293
- tolerance level (*see* radiation and maximum permissible level)
 instrument 231
 total cross-sections, atomic 104
 function of atomic weight 115
 transport equation 43
 transport mean free path 161
 transport of radioactive materials 306, 307, 324-327
 transport theory 158
 true absorption 28
 tube source, current and flux from 226, 227
 tungsten, gamma absorption 34, 35, 38, 39
 gamma build-up factors 49, 52
 removal cross-section 180
 two-group calculations 236, 247
- U
- Ultra-fast neutrons (*see* neutrons)
 uncertainty principle 112
 uncollided flux 20, 285, 286
 units (definitions of)
 absorbed dose 5
 atomic mass unit 331
 barn 20, 104
 curie 304, 332
 "energy unit" 15
 eV 17
 gram rad 5
 gram röntgen 15
 integral absorbed dose 5
 MeV 17
 millicurie 304
 "N" unit 15
 "n" unit 15
 photon intensity 4
 rad 5
 radiation length 82
 rem 6
 rep 14
 röntgen 4, 90-95, 332
 rutherford 304, 332
 watt 245, 331
 uranium, gamma absorption 34, 35, 38, 39
 gamma build-up 49
 gammas from neutron capture 236
 fission neutrons from 147
 fuel elements, shielding 305, 306
 removal cross-section 180
 resonance absorption 236, 238
- V
- Value, relative, of shielding materials 282-284
 velocity, distribution, thermal neutrons 105

- viscosity, organic liquids, radiation effect 293-294
- voids in neutron shields 197-210
- volume, conversion factors 332
- W
- Water, activation of 140, 311-313
- as shielding window 318
- attenuation of BSR radiations 320
- biological doses in shield 285
- borated 287, 288
- build-up factors (gamma) 45-51, 54
- in reactors 244
- capture gammas in 135, 285-290
- cement ratio 270
- energy reflection build-up factors 60
- gamma absorption 38, 39
- metal shields 182, 231, 284-292
- neutron attenuation (fission) 106, 178, 179, 190, 191, 284-296
- (1 MeV) 313
- (pile neutrons) 106, 320
- neutron diffusion 168
- neutron slowing-down 175
- permissible contamination 12
- radiation decomposition 290, 291
- use in beam traps 315
- wave, de Broglie 104
- wave nature of matter 104
- wavelength, Compton 24, 332
- gamma rays 332
- neutron 101, 104, 120, 332
- watt 245, 331
- weight of shield, effect of density 279
- width (gamma, neutron, partial, etc.) 112, 113, 115, 130
- windows, shielding 318-324
- Windscale reactor 230, 234, 235, 280, 319
- wood, as a neutron shield 292-294
- X
- X rays (*see also* gamma radiation)
- biological effects 1
- characteristic 78
- following orbital electron capture 144
- permissible flux 4, 9, 10
- Y
- Yttrium-90, shielding for 327
- Z
- Zinc salts, as liquid windows 321, 324